

Modification indices and residuals

1. Introduction

In practice, many multivariate data sets are repeated measurements over time. For these data sets, the researcher is often interested in modeling the growth that occurs over time. In this regard, linear and nonlinear latent growth curves may be used if the researcher is interested in modeling the growth parameters such as, for example, the intercept and the slope of the linear latent growth curve. Duncan et al (1999) provides a comprehensive treatment of latent growth curve analysis.

LISREL 8.7 for Windows (Jöreskog & Sörbom 2005) may be used to fit linear and nonlinear latent growth curves to repeated measurements over time. Traditional statistical methods such as Maximum Likelihood (ML), Robust Maximum Likelihood (RML), Weighted Least Squares (WLS), Diagonally Weighted Least Squares (DWLS), Generalized Least Squares (GLS) and Un-weighted Least Squares (ULS) are available for complete repeated measurements over time while the Full Information Maximum Likelihood (FIML) method is available for repeated measurements over time with missing values.

In this note, LISREL 8.7 for Windows is used to illustrate the use of modification indices and residuals in structural equation modeling. A linear Latent Growth Curve (LGC) model with an explanatory latent variable is used to demonstrate the use of modification indices. Thereafter, the use of residuals is illustrated by using a measurement model.

2. Modification indices

The data

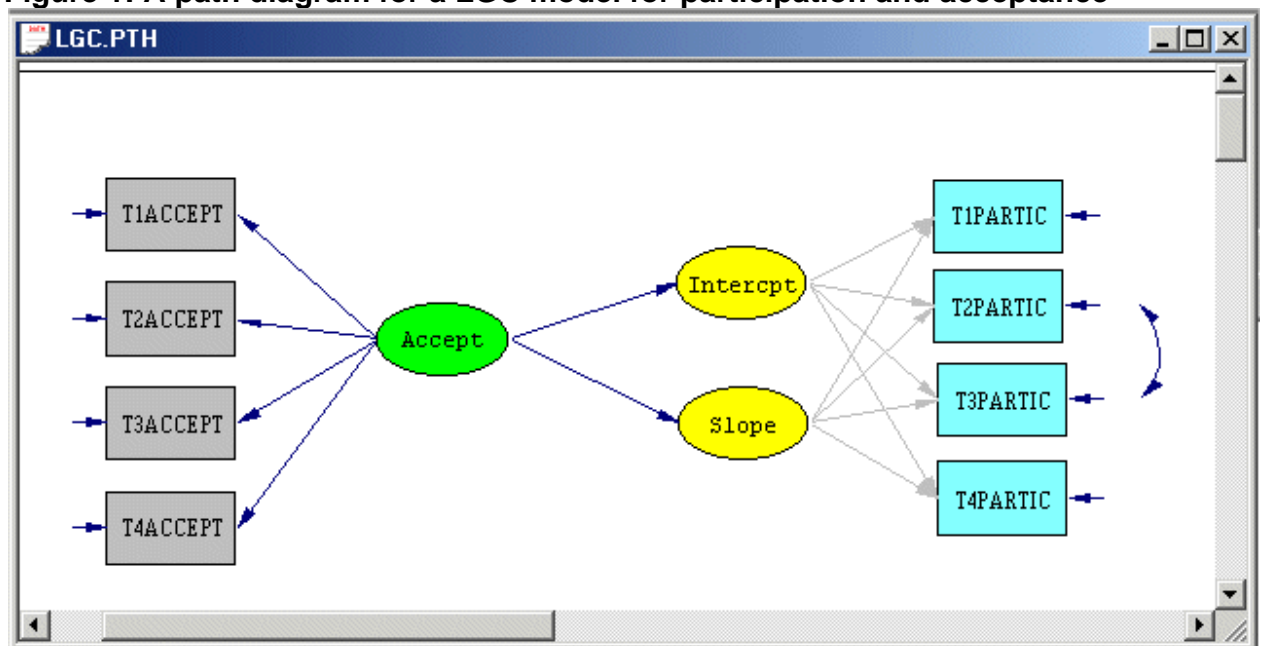
The data set to be considered emanates from a longitudinal study on the growth of the participation of cancer patients in cancer treatment programs. A sample of 374 cancer patients was monitored over a period of four quarters. At the end of each quarter, a participation score was computed for each patient. These four scores are labeled as T1PARTIC, T2PARTIC, T3PARTIC and T4PARTIC respectively. In addition, a cancer acceptance score was computed for each patient. The four corresponding scores are labeled as T1ACCEPT, T2ACCEPT, T3ACCEPT and T4ACCEPT respectively. A small portion of the data set is shown in the following PSF window.

	T1ACCEPT	T2ACCEPT	T3ACCEPT	T4ACCEPT	T1MUTBF	T1TCHCHD	T1PART
1	-0.006	-0.072	0.919	-0.046	2.692	3.696	3.
2	0.695	0.210	0.509	-0.723	2.759	2.892	3.
3	-0.245	-0.322	-1.110	-0.632	1.430	3.337	3.
4	0.001	0.739	0.557	-0.231	1.933	2.706	2.
5	0.628	0.841	-0.442	0.918	3.723	3.361	2.
6	-0.315	0.323	0.630	0.049	0.634	2.520	2.
7	-1.368	-1.009	-1.001	-1.373	0.080	3.641	1.
8	-0.618	-1.074	0.324	-1.191	1.307	3.115	2.
9	0.966	0.152	-0.091	0.410	2.861	4.093	3.
10	0.598	0.082	1.359	0.763	1.233	4.052	2.
11	-0.177	0.196	0.969	0.324	1.301	1.758	2.
12	2.087	1.372	1.163	1.728	3.899	4.367	2.
13	0.766	1.497	1.214	1.514	3.009	3.910	4.
14	1.456	1.420	0.381	-0.085	-0.631	4.358	4.
15	-0.329	0.132	-0.445	0.091	3.525	2.267	3.

