



## Two-level ordinal analysis

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#### 3.1.1 The data

The study was designed to test independent and combined effects of a school-based social-resistance curriculum and a television-based program in terms of tobacco use and cessation.

The structure of this study indicates a three-level hierarchical structure. However, for illustration purposes in this chapter we will consider a two-level structure in which students are nested within schools. Data for the first 10 participants on most of the variables used in this section are shown below in the form of a **SuperMix** spreadsheet file, named **tvspfors.ss3**, located in the **Examples\Ordinal** subfolder.

	(A)_School	(B)_Class	(C)_THKSor	(D)_THKSbi	(E)_PreTHK	(F)_CC	(G)_TV	(H)_CC*TV
1	403.00	403101.00	3.00	1.00	2.00	1.00	0.00	0.00
2	403.00	403101.00	4.00	1.00	4.00	1.00	0.00	0.00
3	403.00	403101.00	3.00	1.00	4.00	1.00	0.00	0.00
4	403.00	403101.00	4.00	1.00	3.00	1.00	0.00	0.00
5	403.00	403101.00	4.00	1.00	3.00	1.00	0.00	0.00
6	403.00	403101.00	3.00	1.00	4.00	1.00	0.00	0.00
7	403.00	403101.00	2.00	0.00	2.00	1.00	0.00	0.00
8	403.00	403101.00	4.00	1.00	4.00	1.00	0.00	0.00
9	403.00	403101.00	4.00	1.00	5.00	1.00	0.00	0.00
10	403.00	403101.00	4.00	1.00	3.00	1.00	0.00	0.00

The variables of interest are:

- School indicates the school a student is from (28 schools in total).
- Class identifies the classroom (135 classrooms in total).
- THKSord represents the post-intervention tobacco and health knowledge scaled score, with 4 categories ranging between 1 and 4. The frequency distribution of the post-intervention THKS scores indicated that approximately half the students had scores of 2 or less, and half of 3 or greater. In terms of quartiles, four ordinal classifications were suggested corresponding to 0 – 1, 2, 3, and 4 – 7 correct responses.
- PreTHKS indicates a student's score prior to intervention, *i.e.* the number correct of 7 items.
- CC is a binary variable indicating whether a social-resistance classroom curriculum was introduced, where 0 indicates "no" and 1 "yes."
- TV is an indicator variable for the use of media (television) intervention, with a "1" indicating the use of media intervention, and "0" the absence thereof.
- CC\*TV was constructed by multiplying the variables TV and CC, and represents the CC by TV interaction.

In this chapter we will explore a random intercept model using the ordinal variable THKSord as outcome. In previous analyses of this data, the post-intervention score was assumed to be a continuous variable. In contrast, here categories are created and the implied data collapse may lead to a loss of information and thus results may differ from those obtained previously.

### 3.1.1.1 Exploring the data

The focus in this chapter is on the influence of the intervention on the tobacco health knowledge scores of the students, as represented by the ordinal outcome variable THKSord. A cross-tabulation of the variables CC, TV, and THKSord are given in Table 6.1 below.

In general, students not exposed to the social-resistance classroom curriculum (CC = 0) seem to have less health knowledge than those students exposed to the social-resistance classroom curriculum (CC = 1), regardless of their exposure to media intervention. The opposite is true for students from groups assigned the social-resistance classroom curriculum (CC = 1).

**Table 6.1: Crosstabulation of CC, TV and THKSord**

TV		CC		Total	
		0	1		
0	THKSord	1	117	62	179
		2	129	78	207
		3	89	106	195
		4	86	134	220
	Total	421	380	801	
1	THKSord	1	110	66	176
		2	105	86	191
		3	91	114	205
		4	110	117	227
	Total	416	383	799	

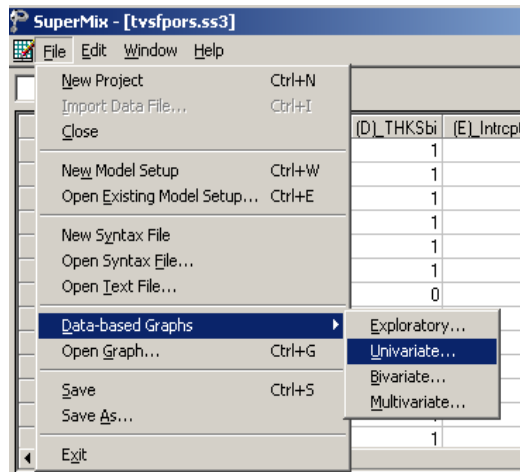
The trend is also apparent when the post-intervention scores are expressed as proportions (see Table 6.2).

**Table 6.2: Observed proportion of high post-intervention scores**

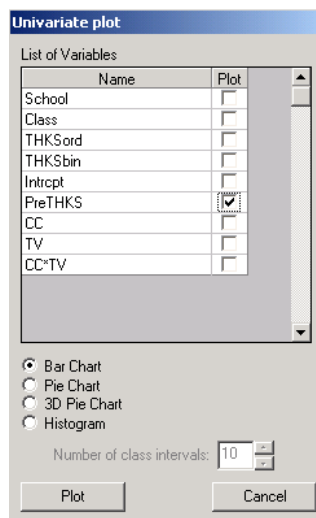
TV		CC		Total	
		0	1		
0	THKSord	1	0.0731	0.0388	0.1119
		2	0.0806	0.0488	0.1294
		3	0.0556	0.0663	0.1219
		4	0.0538	0.0838	0.1375
	Total	0.2631	0.2375	0.5006	
1	THKSord	1	0.0688	0.0413	0.1100
		2	0.0656	0.0538	0.1194
		3	0.0569	0.0713	0.1281
		4	0.0688	0.0731	0.1419
	Total	0.2600	0.2394	0.4994	

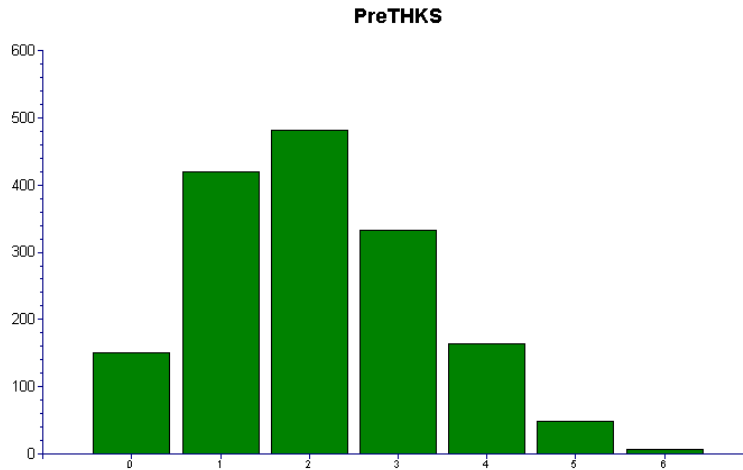
First, notice that the outcome variable THKSord has a skewed distribution. By combining the proportions per category over interventions, we find that 0.2219 of the 1600 students had a value of 1 for THKSord, 0.2488 had a value of 2, 0.25 had a value of 3, and 0.2794 a value of 4 for THKSord. The monotonic increase in the proportion observed in each category of THKSord indicates that it would be inappropriate to try to fit a continuous model to the data.

The pre-intervention scores of the students may be used as a covariate in the analysis. To get some idea of the relationship between the scale score PreTHKS and the post-intervention score THKSord, an exploratory graph may be useful. To take a closer look at the distribution of PreTHKS, select the **Data-based Graphs, Univariate...** option from the **File** menu after opening the SuperMix spreadsheet **tvspfors.ss3**.



The **Univariate plot** dialog box is activated. Select the variable PreTHKS, and request a **Bar Chart**. Click **Plot**.

