Three-level logistic regression model

3.1.1 Introduction

Having fitted 2-level models where students were nested within either classrooms or schools thus far, we now consider a 3-level model with both classroom and school defining levels of the hierarchy.

3.1.1.1 The model

The level-1 and level-2 models are the same as for the previous two models, as shown below.

Level 1 model $(k = 1, \ldots, n_{ij})$:

$$\text{THKSbin}_{ijk} = b_{0ij} + b_{1ij}\text{PRETHKS}_{ijk} + e_{ijk}$$

Level-2 model $(j = 1, \ldots, n_i)$:

$$b_{0ij} = b_{00i} + b_{01i}\text{CC}_{ij} + b_{02i}\text{TV}_{ij} + b_{03i}(\text{CC}_{ij} \times \text{TV}_{ij}) + v_{0ij}$$
$$b_{1ij} = b_{10i}$$

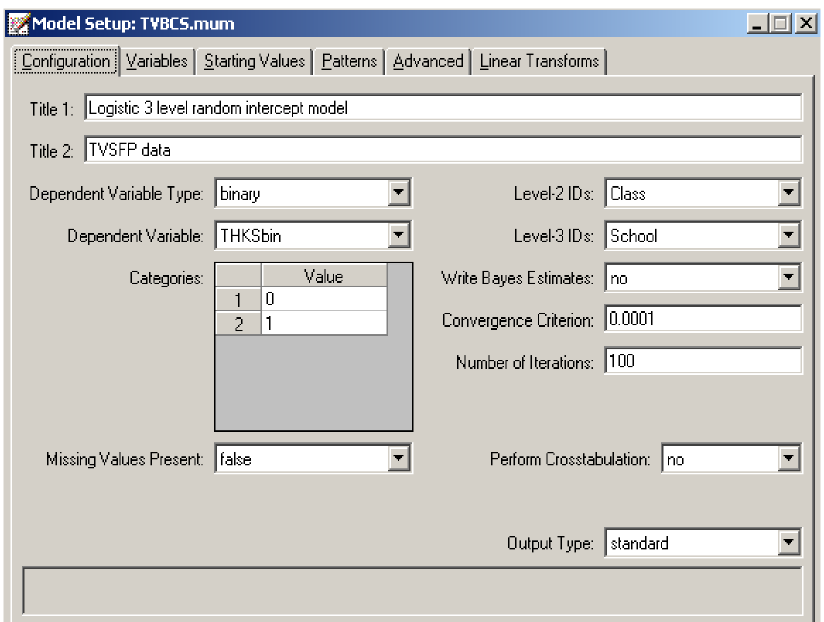
With classrooms nested within schools, however, a third level of the hierarchy is defined. At this level, the level-2 coefficients become outcomes again, and can potentially vary over the schools (level-3 units). In the current model, we allow only the intercept to vary randomly over the schools.

Level-3 model $(i = 1, \ldots, N)$

$$
\begin{aligned}
b_{00i} &= \beta_0 + v_{0i} \\
b_{01i} &= \beta_1 \\
b_{02i} &= \beta_2 \\
b_{03i} &= \beta_3 \\
b_{10i} &= \beta_4
\end{aligned}
$$

3.1.1.2 Setting up the analysis

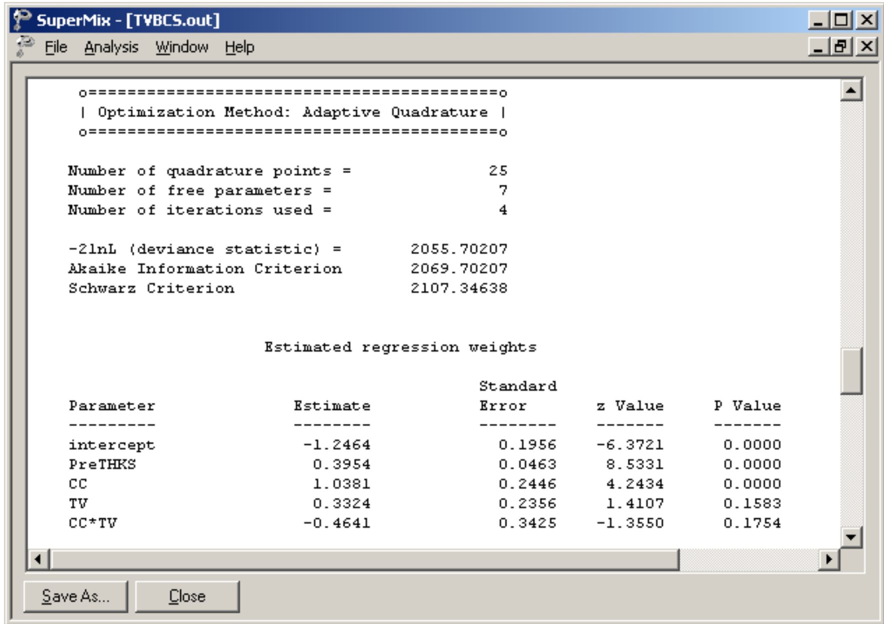
We modify our model setup saved to the syntax file **TVBS.mum** by first using the **Open Existing Model Setup** option on the **File** menu to retrieve the syntax file. Then click on **File, Save** as to save the model setup in a new file, such as **TVBCS.mum**. Next, select CLASS as the **Level-2 ID** and SCHOOL as the **Level-3 IDs** as shown below. We now have both level-2 and level-3 IDs selected.