The model

We consider the four quarterly participation scores and four quarterly cancer acceptance scores obtained for the sample of 374 cancer patients. A linear latent growth curve is modeled for the four quarterly participation scores. In addition, cancer acceptance level is modeled as an explanatory effect for the intercept and slope of the latent growth curve. A path diagram for the corresponding LGC model for cancer Participation and Acceptance is shown in Figure 1.

Figure 1: A path diagram for a LGC model for participation and acceptance



The Analysis

- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the file **LGC.SPL** in the **WORKSHOP** folder of LISREL 8.7 for Windows.
- Click on the **Open** push button to open the following text editor window.

```

LGC.spl
Raw Data from File LGC.PSF
Latent Variables: Intercpt Slope Accept
Relationships:
T1PARTIC = 1*Intercpt 1*Slope
T2PARTIC = 1*Intercpt 2*Slope
T3PARTIC = CONST 1*Intercpt 3*Slope
T4PARTIC = CONST 1*Intercpt 4*Slope
Let the Errors of T2PARTIC and T3PARTIC Correlate
T1ACCEPT T2ACCEPT T3ACCEPT T4ACCEPT = Accept
Slope = CONST Accept
Intercpt = CONST Accept
Let the Errors of Intercpt and Slope Correlate
LISREL Output: ND=3 SC MI
Path Diagram
End of Problem*

```

Line 1 specifies the raw data source.

Line 2 specifies labels for the latent variables of the model.

Lines 4-8 specify the linear latent growth curve model for the 4 participation scores.

Line 9 specifies the 4 acceptance scores as indicators of the latent variable cancer acceptance over time.

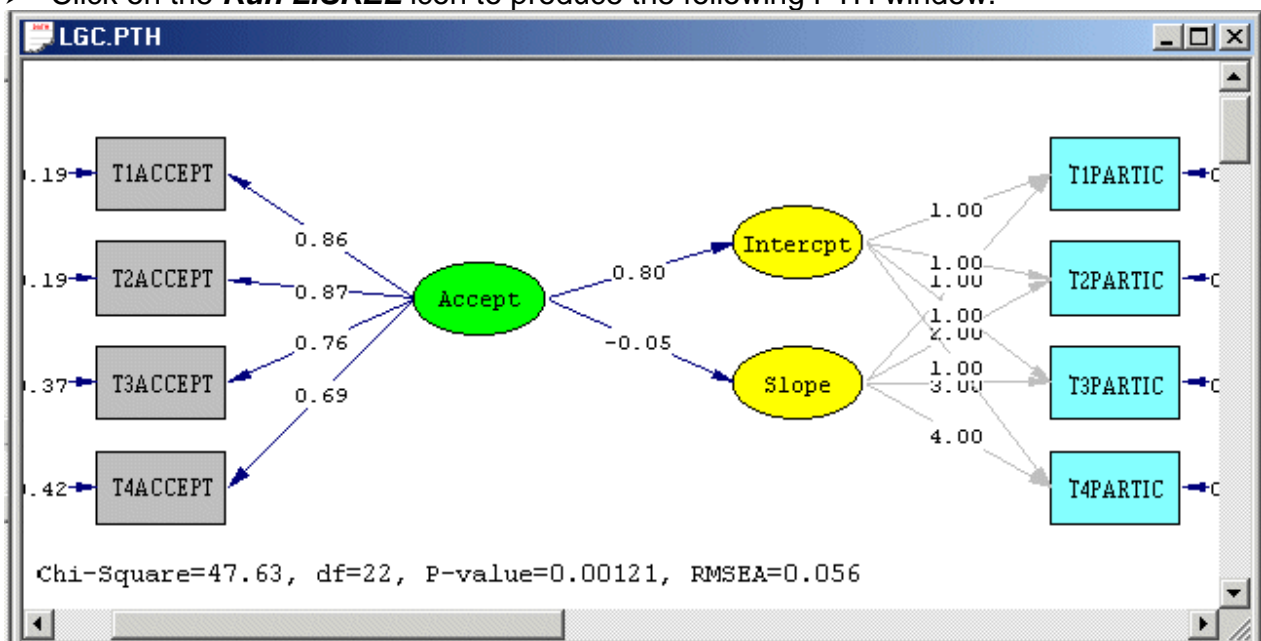
Lines 10-12 specify the structural model for the intercept and slope of the latent growth curve.

Line 13 requests the results in the terms of the LISREL model for the model in Figure 1.

Line 14 requests a path diagram (PTH) file.

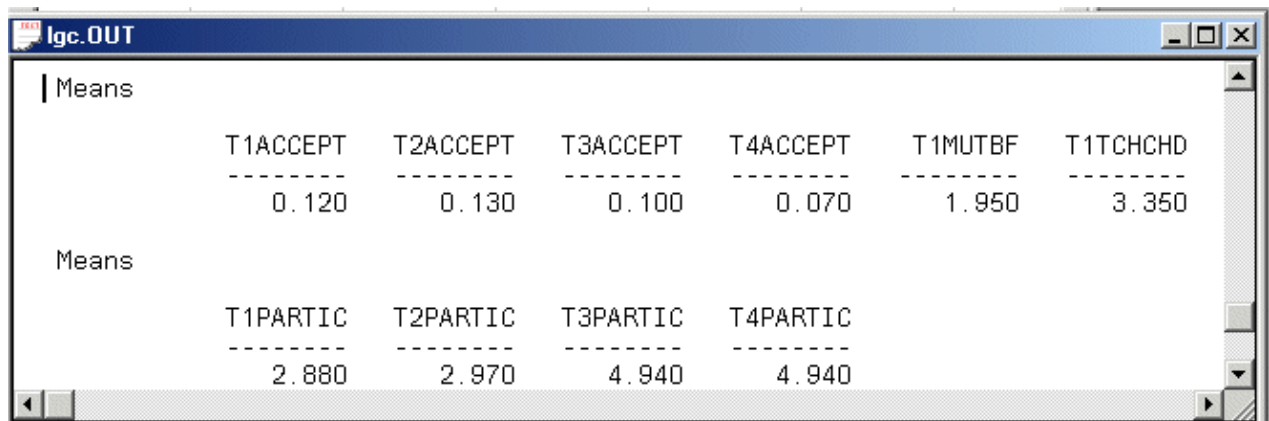
Line 15 indicates the end of the SIMPLIS commands to be processed.