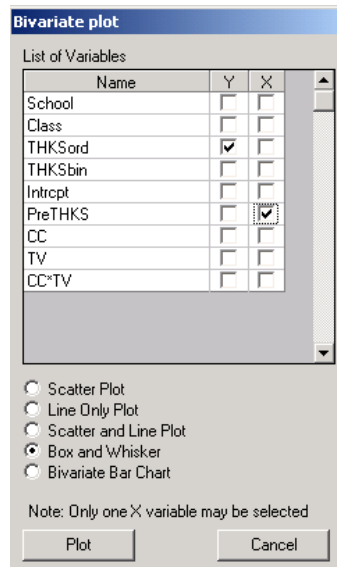




**Figure 6.1: Distribution of the PreTHKS scores**

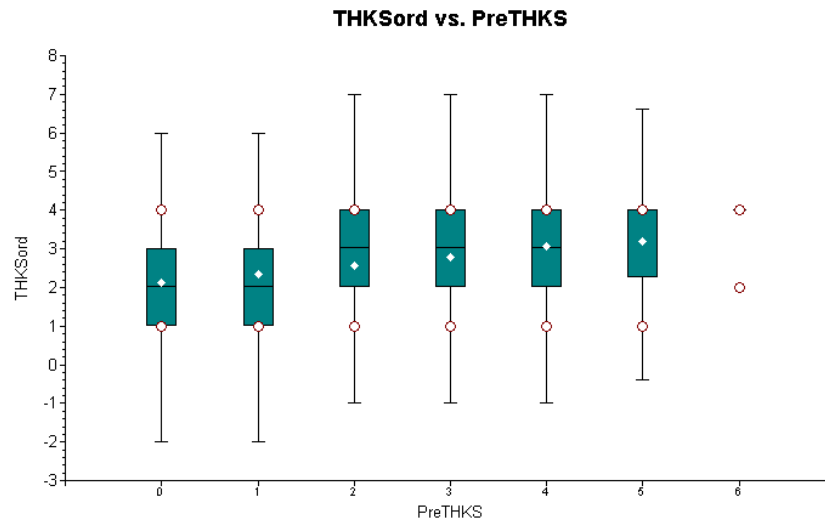
Figure 6.1 is obtained. In contrast to the outcome variable THKSord, the distribution of the PreTHKS score has a lower mean, with very few students exhibiting extensive knowledge on the subject matter (PreTHKS = 5 or PreTHKS = 6).

We now take a closer look at the distribution of the outcome variable at each distinct pre-intervention score value by utilizing the **Data-based Graphs, Bivariate** option on the **File** menu. By default, a bar chart will be produced. Select the variable THKSord in the **Y** column and the variable PreTHKS in the **X** column, and request a **Box and Whisker** plot before clicking the **Plot** button.



The figure below shows a reasonably steady increase in the mean THKSord with increasing PreTHKS scores. This seems to be expected: students with more initial knowledge ending up having higher post-intervention scores as well. Note that only 55 of the 1600 observations

showed a score of 5 or higher on the pre-intervention score, and that no student obtained a post-intervention score of 7 out of 7.



**Figure 6.2: Box-and-whisker plot of THKSord for values of PreTHKS**

Finally, we also take a look at the mean pre-intervention scores of the students for each of the four subgroups. These are summarized in Table 6.3 below, and show that the mean pre-intervention scores do not differ much.

**Table 6.3: Mean pre-intervention scores**

Study condition	Mean
CC = 0, TV = 0	2.152
CC = 0, TV = 1	2.087
CC = 1, TV = 0	2.050
CC = 1, TV = 1	1.979

### 3.1.2 A multilevel ordinal model with logistic link function

#### 3.1.2.1 The proportional odds model

The model we use for the analysis of ordinal data is based on McCullagh's (1980) proportional odds model, which characterizes the ordinal responses in  $C$  categories in terms of  $C - 1$  cumulative category comparisons, specifically  $C - 1$  cumulative logits. The McCullagh model can be written as

$$\log \left[ \frac{P(y \leq c)}{1 - P(y \leq c)} \right] = \gamma_c - \mathbf{x}'\boldsymbol{\beta}$$

where

- $c = 1, \dots, C-1$  for the  $C$  categories of the ordinal outcome
- $\mathbf{x}$  is the vector of explanatory variables, plus the intercept
- $\gamma_c$  represent the threshold parameter(s); and reflect the cumulative odds when  $\mathbf{x} = 0$ .

The positive association between a predictor variable  $x$  and the ordinal outcome variable  $y$  is reflected by  $\beta$ . It is assumed that the effect of  $x$  is the same for each of the cumulative odds ratios.

To illustrate, consider a model with a single predictor  $x$ . The odds that the response is less than or equal to  $c$  (for any fixed  $c$ ) is divided by  $e^\beta$  for every unit change in  $x$ , as shown below:

$$\left[ \frac{P(y \leq c)}{1 - P(y \leq c)} \right] = \exp(\gamma_c - x\beta) = \frac{e^{\gamma_c}}{(e^\beta)^x}.$$

On the other hand, the odds that the response is greater than or equal to  $c$  (again for a fixed  $c$ ) is multiplied by  $e^\beta$  for every unit change in  $x$ :

$$\left[ \frac{1 - P(y \leq c)}{P(y \leq c)} \right] = e^{-\gamma_c} \times (e^\beta)^x.$$

To motivate the ordinal regression model, it is often assumed that there is an unobservable latent variable ( $y^*$ ) which is related to the actual response through the "threshold concept." An example of this is when respondents are asked to rate their agreement with a given statement using the categories "Disagree," "Neutral," "Agree." These three options leave no room for any other response, though one can argue that these are three possibilities along a continuous scale of agreement that would also make provision for "Strongly Agree" and "Disagree somewhat." The ordinal responses captured in  $y$  and the latent continuous variable  $y^*$  are linked through some fixed, but unknown, thresholds.

For the dichotomous model, one threshold value is assumed, while for the ordinal model, a series of threshold values  $\gamma_0, \gamma_1, \gamma_2, \dots, \gamma_C$ , where  $C$  equals the number of ordered categories,  $\gamma_0 = -\infty$ , and  $\gamma_C = \infty$ , is assumed. Here, a response occurs in category  $c$  ( $Y = c$ ) if the latent response process  $y$  exceeds the threshold value  $\gamma_{c-1}$ , but does not exceed the threshold value  $\gamma_c$ . The cumulative probabilities are given in terms of the cumulative logits with  $C-1$  strictly increasing model thresholds  $\gamma_1, \gamma_2, \dots, \gamma_{C-1}$ . In the current case, we will thus have  $C-1 = 3$  cumulative probabilities, given in terms of 3 thresholds  $\gamma_1, \gamma_2$  and  $\gamma_3$ . The

thresholds represent the marginal response probabilities in the  $C$  categories. We will illustrate the use of the logistic link function in this example.

To set the location of the latent variable, it is common to set a threshold to zero. Usually, the first of the threshold parameters ( $\gamma_1$ ) is set to zero. Alternatively, the model intercept ( $\beta_0$ ) is set to zero and  $C-1$  thresholds are estimated.

### 3.1.2.2 The mixed-effect ordinal logistic regression model

A limitation of the model specified in the previous section is that it is assumed that the effect of covariates is the same across the cumulative logits. To overcome this limitation, an extension of the mixed-effects ordinal logistic regression model to allow for nonproportional odds for a set of regressors was developed by Hedeker & Mermelstein (1998). This generalization of the proportional odds model can be formulated as

$$\log \left[ \frac{P(y \leq c)}{1 - P(y \leq c)} \right] = \gamma_c - [\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i].$$

In this model, as in the proportional odds model, the origin of the latent variable  $y$  is set by setting the first threshold,  $\gamma_1$ , equal to zero. It is assumed that  $\mathbf{v}_i : NID(\mathbf{0}, \boldsymbol{\Sigma}_v)$ . The unit of measurement is  $\sigma = \pi / \sqrt{3}$ .