Keep all the other settings unchanged. Save the changes to the file **TVBCS.mum** and select the **Run** option on the **Analysis** menu to run the analysis.

3.1.1.3 Discussion of results

The portions of the output file **TVBCS.out** containing the estimates of the fixed and random coefficients in the current model are shown below.

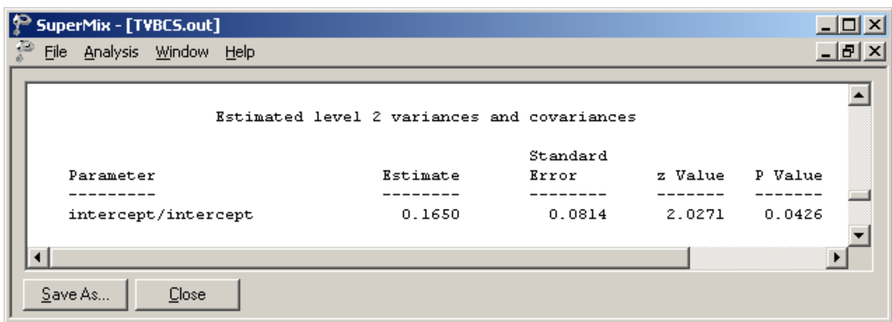


SuperMix - [TVBCS.out]

File Analysis Window Help

```
 o=========================================o
 | Optimization Method: Adaptive Quadrature |
 o=========================================o

Number of quadrature points =              25
Number of free parameters =                 7
Number of iterations used =                 4

-2lnL (deviance statistic) =       2055.70207
Akaike Information Criterion       2069.70207
Schwarz Criterion                  2107.34638


                    Estimated regression weights

                                           Standard
Parameter              Estimate            Error        z Value      P Value
---------              --------            --------     -------      -------
intercept               -1.2464             0.1956      -6.3721       0.0000
PreTHKS                  0.3954             0.0463       8.5331       0.0000
CC                       1.0381             0.2446       4.2434       0.0000
TV                       0.3324             0.2356       1.4107       0.1583
CC*TV                   -0.4641             0.3425      -1.3550       0.1754
```

Save As... Close

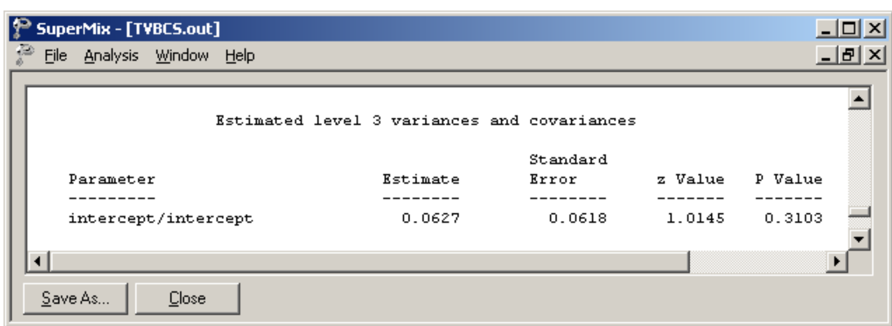


SuperMix - [TVBCS.out]

File Analysis Window Help

```
               Estimated level 2 variances and covariances

                                                Standard
Parameter                       Estimate        Error        z Value    P Value
---------                       --------        --------     -------    -------
intercept/intercept               0.1650          0.0814      2.0271     0.0426
```

Save As... Close



SuperMix - [TVBCS.out]