- Use the **Save As** option on the **File** menu to save the SIMPLIS syntax file as **LGC.spl**.
- Click on the **Run LISREL** icon to produce the following PTH window.



Adjusting the model

- Select the **Close all** option on the **Windows** menu to close all open windows.
- Select the **Open** option on the **File** menu of the root window to load the **Open** dialog box.
- Select the **Prelis Data (*.psf)** option from the **Files of type:** dropdown list box.
- Browse to locate the file **LGC.PSF** in the **WORKSHOP** subfolder of the LISREL 8.7 for Windows installation folder.
- Click on the file **LGC.PSF** to enter it into the **File name:** string field.
- Click on the **Open** push button to open the PSF window for **LGC.PSF**.
- Select the **Output Options** option on the **Statistics** menu of the PSF window to load the **Output** dialog box.
- Click on the **OK** push button to run PRELIS25.EXE to produce the means shown in the following output window.



The screenshot shows a window titled "lgc.OUT" with a scrollable text area. The text displays means for several variables, organized into two sections. The first section lists T1ACCEPT, T2ACCEPT, T3ACCEPT, T4ACCEPT, T1MUTBF, and T1TCHCHD. The second section lists T1PARTIC, T2PARTIC, T3PARTIC, and T4PARTIC. Each variable name is followed by a horizontal line and its corresponding mean value.

Variable	Mean
T1ACCEPT	0.120
T2ACCEPT	0.130
T3ACCEPT	0.100
T4ACCEPT	0.070
T1MUTBF	1.950
T1TCHCHD	3.350
T1PARTIC	2.880
T2PARTIC	2.970
T3PARTIC	4.940
T4PARTIC	4.940

The results above indicate that the mean participation scores for quarters 3 and 4 are identical up to 3 decimal places. Consequently, it appears that no significant growth occurred from quarter 3 to quarter 4. This implies that it may be sensible to incorporate this result in the linear latent growth curve model for the 4 participation scores. This result may be incorporated as follows.

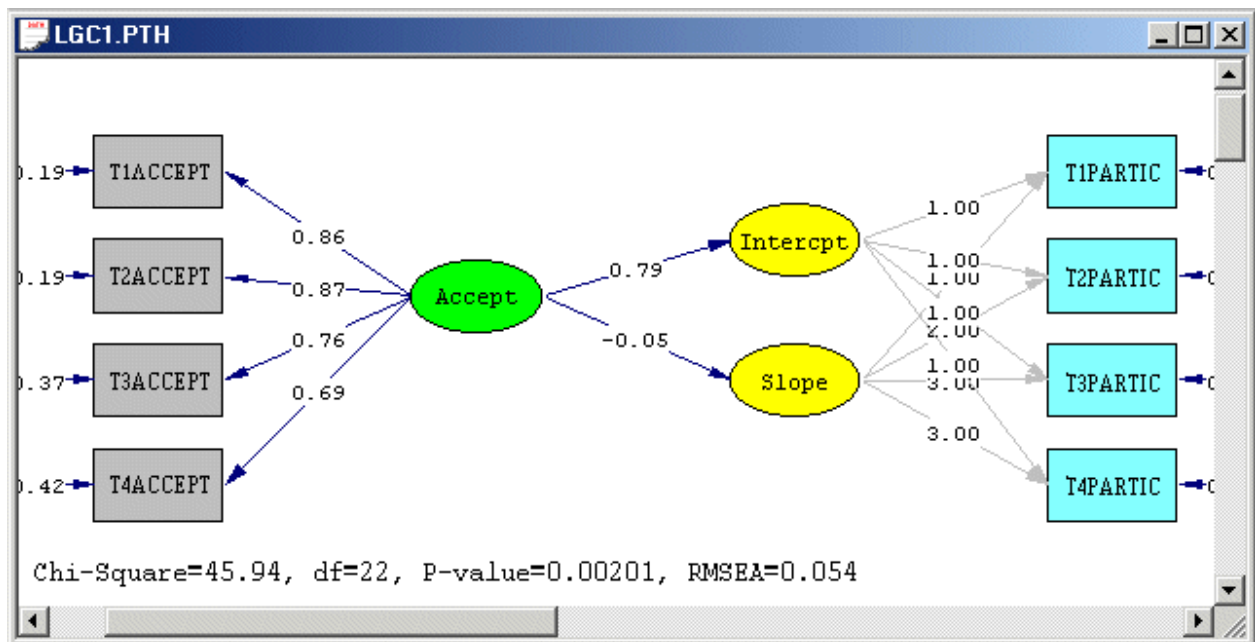
- Select the **Close all** option on the **Windows** menu to close all open windows.
- Select the **Open** option on the **File** menu of the root window to load the **Open** dialog box.
- Browse to locate the file **LGC.SPL** in the **WORKSHOP** subfolder of the LISREL 8.7 for Windows installation folder.
- Click on the file **LGC.SPL** to enter it into the **File name:** string field.
- Click on the **Open** push button to open the text editor window for **LGC.SPL**.
- Change the coefficient of the latent variable **Slope** from "4" to "3" for the fourth participation score.
- Select the **Save As** option on the **File** menu of the text editor window to load the **Save As** dialog box.
- Enter the name **LGC1.SPL** in the **File name:** string field.
- Click on the **Save** push button to produce the following text editor window.

```

LGC1.spl
Raw Data from File LGC.PSF
Latent Variables: Intercpt Slope Accept
Relationships:
T1PARTIC = 1*Intercpt 1*Slope
T2PARTIC = 1*Intercpt 2*Slope
T3PARTIC = CONST 1*Intercpt 3*Slope
T4PARTIC = CONST 1*Intercpt 3*Slope
Let the Errors of T2PARTIC and T3PARTIC Correlate
T1ACCEPT T2ACCEPT T3ACCEPT T4ACCEPT = Accept
Slope = CONST Accept
Intercpt = CONST Accept
Let the Errors of Intercpt and Slope Correlate
LISREL Output: ND=3 SC MI
Path Diagram
End of Problem*

```

➤ Click on the **Run LISREL** icon to produce the following PTH window.



