Two-level crossed and nested model (HCM2)

Within-Cell Model

$$Y_{ijk} = \sum_{p=0}^{P-1} a_{pijk}\pi_{pjk} + e_{ijk} \tag{1}$$

Typically, $a_{0ijk} = 1$ for all *i*,*j*,*k*.

Between-Cell Model

$$\begin{aligned}\pi_{pjk} = \theta_p &+ \sum_{q_p=1}^{Q_p}(\gamma_{pq_p} + c_{pq_pk})W_{pq_pj} + \sum_{s_p=1}^{S_p}(\beta_{ps_p} + b_{ps_pj})X_{ps_pk} \\ &+ \sum_{p=1}^{P}\sum_{r_p=1}^{R_p}\delta_{pr_p}H_{pr_pjk} + b_{p0j} + c_{p0k}\end{aligned} . \tag{2}$$

- Note there are P equations in the between-cell model. Any random term c_{pq_pk}, b_{ps_pj} may be constrained to be zero.
- The number of row-level predictors across all equations having fixed row intercepts is Q^F. The number of column-level predictors across all equations having fixed column intercepts is S^F.
- The total number of row-by-column predictors in H is $R = \sum_{p=1}^{P} R_p$.
- The total number of random row effects (including intercepts) is JQ^r.
- The total number of random column effects (including intercepts) is KS^r.

Degrees of Freedom

1. For any γ_{pq_p} in an equation having a random row intercept b_{p0j} ,df= $J - Q_p - 1$.
2. For any β_{ps_p} in an equation having a random row intercept c_{p0k}, df= $K - S_p - 1$.
3. For all other coefficients, df= $N - JQ^R - KS^r - Q^F - S^F - R$.