File Analysis Window Help

```
               Estimated level 3 variances and covariances

                                                Standard
Parameter                       Estimate        Error        z Value    P Value
---------                       --------        --------     -------    -------
intercept/intercept               0.0627          0.0618      1.0145     0.3103
```

Save As... Close

Table 4.8: Comparison of results for three models with binary variable THKSbin as outcome

Coefficient		2-level: CLASS as ID	2-level: SCHOOL as ID	3-level
Fixed effects:				
Intercept	estimate	–1.2535	–1.228	–1.2465
	standard error	0.1695	0.1949	0.1957
PRETHKS	estimate	0.401	0.3871	0.3954
	standard error	0.0461	0.0451	0.0463
CC	estimate	0.9883	1.0893	1.0383
	standard error	0.1973	0.2454	0.2448
TV	estimate	0.287*	0.3741*	0.3325*
	standard error	0.192	0.235	0.2358
CCxTV	estimate	–0.369*	–0.5578*	–0.4644*
	standard error	0.2774	0.3403	0.3427
Random effects:				
Var(between classrooms)	estimate	0.2193		0.1649
	standard error	0.0802		0.0813
Var(between schools)	estimate		0.1065	0.063*
	standard error		0.0578	0.0616

*: Not significant at 5% level of significance.

Results for this model are compared to those obtained using the two 2-level models in Table 4.8. Generally, there is close agreement between the models in terms of both the sign and size of the effects. Note that the only intervention method that consistently has an estimated coefficient significantly different from zero is CC. While use of the media intervention (TV) can positively influence the post-intervention score, it seems clear that using both methods simultaneously does not have any real benefits.

3.1.1.4 Interpreting the adaptive quadrature results

3-level ICCs

Intraclass correlation coefficients can be obtained for the three-level dichotomous outcome model. As mentioned earlier, it is assumed that the level-1 error variance is equal to $\pi^2/3$ for the logistic link function if the model is true (see, *e.g.*, Hedeker & Gibbons (2006), p. 157). Using this approximation, the formulae for the standard ICCs can be adjusted.

From the output for the random effects, we have

$$\text{Level-1: estimated}\left(\text{error var}\right) = \pi^2/3{=}3.2899$$
$$\text{Level-2: estimated}\left(\text{class var}\right) = 0.1649$$
$$\text{Level-3: estimated}\left(\text{school var}\right) = 0.0630.$$

Based on this information, we can calculate the ICC as shown below.

Similarity of students within the same school:

$$ICC = \frac{\sigma^2_{v(3)}}{\sigma^2_{v(3)} + \sigma^2_{v(2)} + \sigma^2_e} = \frac{0.063}{0.063 + 0.1649 + 3.28986}$$

$$= 0.0179.$$

Similarity of students within the same classrooms (and schools):

$$ICC = \frac{\sigma^2_{v(2)}}{\sigma^2_{v(3)} + \sigma^2_{v(2)} + \sigma^2_e} = \frac{0.1649}{0.063 + 0.1649 + 3.28986}$$

$$= 0.04688.$$

Similarity of classes within the same school:

$$ICC = \frac{\sigma^2_{v(2)}}{\sigma^2_{v(3)} + \sigma^2_{v(2)}} = \frac{0.1649}{0.063 + 0.1649}$$

$$= 0.7236.$$

Estimated unit-specific and population-average probabilities

Under the assumption that $\mathbf{v}_i$, $\mathbf{v}_{ij}$ and ε_{ijk} are independently distributed, it follows that for the three-level model the design effect is defined as

$$d_{ijk} = \frac{(\sigma^2_{v(3)} + \sigma^2_{v(2)} + \sigma^2_e)}{\sigma^2_e} = 1.0692.$$

The estimated unit-specific probabilities are calculated using

$$\hat{\eta}_{ijk} = -1.2465 + 1.0383 \times \mathrm{CC}_i + 0.3325 \times \mathrm{TV}_i - 0.4.644 \times \mathrm{CC}_i \times \mathrm{TV}_i$$
$$+0.3954 \times \mathrm{PreTHKS}_{ijk}$$

and

$$\mathrm{Prob}(\mathrm{THKSbin} = 1 \,|\, \boldsymbol{\beta}) = \frac{1}{1 + e^{-\eta_{ijk}}}$$

The estimated population-average probabilities (Hedeker & Gibbons, 2006) are obtained in a similar fashion as the unit-specific probabilities after replacing $\hat{\eta}_{ijk}$ with $\hat{\eta}_{ijk}^{*} = \hat{\eta}_{ijk} / \sqrt{d_{ijk}}$ in the second of the equations shown above.