- Close the **PTH** window for **LGC1.PTH**.
- The modification indices for the latent growth curve model are shown in the following two text editor windows.

Modification Indices for LAMBDA-Y		
	Intercept	Slope
T1PARTIC	1.611	0.816
T2PARTIC	1.611	0.816
T3PARTIC	0.279	0.002
T4PARTIC	1.334	0.077

Modification Indices for THETA-EPS				
	T1PARTIC	T2PARTIC	T3PARTIC	T4PARTIC
T1PARTIC	- -			
T2PARTIC	- -	- -		
T3PARTIC	0.059	- -	- -	
T4PARTIC	0.031	0.002	0.002	- -

Modification Indices for THETA-DELTA-EPS				
	T1PARTIC	T2PARTIC	T3PARTIC	T4PARTIC
T1ACCEPT	6.911	2.017	0.996	2.837
T2ACCEPT	1.021	1.707	2.550	0.496
T3ACCEPT	0.860	1.961	7.036	1.455
T4ACCEPT	4.064	0.092	1.149	11.767

Modification Indices for THETA-DELTA				
	T1ACCEPT	T2ACCEPT	T3ACCEPT	T4ACCEPT
T1ACCEPT	- -			
T2ACCEPT	0.783	- -		
T3ACCEPT	1.343	0.143	- -	
T4ACCEPT	0.752	1.714	0.079	- -

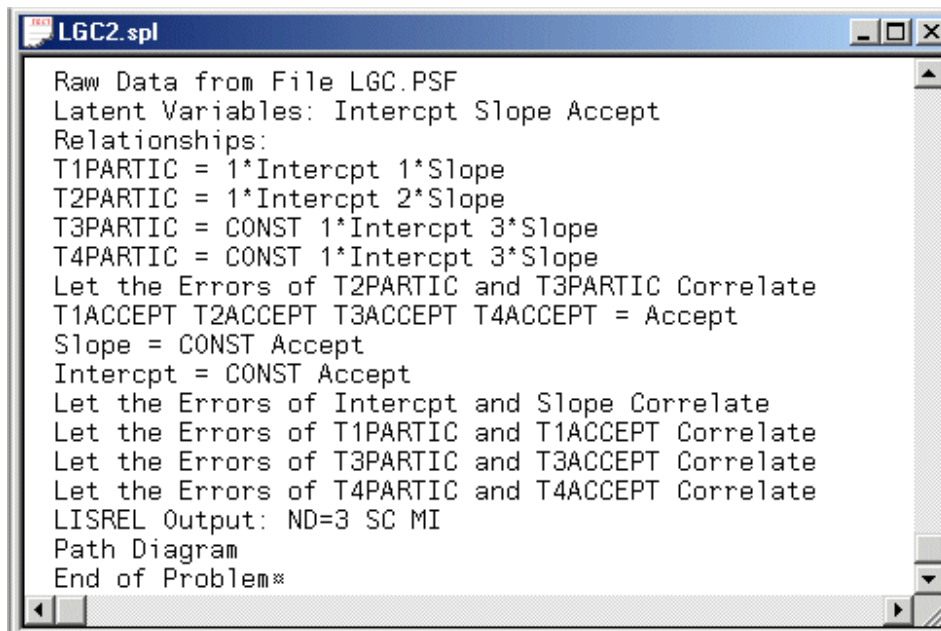
Modification Indices for TAU-X				
	T1ACCEPT	T2ACCEPT	T3ACCEPT	T4ACCEPT
	0.376	1.261	0.019	0.271

Modification Indices for KAPPA	
Accept	
	6.491

Maximum Modification Index is 11.77 for Element (4,4) of THETA DELTA-EPSILON

We note that the largest modification indices are those for the measurement error covariances between the participation and acceptance indicators at quarters 1, 2 and 4. These correlated measurement errors may be incorporated in the model in Figure 1 as follows.

- Select the **Close all** option on the **Windows** menu to close all open windows.
- Select the **Open** option on the **File** menu of the root window to load the **Open** dialog box.
- Browse to locate the file **LGC1.SPL** in the **WORKSHOP** subfolder of the LISREL 8.7 for Windows installation folder.
- Click on the file **LGC1.SPL** to enter it into the **File name:** string field.
- Click on the **Open** push button to open the text editor window for **LGC1.SPL**.
- Insert the following lines
Let the Errors of T1PARTIC and T1ACCEPT Correlate
Let the Errors of T3PARTIC and T3ACCEPT Correlate
Let the Errors of T4PARTIC and T4ACCEPT Correlate
just before the **LISREL Output** command.
- Select the **Save As** option on the **File** menu of the text editor window to load the **Save As** dialog box.
- Enter the name **LGC2.SPL** in the **File name:** string field.
- Click on the **Save** push button to produce the following text editor window.