For this model, the category probabilities are defined as

$$P(y_{ij} \leq c) = \psi(\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i))$$

and

$$P(y_{ij} = c) = \psi(\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)) - \psi(\gamma_{c-1} - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i))$$

where the cumulative standard logistic distribution function is

$$\psi(\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)) = \frac{1}{1 + \exp[-\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)]}.$$

Various link functions may be used with this model. If we define  $G^{-1}[P(y_{ij} \leq c)]$  as

$$G^{-1}[P(y_{ij} \leq c)] = \gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i),$$

or, equivalently,

$$P(y_{ij} \leq c) = G[\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)],$$



three types of models can easily be fitted:

- Using  $G^{-1}(P) = \log[P/(1-P)]$  will give a cumulative logit model, *i.e.* a proportional odds model,
- using  $G^{-1}(P) = \Phi^{-1}[P/(1-P)]$  will produce a cumulative probit model, and
- using  $G^{-1}(P) = \log[-\log(1-P)]$ , the so-called complementary log-log link, will give a proportional hazards model.

For more on the use of link functions, please see Section 4.1.1.

### 3.1.2.3 A general multilevel ordinal model

The multilevel ordinal model is defined in terms of the cumulative probability  $P(y_{ij} \leq c)$  where  $c$  denotes the category of interest. The level-1 model is written in terms of the cumulative logits, as shown below.

**Level-1 model:**

$$\log \left[ \frac{P(y_{ij} \leq c)}{1 - P(y_{ij} \leq c)} \right] = \gamma_c - [\mathbf{x}_{ij}' \mathbf{b}_i].$$

where  $\mathbf{x}_{ij}$  represent the values of the covariates corresponding to level-1 unit  $j$  nested within level-2 unit  $i$ .

**Level-2 model:**

If all the elements of the coefficient vector  $\mathbf{b}_i$  are allowed to vary randomly across level-2 units, then

$$\mathbf{b}_i = \boldsymbol{\beta} + \mathbf{v}_i,$$

which models the level-2 effects as a function of an overall mean  $\boldsymbol{\beta}$  and a unique random component  $\mathbf{v}_i : NID(\mathbf{0}, \boldsymbol{\Sigma}_v)$ . The latter is also referred to as the level-2 residuals and indicates the extent to which a given level-2 unit differs from the average, as estimated by the first part of the level-2 model.

Note that the level-2 model does not depend on the response variable. As the regression coefficients  $\beta_0, \beta_1, \beta_2$  and  $\beta_3$  are without subscript, it is assumed that they do not vary across the categories and hence that the relationship between the predictor variables and the

cumulative logits is not dependent on  $c$ . McCullagh (1980) referred to this as the assumption of identical odds ratios across the  $C - 1$  categories.

In practice, a subset of the coefficients  $\mathbf{b}_i$  are assumed to have fixed, but unknown, values. For example, a random intercept-and-slope model with 2 predictors of which the first has a random slope would have a level-2 model of the form

$$\begin{aligned} b_{0i} &= \beta_0 + v_{0i} \\ b_{1i} &= \beta_1 + v_{1i} \\ b_{2i} &= \beta_2 \end{aligned}$$

In this model, only the first two coefficients are assumed to vary randomly across the level-2 units.

Another characteristic of the current model is that a positive coefficient for a regressor indicates that the odds that the response is greater than or equal to  $c$  increases with an increase in regressor values. However, another formulation as shown below, in which the regression parameters  $\beta$  are identical but of opposite sign, is commonly used in survival analysis models (see Chapter 8):

$$\log \left[ \frac{P_{ijc}}{1 - P_{ijc}} \right] = \gamma_c + [\mathbf{x}_i' \mathbf{b}_i] \quad (c = 1, \dots, C - 1).$$

#### 3.1.2.4 An ordinal model with 2 covariates and an interaction term

As in the case of the binary variable THKSbin, we intend to explore the relationship between the type of intervention, the pre-intervention scores of students and the ordinal outcome variable THKSord. We do so using a 2-level model, with students nested within schools.

##### Level-1 model:

At the first level, the pre-intervention score is used as predictor.

$$\log \left[ \frac{P(\text{THKSord}_{ij} \leq c)}{1 - P(\text{THKSord}_{ij} \leq c)} \right] = \gamma_c - [b_{0i} + b_{1i} \text{PreTHKS}_{ij}] \quad (j = 1, \dots, n_i \text{ subjects})$$

##### Level-2 model:

At the school level, the types of intervention (represented by the dummy variables CC and TV) are used to explain differences in the intercepts of the groups. In addition, the interaction between CC and TV is included in the model.

$$b_{0i} = \beta_0 + \beta_2 CC_i + \beta_3 TV_i + \beta_4 (CC*TV)_i + v_{0i} \quad (i = 1, \dots, N \text{ groups})$$

$$b_{1i} = \beta_1$$

It is assumed that  $v_{0i} : NID(0, \sigma_v^2)$ .

The model can also be formulated in a single expression as:

$$\log \left[ \frac{P(\text{THKSord}_{ij} \leq c)}{1 - P(\text{THKSord}_{ij} \leq c)} \right]$$

$$= \gamma_c - [\beta_0 + \beta_1 \text{PreTHKS}_{ij} + \beta_2 CC_i + \beta_3 TV_i + \beta_4 (CC*TV)_i + v_{0i}]$$

Recall that the outcome variable has 4 categories. There are thus 3 thresholds. In this model

- $0 - \beta_0$  (remember that  $\gamma_1 = 0$  for identification purposes) is the first logit (category 1 vs. categories 2 to 4) for groups with no intervention ( $CC = TV = 0$ ). This logit is adjusted for the effect of PreTHKS.
- $\gamma_2 - \beta_0$  is the second logit, representing categories 1 and 2 vs. categories 3 and 4, for groups with no intervention ( $CC = TV = 0$ ). This logit is also adjusted for the effect of PreTHKS.
- $\gamma_3 - \beta_0$  is the third logit, representing categories 1 to 3 vs. category 4, for the same groups and again adjusted for the effect of PreTHKS.
- The coefficient  $\beta_1$  represents the effect of PreTHKS on THKSord.
- The coefficient  $\beta_2$  denotes the PreTHKS adjusted logit differences between  $CC = 1$  and  $CC = 0$  (for  $TV = 0$ ).
- The coefficient  $\beta_3$  denotes the PreTHKS adjusted logit differences between  $TV = \text{yes}$  and  $TV = \text{no}$  (for  $CC = 0$ ).
- The coefficient  $\beta_4$  is the adjusted difference in logit attributable to interaction between  $CC$  and  $TV$  ( $CC * TV$ ).
- The random school deviation is represented by  $v_{0i}$ . Note that we assume a single, fixed and thus common PreTHKS slope over the level-2 units.
- The interpretation of the coefficients is dependent on the coding of the variables used in the model.

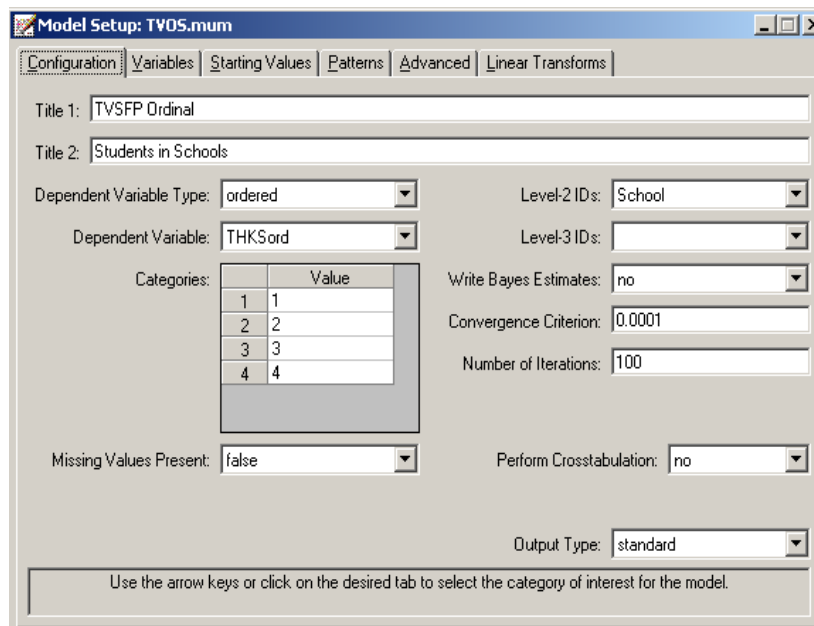
### 3.1.2.5 Setting up the analysis

Using the data in **tvspors.ss3**, we consider the situation where students are nested within schools and fit a two-level model with the ordinal variable THKSord as outcome. We wish to

examine the relationships between the outcome and the two intervention methods employed, simultaneously taking students' pre-intervention scores into account. To do so, we use the model described above with schools as the level-2 units.