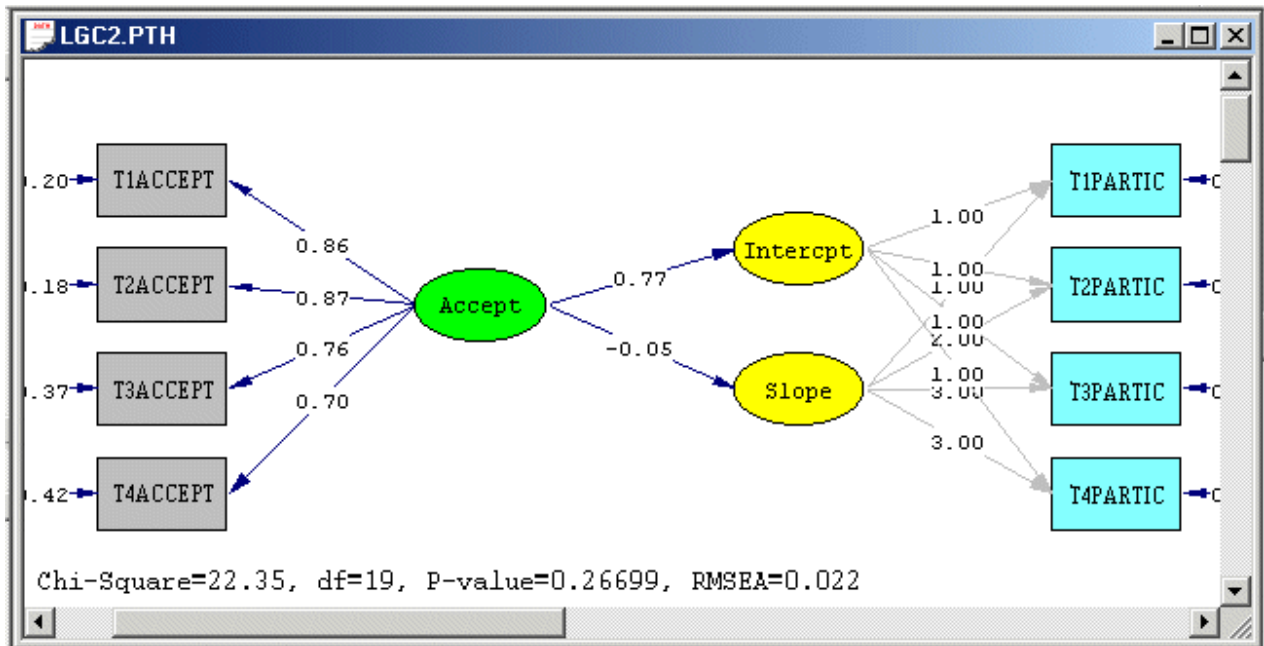


```

Raw Data from File LGC.PSF
Latent Variables: Intercpt Slope Accept
Relationships:
T1PARTIC = 1*Intercpt 1*Slope
T2PARTIC = 1*Intercpt 2*Slope
T3PARTIC = CONST 1*Intercpt 3*Slope
T4PARTIC = CONST 1*Intercpt 3*Slope
Let the Errors of T2PARTIC and T3PARTIC Correlate
T1ACCEPT T2ACCEPT T3ACCEPT T4ACCEPT = Accept
Slope = CONST Accept
Intercpt = CONST Accept
Let the Errors of Intercpt and Slope Correlate
Let the Errors of T1PARTIC and T1ACCEPT Correlate
Let the Errors of T3PARTIC and T3ACCEPT Correlate
Let the Errors of T4PARTIC and T4ACCEPT Correlate
LISREL Output: ND=3 SC MI
Path Diagram
End of Problem*

```

- Click on the **Run LISREL** icon to produce the following PTH window.



The modification indices for the analysis are listed in the following two text editor windows.

LGC2.OUT

Modification Indices for LAMBDA-Y				
	Intercept	Slope		
T1PARTIC	2.702	2.118		
T2PARTIC	2.702	2.118		
T3PARTIC	0.013	0.280		
T4PARTIC	0.859	0.034		

Modification Indices for THETA-EPS				
	T1PARTIC	T2PARTIC	T3PARTIC	T4PARTIC
T1PARTIC	--	--	--	--
T2PARTIC	--	--	--	--
T3PARTIC	0.583	--	--	--
T4PARTIC	0.564	0.066	0.066	--

Modification Indices for THETA-DELTA-EPS				
	T1PARTIC	T2PARTIC	T3PARTIC	T4PARTIC
T1ACCEPT	--	0.080	4.255	0.888
T2ACCEPT	0.006	0.205	1.907	0.199
T3ACCEPT	0.076	1.545	--	0.025
T4ACCEPT	1.632	0.001	0.027	--

Modification Indices for THETA-DELTA

	T1ACCEPT	T2ACCEPT	T3ACCEPT	T4ACCEPT
T1ACCEPT	- -			
T2ACCEPT	0.252	- -		
T3ACCEPT	0.933	0.212	- -	
T4ACCEPT	0.058	0.966	0.098	- -

Modification Indices for TAU-X

	T1ACCEPT	T2ACCEPT	T3ACCEPT	T4ACCEPT
	0.366	1.284	0.026	0.323

Modification Indices for KAPPA

Accept
6.476

Maximum Modification Index is 6.48 for Element (1,1) of KAPPA

From the results above, it is evident that none of the modification indices suggests further refining of the model.