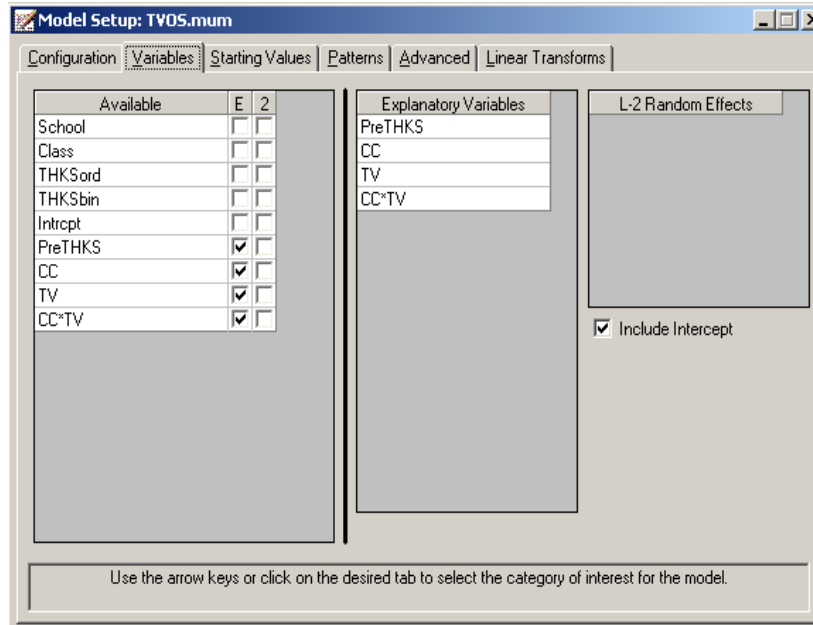
Use the **File, Open Spreadsheet** option to activate the display of an **Open** dialog box. Browse for the file **tvspors.ss3** in the **Examples\Ordinal** folder. Select the file and click the **Open** button to return to the main SuperMix window, where the contents of the SuperMix system file are displayed. We are now ready to provide model specifications.

We use the SuperMix interface to provide the model specifications. From the main menu bar, select the **File, New Model Setup** option. The **Configuration** tab of the **Model Setup** dialog box is displayed by default.

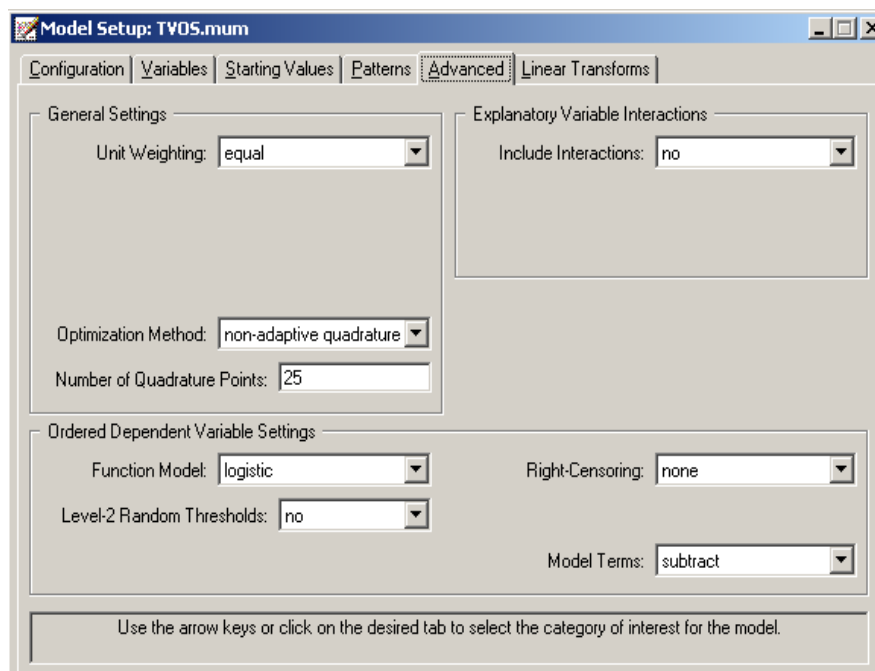


Category	Value
1	1
2	2
3	3
4	4

Start by selecting the ordinal outcome variable THKSord from the **Dependent Variable** drop-down list box. The type of outcome is specified as ordered using the drop-down list box in the **Dependent Variable Type** field. Once this selection is made, the **Categories** field is displayed. The School identification variable is used to define the hierarchical structure of the data, and is selected as the **Level-2 ID** from the **Level-2 IDs** drop-down list box. A title for the analysis is entered in the **Title** fields. A convergence criterion of 0.0001 is requested. By default, the maximum number of iterations allowed is 100. Default settings for all other options associated with this tab are used. Proceed to the **Variables** tab by clicking on this tab.



The **Variables** tab is used to specify the fixed and random effects to be included in the model. Start by selecting the explanatory (fixed) variables using the drop-down list box next to the first row in the **Explanatory Variables** box. After selecting all the explanatory variables, the random effect(s) at level 2 must be selected. In this case, we wish to allow only the intercept to vary randomly over the schools. By default, the intercept is assumed to vary randomly over higher levels of the hierarchy as indicated by the checked boxes for the **Include Intercept** options.



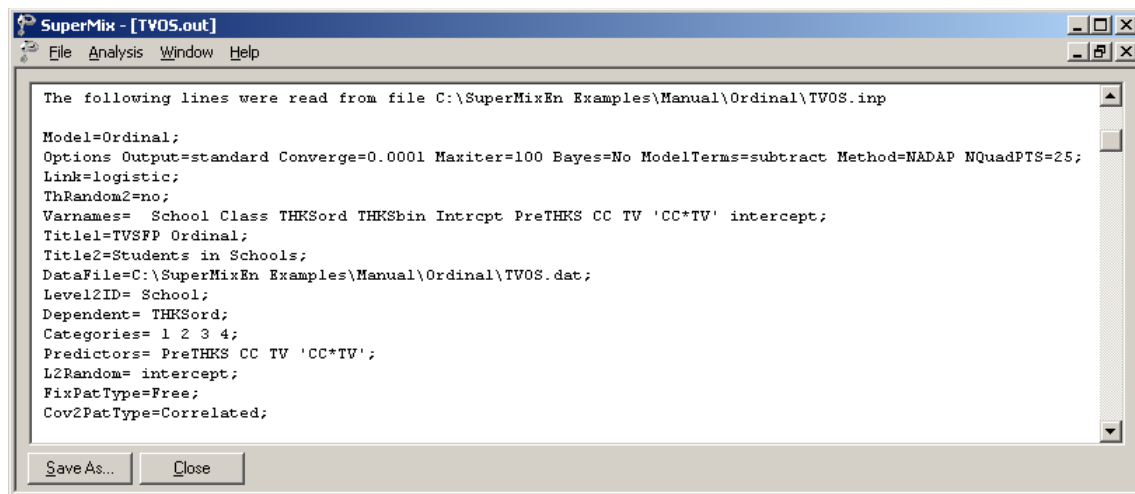
We opt to increase the number of quadrature points to be used during estimation. To do so, select the **Advanced** tab and change the **Number of Quadrature Points** field to 25. We also request the use of a **logistic link** function from the **Function model** drop-down list box.

Before running the analysis, the model specifications have to be saved. Select the **File, Save** option, and provide a name for the model specification file, for example **TVOS.mum**. Run the analysis by selection the **Run** option from the **Analysis** menu.

### 3.1.2.6 Discussion of results

Portions of the output file **TVOS.out** are shown below.