3. Residuals

The data

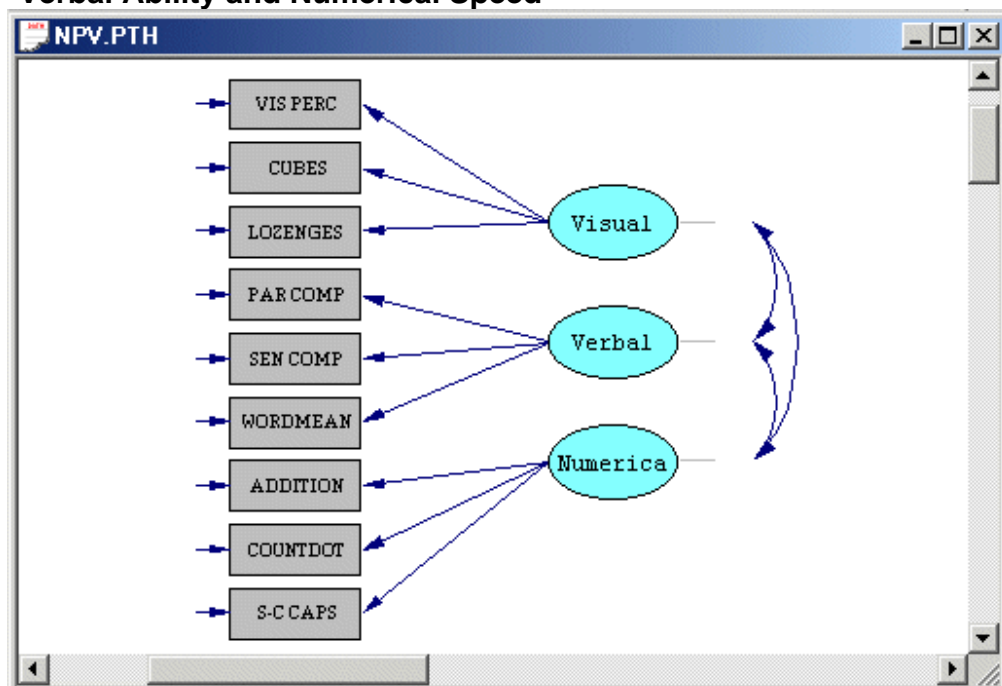
The data are the scores on nine psychological tests of 145 seventh- and eighth-grade students at the Grant-White school in Chicago. The raw data are listed in the PRELIS System File (PSF) *NPV.PSF* in the *WORKSHOP* subfolder of the LISREL 8.7 for Windows installation folder. The first couple of lines of this file are shown in the following PSF window.

	VIS PERC	CUBES	LOZENGES	PAR COMP	SEN COMP	WORDMEAN	ADDITK
1	23.000	19.000	4.000	10.000	17.000	10.000	69.
2	33.000	22.000	17.000	8.000	17.000	10.000	65.
3	34.000	24.000	22.000	11.000	19.000	19.000	50.
4	29.000	23.000	9.000	9.000	19.000	11.000	114.
5	16.000	25.000	10.000	8.000	25.000	24.000	112.
6	30.000	25.000	20.000	10.000	23.000	18.000	94.
7	36.000	33.000	36.000	17.000	25.000	41.000	129.
8	28.000	25.000	9.000	10.000	18.000	11.000	96.
9	30.000	25.000	11.000	11.000	21.000	8.000	103.
10	20.000	25.000	6.000	9.000	21.000	16.000	89.
11	27.000	26.000	6.000	10.000	16.000	13.000	88.
12	32.000	21.000	8.000	1.000	7.000	11.000	103.
13	38.000	31.000	12.000	10.000	11.000	14.000	83.
14	17.000	21.000	6.000	5.000	10.000	10.000	99.
15	34.000	28.000	24.000	14.000	22.000	26.000	49.

The measurement model

We consider nine psychological tests that were administered to 145 seventh- and eighth-grade students. These nine psychological tests are theoretically constructed to measure the visual perception, verbal ability and numerical speed of seventh- and eighth-grade children. A path diagram for the corresponding measurement model for visual perception, verbal ability and numerical speed is shown in Figure 2.

Figure 2: A path diagram for a measurement model of Visual Perception, Verbal Ability and Numerical Speed



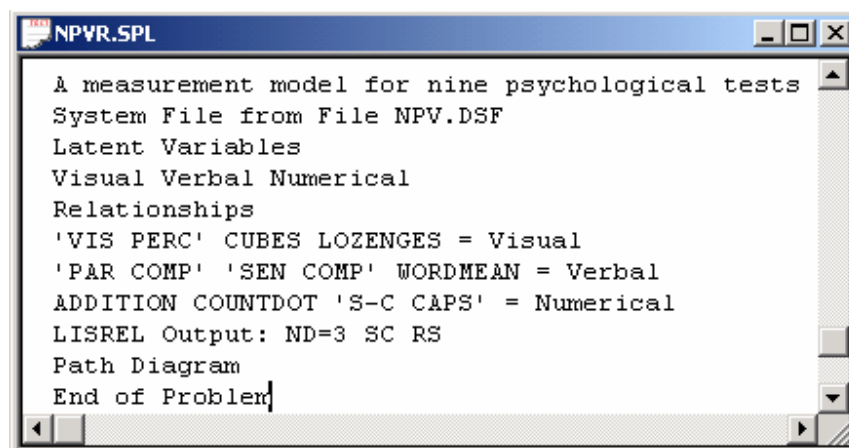
Fitting the measurement model to the data

Generating the Data System File (DSF)

- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the **WORKSHOP** folder of the LISREL 8.7 for Windows installation folder.
- Select the **PRELIS Data (*.psf)** option from the **Files of type** dropdown list box.
- Select the file **NPV.PSF**.
- Click on the **Open** push button to open the PSF window for **NPV.PSF**.
- Select the **Output Options** option on the **Statistics** menu to load the **Output** dialog box.
- Check the **Save to File** checkbox in the **Asymptotic Covariance Matrix** section.
- Enter the name **NPV.ACM** in the string field in the **Asymptotic Covariance Matrix** section.
- Click on the **OK** push button to run PRELIS27.EXE to generate the text editor window containing the output file **NPV.OUT**.

The SIMPLIS syntax file

- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the file **NPVR.SPL** in the **WORKSHOP** folder of LISREL 8.7 for Windows.
- Click on the **Open** push button to open the following text editor window.



```
NPVR.SPL
A measurement model for nine psychological tests
System File from File NPV.DSF
Latent Variables
Visual Verbal Numerical
Relationships
'VIS PERC' CUBES LOZENGES = Visual
'PAR COMP' 'SEN COMP' WORDMEAN = Verbal
ADDITION COUNTDOT 'S-C CAPS' = Numerical
LISREL Output: ND=3 SC RS
Path Diagram
End of Problem
```

Line 1 provides a title for the analysis.