#### Program information and syntax



The screenshot shows a window titled "SuperMix - [TVOS.out]" with a menu bar containing "File", "Analysis", "Window", and "Help". The main text area displays the following code:

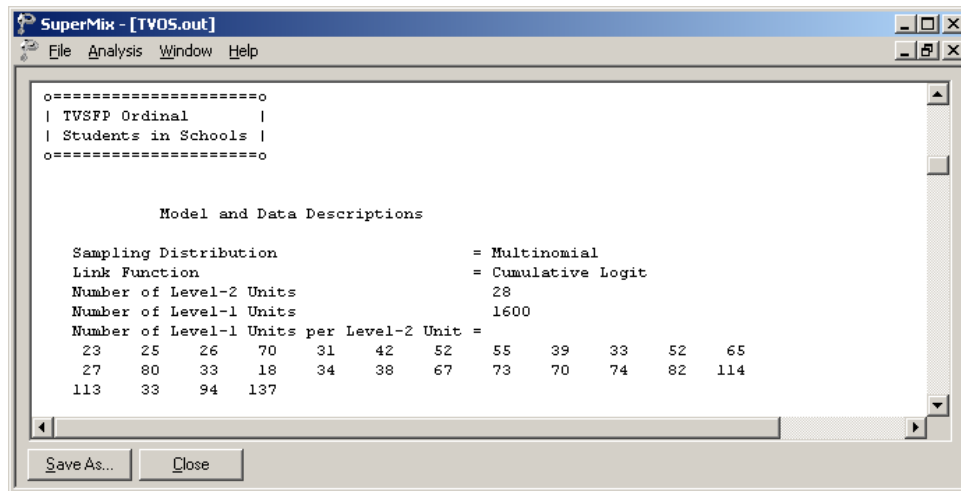
```
The following lines were read from file C:\SuperMixEn Examples\Manual\Ordinal\TVOS.inp

Model=Ordinal;
Options Output=standard Converge=0.0001 Maxiter=100 Bayes=No ModelTerms=subtract Method=NADAP NQuadPTS=25;
Link=logistic;
ThRandom2=no;
Varnames= School Class THKSord THKSbin Intrcpt PreTHKS CC TV 'CC*TV' intercept;
Title1=TVSFP Ordinal;
Title2=Students in Schools;
DataFile=C:\SuperMixEn Examples\Manual\Ordinal\TVOS.dat;
Level2ID= School;
Dependent= THKSord;
Categories= 1 2 3 4;
Predictors= PreTHKS CC TV 'CC*TV';
L2Random= intercept;
FixPatType=Free;
Cov2PatType=Correlated;
```

At the bottom of the window, there are two buttons: "Save As..." and "Close".

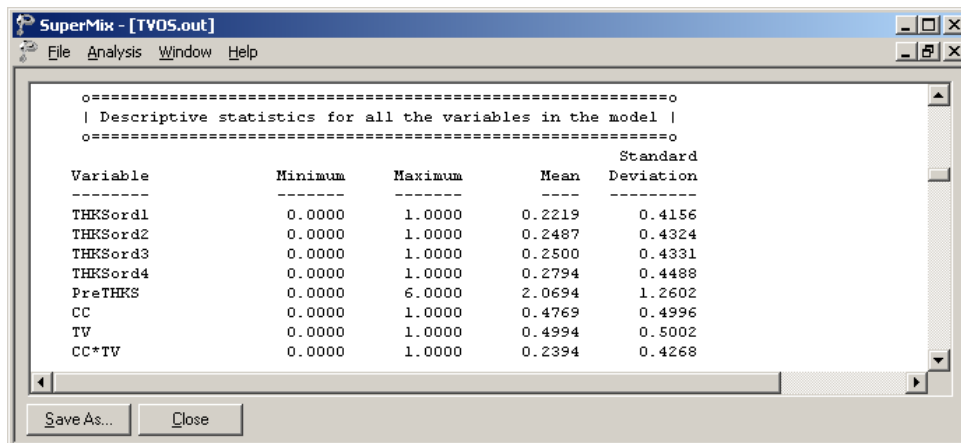
At the top of the file, the syntax saved to the **TVOS.mum** file is shown. The first part states the selection of iteration control options, requests for Bayes residuals, and the specifications necessary to define the model fitted as an ordinal model with logistic link function. The second part of the syntax provides information on the structure of the data, the name and structure of the outcome variable, and the predictors included in the model. Note that this part now also includes information on the categories of the outcome variable and the link function selected.

The next section contains a description of the model specifications. The use of a logistic response function (logit link function), with the assumption of a normal distribution of random effects is indicated. It is also noted that covariate and random effect means are subtracted from the thresholds, implying that a positive coefficient indicates a positive association between the outcome and the predictor in question. To add the covariate and random effect means instead of using the default subtract setting, the **add** option must be selected in the **Model Terms** field on the **Advanced** tab of the **Model Setup** dialog box.



## Descriptive statistics

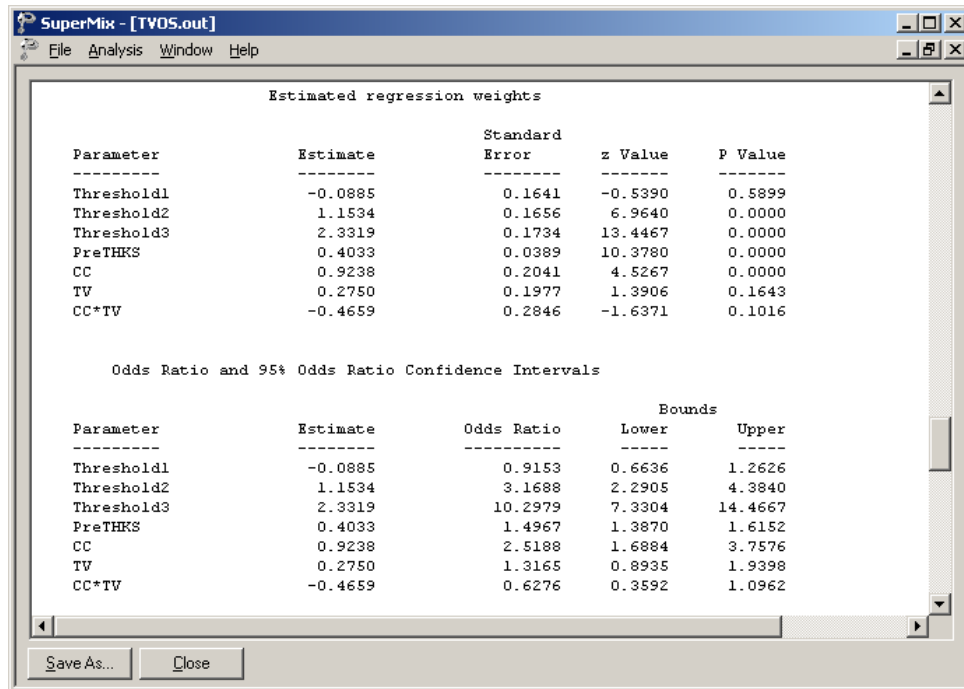
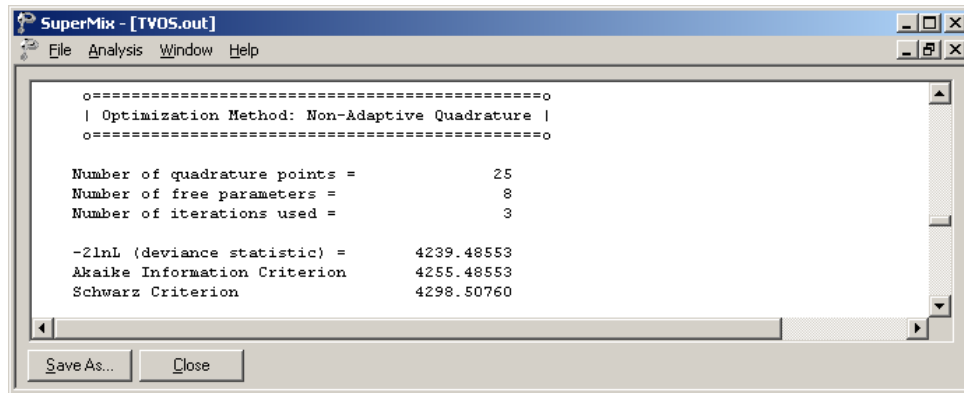
After the observation counts, descriptive statistics for all variables included in the model are followed by a frequency table for the categories of the outcome variable.



## Fixed effects results

The output describing the estimated parameters after convergence is shown next. Two iterations were required to obtain convergence, using 25 quadrature points per dimension. The likelihood function value at convergence as well as the deviance are also given, and may be used to compare a set of nested models. The estimates are shown in the column with heading Estimate, and correspond to the coefficients  $\beta_0, \beta_1, \dots, \beta_4$  in the model specification. Significant effects of PreTHKS and CC are observed. With the exception of the CC \*TV interaction term, positive relationships between the predictors and the ordinal outcome variable are indicated by these results. We also note that the coefficient associated with the

curriculum-based intervention (CC) is almost three times the size of the estimated coefficient for media intervention (TV).



The alternative parameterization, setting threshold = 0 is shown next. The estimates of  $\gamma_2$  and  $\gamma_3$  are 1.242 and 2.420 respectively – recall that for identification purposes  $\gamma_1$  was set to zero.



SuperMix - [TV05.out]

File Analysis Window Help

Alternative Parameterization, setting Threshold1= 0

Estimated regression weights

Parameter	Estimate	Standard Error	z Value	P Value
intcept	0.0885	0.1641	0.5390	0.5899
Threshold2	1.2418	0.0571	21.7663	0.0000
Threshold3	2.4204	0.0748	32.3401	0.0000

Odds Ratio and 95% Odds Ratio Confidence Intervals

Parameter	Estimate	Odds Ratio	Bounds	
			Lower	Upper
intcept	0.0885	1.0925	0.7920	1.5070
Threshold2	1.2418	3.4619	3.0956	3.8715
Threshold3	2.4204	11.2503	9.7153	13.0278

Save As... Close

## Random effects results

The last part of the output file contains information on the random effects and calculation of the intraclass correlation coefficient. The variation in intercept over schools is estimated at 0.0735, with the associated  $p$ -value of 0.055 indicating its statistical significance.

SuperMix - [TV05.out]

File Analysis Window Help

Estimated level 2 variances and covariances

Parameter	Estimate	Standard Error	z Value	P Value
intercept/intercept	0.0735	0.0383	1.9189	0.0550

Save As... Close

SuperMix - [TV05.out]

File Analysis Window Help

Calculation of the intraclass correlation

```

-----
residual variance = pi*pi / 3 (assumed)
cluster variance = 0.0735

intraclass correlation = 0.0735 / ( 0.0735 + (pi*pi/3) ) = 0.022

```

Save As... Close

In the case of the fixed effects, a 2-tailed  $p$ -value is used, as the alternative hypothesis considered here is of the form  $H_1 : \beta \neq 0$ . As variances are constrained to be elements of the

interval  $[0, +\infty)$ , the  $p$ -values used for these effects are 1-tailed. If the model is true, it is assumed that the level-1 error variance,  $\sigma_e^2$ , is equal to  $\pi^2/3$  for the logistic link function, where  $\pi$  represents the constant 3.141592654 (see, *e.g.*, Hedeker & Gibbons (2006), p. 157).

Finally, the calculation of the intraclass correlation is shown. In this calculation it is assumed that the residual variation,  $\sigma_e^2$ , is equal to  $\pi^2/3$ . The value of 0.022 indicates that almost all variation is attributable to students, rather than to the schools.

### 3.1.2.7 Interpreting the output

#### Model-based graphs

Activate the **Model Setup** window by clicking on it. Using the **Plot Equations for Outcome Variable** dialog box that appears when the **File, Model-based Graphs, Equations** option is selected, we can graphically depict the trend in post-intervention scores as a function of pre-intervention scores, taking the type of intervention into account. The dialog box below shows the selection of the predictor PreTHKS. Grouping of plots by the categories of CC is requested, while marking of the plots by TV is indicated by the selection in the **Mark** column. Two graphs will thus be displayed on the same set of axes: one for each value of the indicator variable TV. By default, all variables present in the model, but not selected for inclusion in the graph, will be assumed to have a value of 0. In the current situation, this means that CC\*TV is kept constant at zero. In effect, the graphs are for students from schools where only one of the interventions was administered; students from schools where both were implemented would have a value of 1 on the variable CC\*TV.

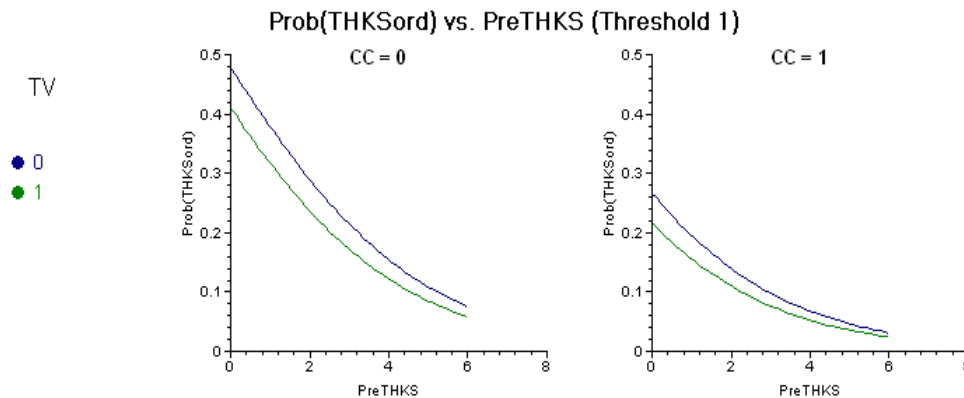
Name	Predictor	Group	Mark
PreTHKS	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
CC	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
TV	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
CC*TV	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
intcept	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
School	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Remaining predictors fixed at 0  
 Remaining predictors fixed at their means

Note: Only one variable may be selected for grouping and only one for marking.

Plot Cancel

Figure 6.3 shows slightly modified versions of the graphs obtained when the **Plot** button is clicked. For publication purposes, the line type associated with the value  $TV = 1$  was changed to a dotted line. This was accomplished by clicking on the top line to activate the **Plot Parameters** dialog box and changing the line parameters so that the color is green and the style is dotted rather than solid. The plots show that the curriculum-based intervention had a larger effect on the post-intervention score: the intercept in the case where  $CC = 1$  is appreciably higher than when  $CC = 0$ . In both graphs, the solid line indicates the absence of media-based intervention. The use of media-based intervention seems to have had a positive, albeit small, effect on the outcome.



**Figure 6.3: Model-based graphs of THKSord by PreTHKS for groups**

### Interpretation of fixed effect estimates

The outcome variable has four categories, and there are thus 3 thresholds. The coefficient  $\beta_1$ , representing the effect of PreTHKS on THKSord, is estimated as 0.4033. The PreTHKS adjusted logit differences between  $CC = 1$  and  $CC = 0$  (keeping  $TV = 0$ ) is estimated as  $\hat{\beta}_2 = 0.9238$ , in contrast with the PreTHKS adjusted logit differences between  $TV = \text{yes}$  and  $TV = \text{no}$  (keeping  $CC = 0$ ) which is estimated as  $\hat{\beta}_3 = 0.2750$ . The coefficient  $\beta_4$  denotes the adjusted difference in logit attributable to the interaction between  $CC$  and  $TV$  ( $CC * TV$ ) and is estimated at  $-0.4659$ , which diminishes the combined effects of  $CC$  and  $TV$ .

### Logits for groups with no intervention

The first logit for groups with no intervention, for category 1 vs. categories 2 to 4, is  $\hat{\gamma}_1 = -0.0885$ . The second logit for the same group, for categories 1 and 2 vs. 3 and 4, can be calculated as  $\hat{\gamma}_2 = 1.1534$ . The third and final logit for this group, for categories 1 to 3 vs. 4,

is  $\hat{\gamma}_3 = 2.3319$ . All of the logits are adjusted for the effect of the pre-intervention score PreTHKS.