Line 2 specifies the data source.

Lines 3 to 4 specify labels for the latent variables.

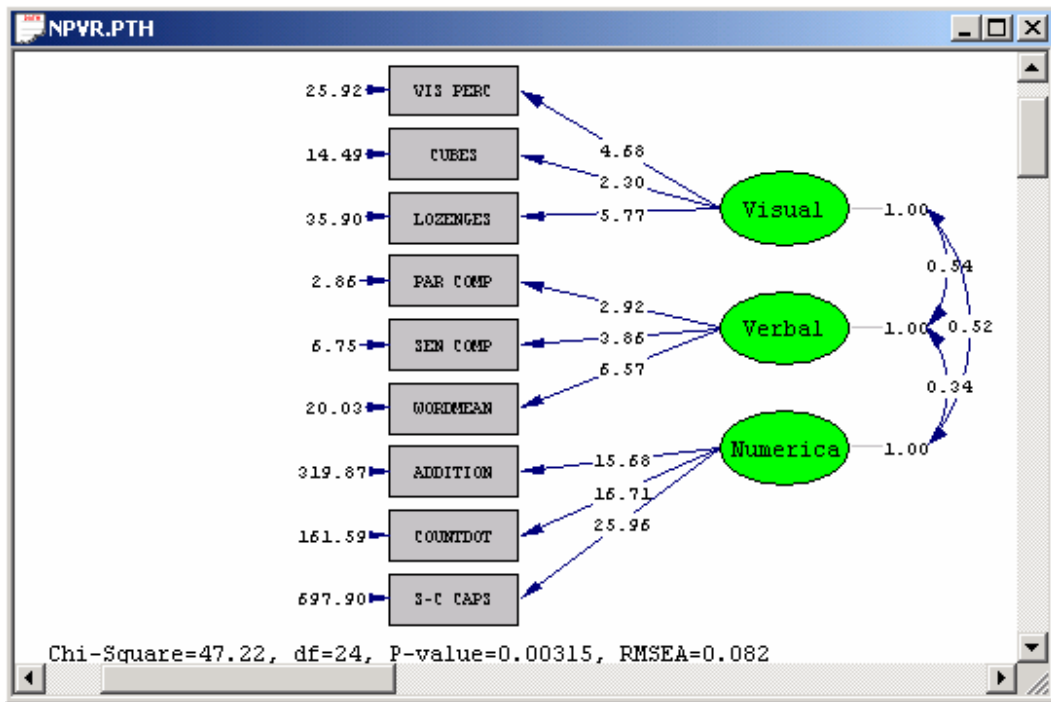
Lines 5 to 8 specify the measurement model to be fitted to the data.

Line 9 specifies that the results should be printed in terms of the LISREL model for the measurement model in Figure 2. It also requests three decimal places, the completely standardized solution and an analysis of the residuals.

Line 10 requests a path diagram.

Line 11 indicates no more SIMPLIS commands are to be processed.

- Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



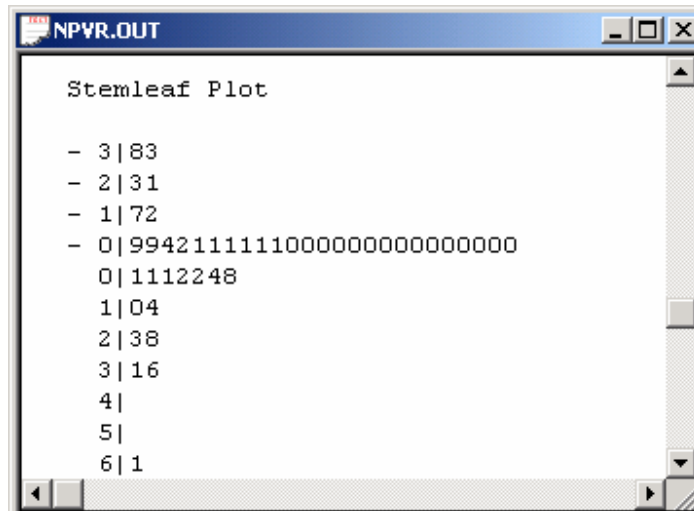
The output file, *NPVR.OUT*, contain the results of the residual analysis. The fitted residuals are displayed in the following text editor window.

Fitted Residuals

	VIS PERC	CUBES	LOZENGES	PAR COMP	SEN COMP	WORDMEAN
VIS PERC	0.000					
CUBES	-0.728	0.000				
LOZENGES	-1.189	2.172	0.000			
PAR COMP	0.583	-0.206	0.094	0.000		
SEN COMP	0.183	-1.490	-0.934	0.012	0.000	
WORDMEAN	0.814	-1.276	2.472	-0.021	-0.001	0.000
ADDITION	-21.247	-11.820	-32.560	1.371	7.753	-0.835
COUNTDOT	3.741	-4.402	-8.786	-9.053	-2.345	-16.672
S-C CAPS	61.108	9.615	36.404	13.819	27.590	22.697

	ADDITION	COUNTDOT	S-C CAPS
ADDITION	0.000		
COUNTDOT	31.196	0.000	
S-C CAPS	-38.442	-22.887	0.000

An inspection of the residuals above reveals that the largest absolute fitted residual is 61.108 for the covariance between S-C CAPS and VIS PERC. A stem-and-leaf plot of the fitted residuals is shown in the text editor window below.



The text editor window below contains the standardized fitted residuals.