### Logits for groups with classroom curriculum intervention (CC = 1)

Turning to the groups with classroom curriculum intervention (CC = 1), logits can be obtained in similar fashion:

$$\hat{\gamma}_1 - \hat{\beta}_2 = -0.0885 - 0.9238 = -1.0123$$

$$\hat{\gamma}_2 - \hat{\beta}_2 = 1.1534 - 0.9238 = 0.2296, \text{ and } \hat{\gamma}_3 - \hat{\beta}_2 = 1.4081.$$

### Logits for groups with media intervention (TV = 1)

For the groups where media intervention was employed, the logits are:

$$\hat{\gamma}_1 - \hat{\beta}_3 = -0.3635, \hat{\gamma}_2 - \hat{\beta}_3 = 0.8784, \text{ and } \hat{\gamma}_3 - \hat{\beta}_3 = 2.0569.$$

### Estimated outcomes for groups: unit-specific results

To evaluate the expected effect of the CC, TV, CC\*TV, and PreTHKS variables we use the expression below:

$$\log \left[ \frac{\hat{P}(\text{THKSord}_{ij} \leq c)}{1 - \hat{P}(\text{THKSord}_{ij} \leq c)} \right] = \hat{\gamma}_c - \left[ \hat{\beta}_2 \text{CC}_i + \hat{\beta}_3 \text{TV}_i + \hat{\beta}_4 (\text{CC} * \text{TV})_i + \hat{\beta}_1 \text{PreTHKS}_{ij} \right]$$

The variable PreTHKS is treated as a continuous variable in this example, although it too is originally a scale score. In order to facilitate comparison of treatment groups, the mean PreTHKS score for groups can be used to obtain the logits. The mean PreTHKS scores for each of the four treatment groups were given in Table 6.3. This table is reproduced below.

**Table 6.4: Mean pre-intervention scores**

Study condition	Proportion
CC = 0, TV = 0	2.152
CC = 0, TV = 1	2.087
CC = 1, TV = 0	2.050
CC = 1, TV = 1	1.979

The probabilities for the responses of typical subjects from the group with no intervention (TV = CC = 0) can be obtained using the modified equation

$$\log \left[ \frac{\hat{P}(\text{THKSord}_{ij} \leq c)}{1 - \hat{P}(\text{THKSord}_{ij} \leq c)} \right] = \hat{\gamma}_c - [0.4033(2.1520)] \\ = \hat{\gamma}_c - 0.8679.$$

Let

$$\hat{\eta}_{ijc} = \log \left[ \frac{\hat{P}(\text{THKSord}_{ij} \leq c)}{1 - \hat{P}(\text{THKSord}_{ij} \leq c)} \right].$$

Similar equations for the groups with classroom curriculum intervention and media intervention respectively are then

$$\hat{\eta}_{ijc} = \hat{\gamma}_c - [+0.9238 + 0.4033(2.050)] \\ = \hat{\gamma}_c - 1.7506$$

and

$$\hat{\eta}_{ijc} = \hat{\gamma}_c - [0.2750 + 0.4033(2.087)] \\ = \hat{\gamma}_c - 1.1167.$$

When both intervention methods were employed and thus  $TV = CC = CC*TV = 1$ , we have

$$\hat{\eta}_{ijc} = \hat{\gamma}_c - [0.9238 + 0.2750 - 0.4659 + 0.4033(1.979)] \\ = \hat{\gamma}_c - 1.5310.$$

In this example, the logistic link function was specified, and we can rewrite any formula of the form  $\hat{\eta}_{ijc} = \gamma_c - a$  in the alternative form

$$\hat{P}(\text{THKSord}_{ij} \leq c) = \frac{e^{\gamma_c - a}}{1 + e^{\gamma_c - a}} = \frac{e^{\eta_{ijc}}}{1 + e^{\eta_{ijc}}}.$$

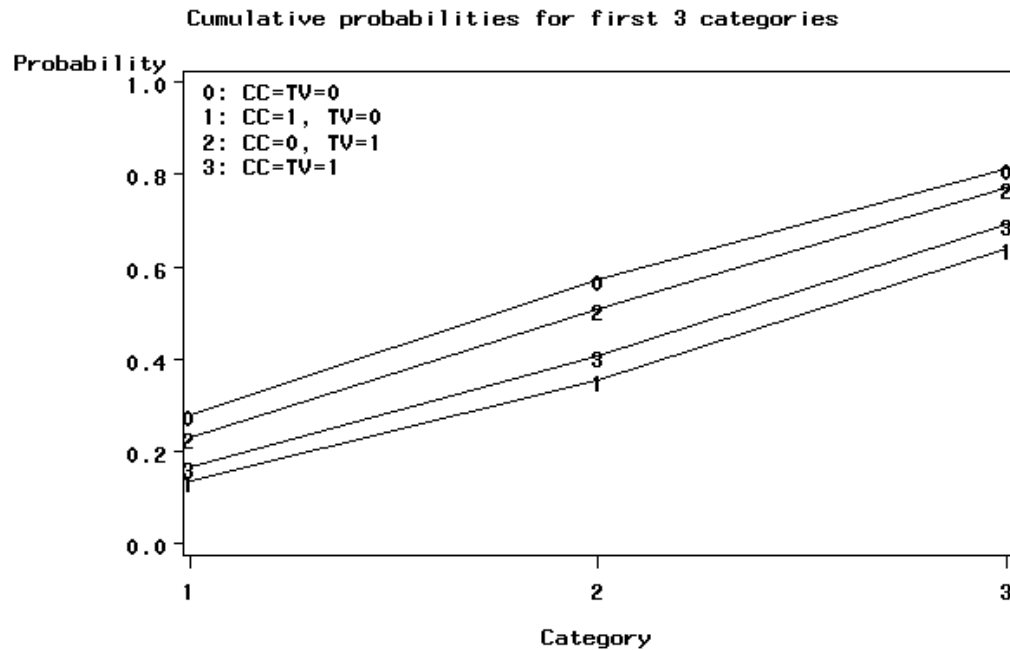
Table 6.5 contains the cumulative response probabilities obtained through substitution in the above formulae for the first three categories of the ordinal outcome THKSord.

The probabilities reported in Table 6.5 are cumulative: for example, the probability of a response in either category 1 or 2 for the group with  $CC = TV = 0$  is equal to 0.5709. The probability of a response in category 1 is 0.2776, and therefore the probability of a response in category 2 is  $0.5709 - 0.2776 = 0.2933$ . Similarly, the estimated response probability of a category 3 response for a respondent from the same group is  $0.8121 - 0.5709 = 0.2412$ . To obtain the category 4 response for a respondent from the first group, the value of the estimated response in categories 1, 2, or 3 has to be subtracted from 1, so that the probability of a response in category 4 for a typical respondent with  $CC = TV = 0$  is  $1 - 0.8121 = 0.1879$ .

The cumulative probabilities for the first 3 categories of the ordinal outcome are plotted in Figure 6.4.

**Table 6.5: Cumulative response probabilities for various groups and categories**

Category	CC	TV	$\hat{\eta}_{ijc} = \gamma_c - \mathbf{x}'\boldsymbol{\beta}$	Probability of response
1	0	0	-0.9564	0.2776
1	1	0	-1.8390	0.1372
1	0	1	-1.2052	0.2306
1	1	1	-1.6195	0.1653
1 or 2	0	0	0.2855	0.5709
1 or 2	1	0	-0.5972	0.3550
1 or 2	0	1	0.0367	0.5092
1 or 2	1	1	-0.3776	0.4067
1, 2 or 3	0	0	1.4640	0.8121
1, 2 or 3	1	0	0.5813	0.6414
1, 2 or 3	0	1	1.2152	0.7712
1, 2 or 3	1	1	0.8009	0.6902



**Figure 6.4: Cumulative response probabilities for categories 1 to 3 of THKSord**

The graph shows two groupings: one representing  $CC = 0$ , regardless of the value of  $TV$ ; and the other  $CC = 1$ , again regardless of the value of  $TV$ . The smallest probability to fall in categories other than category 1 (normal) is for the combination  $CC = TV = 1$ . The fact that the plotted cumulative probability lines for  $CC = 1$  and  $TV = 1$  are close to the line for  $CC =$

TV = 1 suggests that the implementation of media intervention (TV = 1 if implemented) has less impact on the outcome than the use of a classroom curriculum (CC = 1 if implemented).