	VIS PERC	CUBES	LOZENGES	PAR COMP	SEN COMP
VIS PERC	- -				
CUBES	-0.506	0.000			
LOZENGES	- -	2.486	- -		
PAR COMP	0.421	-0.237	0.098	0.000	
SEN COMP	0.141	-2.012	-0.745	0.025	0.000
WORDMEAN	0.235	-0.650	0.675	-0.039	- -
ADDITION	-2.019	-2.068	-3.053	0.259	0.902
COUNTDOT	0.380	-1.154	-0.898	-2.025	-0.317
S-C CAPS	3.956	0.789	1.661	1.412	2.340

	ADDITION	COUNTDOT	S-C CAPS
ADDITION	- -		
COUNTDOT	3.054	0.000	
S-C CAPS	-1.582	-3.295	0.000

From the results above, it is evident that the largest absolute standardized fitted residual is 3.956 for the covariance between S-C CAPS and VIS PERC. This indicates a possible lack of fit of the measurement model in Figure 2 to the data. A careful study of the contents of the S-C CAPS test reveals that it may indicate both visual and numerical ability. Next, we fit the model based on this revelation to the data.

- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the file **NPV1R.SPL** in the **WORKSHOP** folder of LISREL 8.7 for Windows.
- Click on the **Open** push button to open the following text editor window.

```

NPV1R.SPL
A measurement model for nine psychological tests
System File from File NPV.DSF
Latent Variables
Visual Verbal Numerical
Relationships
'VIS PERC' CUBES LOZENGES = Visual
'PAR COMP' 'SEN COMP' WORDMEAN = Verbal
ADDITION COUNTDOT 'S-C CAPS' = Numerical
'S-C CAPS' = Visual
LISREL Output: ND=3 SC RS
Path Diagram
End of Problem

```

Line 1 provides a title for the analysis.

Line 2 specifies the data source.

Lines 3 to 4 specify labels for the latent variables.

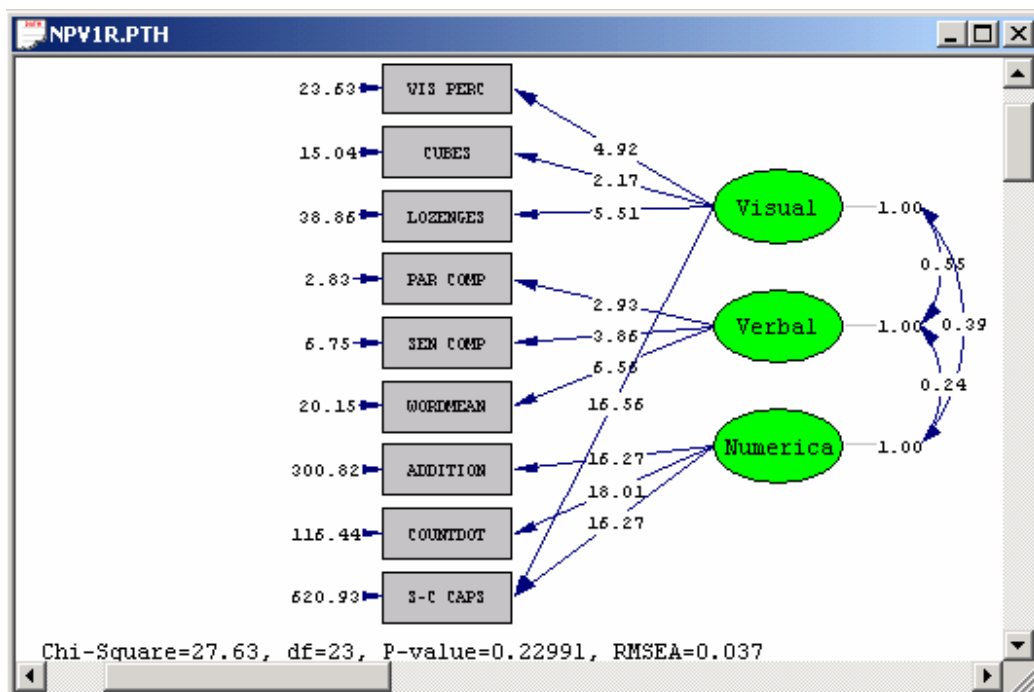
Lines 5 to 9 specify the measurement model to be fitted to the data.

Line 10 specifies that the results should be printed in terms of the LISREL model for the measurement model in Figure 2. It also requests three decimal places, the completely standardized solution and an analysis of the residuals.

Line 11 requests a path diagram.

Line 12 indicates no more SIMPLIS commands are to be processed.

- Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



The standardized residuals are listed in the following text editor window.

Standardized Residuals

	VIS PERC	CUBES	LOZENGES	PAR COMP	SEN COMP	WORDMEAN
VIS PERC	- -					
CUBES	-0.455	0.000				
LOZENGES	-4.392	2.505	- -			
PAR COMP	0.003	-0.111	0.249	0.000		
SEN COMP	-0.461	-1.646	-0.451	-0.002	0.000	
WORDMEAN	-0.130	-0.497	0.754	-0.037	- -	0.000
ADDITION	-1.355	-1.099	-1.748	1.010	1.511	0.585
COUNTDOT	1.027	0.069	0.256	-1.215	0.377	-0.694
S-C CAPS	1.987	-0.981	-0.743	0.124	1.042	-0.215

	ADDITION	COUNTDOT	S-C CAPS
ADDITION	- -		
COUNTDOT	- -	0.000	
S-C CAPS	-0.103	0.055	0.000

Although the standardized residuals for the covariances between LOZENGES and VIS PERC and LOZENGES and CUBES are large, they do not necessarily imply a lack of fit of the model since they all are indicators of visual ability.

References

- Duncan, T.E., Duncan, S.C., Stycker, L.A., Fuzhong, L., & Alpert, A. (1999). *An introduction to latent variable growth curve modeling: Concepts, issues, and applications*. Mahwah, NJ: Erlbaum
- Jöreskog, K. & Sörbom, D. (2005). *LISREL 8.7 for Windows [Computer Software]*. Lincolnwood, IL: Scientific Software International, Inc.