To obtain category probabilities, differences between the cumulative probabilities obtained above are calculated. In other words,

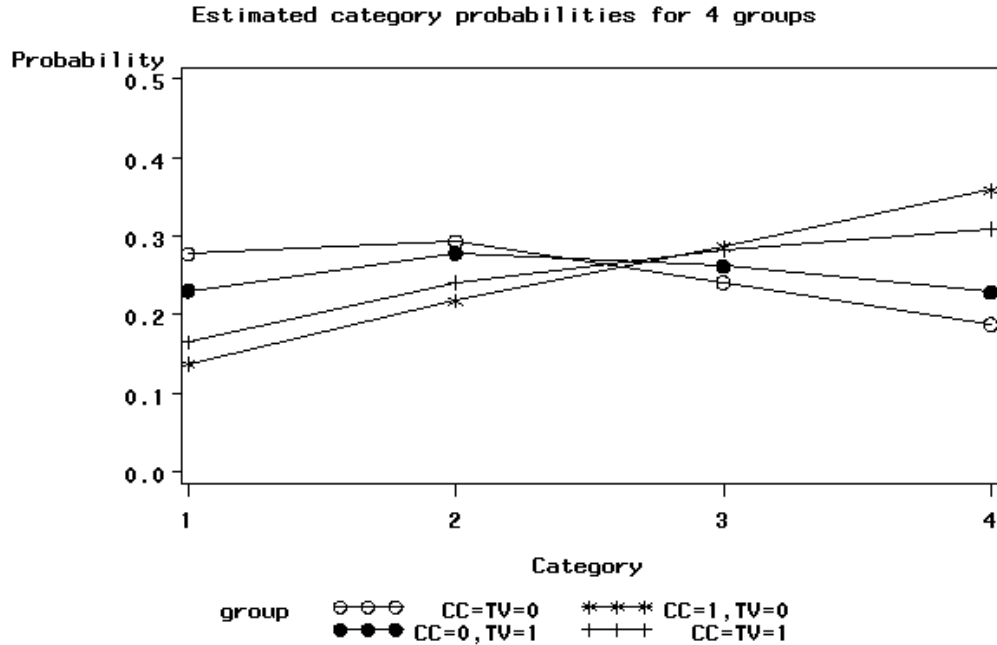
$$\hat{P}(\text{THKSord}_{ij} = c) = \hat{P}(\text{THKSord}_{ij} \leq c) - \hat{P}(\text{THKSord}_{ij} \leq c - 1)$$

The category probabilities are reported in Table 6.6 and are graphically displayed in Figure 6.5.

A typical respondent from the control group (no intervention) was less likely to respond in categories 3 or 4 of the ordinal post-intervention outcome variable. For both this group and the group which was assigned to media intervention only, the most likely response was in category 2 and the least likely response in category 4. In contrast, groups that were subjected to the classroom curriculum intervention, with or without media intervention, were most likely to display a high level of knowledge (*i.e.*, a response in categories 3 or 4), and least likely to respond in the first category. From this graph we conclude that the classroom curriculum intervention was key – groups subjected to the intervention tended to increase in knowledge over the study period.

**Table 6.6: Estimated unit-specific probabilities for THKSord categories**

Category	CC	TV	$\hat{P}(\text{THKSord}_{ij} = c)$
1	0	0	0.2776
	1	0	0.1372
	0	1	0.2306
	1	1	0.1653
2	0	0	0.2933
	1	0	0.2178
	0	1	0.2786
	1	1	0.2414
3	0	0	0.2413
	1	0	0.2864
	0	1	0.2620
	1	1	0.2835
4	0	0	0.1879
	1	0	0.3586
	0	1	0.2287
	1	1	0.3098



**Figure 6.5: Estimated category probabilities for THKSord**

### Estimated outcomes for different groups: population-average results

In the introduction to this section, we defined the latent response variable model as

$$y_{ij} = \mathbf{z}'_{(1)ij} \mathbf{b}_i + \mathbf{x}'_{(1)ij} \boldsymbol{\beta}_{(1)} + e_{ij},$$

making the assumption that  $e_{ij} : i.i.d.(0, \sigma^2)$ . For a probit link function  $\sigma_e^2 = 1$ , and for a logistic link function it is assumed that  $\sigma_e^2 = \pi^2 / 3$ , as indicated in the final lines of the output file. Under the assumption that  $\mathbf{v}_i$  and  $e_{ij}$  are independently distributed, it follows that

$$\sigma_{y_{ij}}^2 = \mathbf{z}'_{ij} \boldsymbol{\Phi} \mathbf{z}_{ij} + \sigma_e^2.$$

Let

$$d_{ij} = \frac{\sigma_{y_{ij}}^2}{\sigma_e^2}.$$

The quantity  $d_{ij}$  is called the design effect. The estimated population-average probabilities (Hedeker & Gibbons, 2006) are obtained in a similar fashion as the unit-specific probabilities, but replacing  $\hat{\eta}_{ijc}$  with  $\hat{\eta}_{ijc}^* = \hat{\eta}_{ijc} / \sqrt{d_{ij}}$ .



From the output, we have  $\text{var}(u_{i0}) = 0.074$ , where  $u_{i0}$  denotes the random intercept coefficient. In this case,  $\mathbf{z}'_{ij} = \mathbf{1}$  and hence, with  $\sigma_e^2 = \pi^2 / 3$  for the logistic link,

$$\sigma_{y_{ij}}^2 = 1 \times 0.074 + (3.1416)^2 / 3 = 3.3639.$$

Therefore

$$d_{ij} = \frac{3.3639}{3.2899} = 1.0225.$$

To obtain the population-average probability estimates, we now replace the  $\eta_{ijc}$  values calculated for the unit-specific case with  $\hat{\eta}_{ijc}^* = \hat{\eta}_{ijc} / \sqrt{d_{ij}}$ . We can compare these estimated population-average probabilities with the observed data for the four groups formed by the categories of TV and CC as shown in Table 6.5 previously. Table 6.7 shows the estimated population-average probabilities.

A comparison of these probabilities with those reported in Table 6.5 shows little difference between the unit-specific and population-average category probabilities for treatment groups. The population-average category probabilities for the first two categories are slightly smaller than the corresponding unit-specific probabilities, while those for category 3 are slightly larger. The extent of differences between unit-specific and population-average results are highly dependent on the "scaling" induced by dividing the  $\eta_{ijc}$ s by  $\sqrt{d_{ij}}$ . In the current example,  $\sqrt{d_{ij}} = 1.0112$  and thus no large differences could be expected. To obtain category probabilities, differences between the cumulative probabilities may be calculated, as illustrated in the case of the unit-specific results.

**Table 6.7: Cumulative response probabilities for various groups and categories**

Category	CC	TV	$\hat{\eta}_{ijc}^* = \hat{\eta}_{ijc} / \sqrt{d_{ij}}$	Probability of response
1	0	0	-0.9564/1.0112	0.2797
1	1	0	-1.8391/1.0112	0.1396
1	0	1	-1.2052/1.0112	0.2330
1	1	1	-1.6195/1.0112	0.1678
1 or 2	0	0	0.2855/1.0112	0.5701
1 or 2	1	0	-0.5972/1.0112	0.3565
1 or 2	0	1	0.0367/1.0112	0.5092
1 or 2	1	1	-0.3776/1.0112	0.4077
1, 2 or 3	0	0	1.4640/1.0112	0.8097
1, 2 or 3	1	0	0.5813/1.0112	0.6399
1, 2 or 3	0	1	1.2152/1.0112	0.7689
1, 2 or 3	1	1	0.8009/1.0112	0.6883

