Scientific Software International

Three-level crossed and nested models (HCM3)

Within-Cell Model

$$Y_{ijkl} = \sum_{p=0}^{P-1} a_{pijkl}\pi_{pjkl} + e_{ijkl} \tag{1}$$

Typically $a_{0ijkl} = 1$ for all *i,j,k,l*.

Between-Cell Model (Within Clusters)

$$\pi_{pjkl} = \theta_{p0l} + \sum_{q_p=1}^{Q_p}(\theta_{pq_pl} + c_{pq_pkl})W_{pq_pj} + \sum_{s_p=1}^{S_p}(\theta_{ps_pl} + b_{ps_pj})X_{ps_pkl} + \sum_{p=1}^{P}\sum_{r_p=1}^{R_p}\theta_{pr_pl}H_{ps_pjkl} + b_{p0j} + c_{p0kl}$$

Between Clusters

$$\theta_{pv_pl} = \delta_{pv_p0} + \sum_{g_{pv_p}=1}^{G_{pv_p}-1}(\delta_{pv_pg_{pv_p}} + b_{pv_pj})D_{pv_pgl} + d_{pv_p0l}, \tag{2}$$

for $v = q, s,\ or\ r.$

Define G^r as the number of random effects per cluster.

Degrees of Freedom

1. Consider first an equation pv_p in (2) in which there is random intercept, that is, d_{pv_p0l} is not constrained to be zero. The degrees of freedom for a fixed effect $\delta_{pv_pg_{pv_p}}$ in that equation will be $L - G_{pv_p} - 1$, assuming the model intercept δ_{pv_p0} is retained in the model. If the model intercept is not retained, df= $L - G_{pv_p}$.

2. Now consider an equation pv_p in (2) in which there is no random intercept, that is, $d_{pv_p 0l}$ is constrained to be zero. Suppose that *pv*=*pq* and b_{p0j} is not constrained to be zero. Then df= $J - LG^r - Q_p - 1$.
3. Again consider an equation pv_p in (2) in which there is no random intercept, that is, $d_{pv_p 0l}$ is constrained to be zero. Suppose that *pv*=*ps* and c_{p0kl} is not constrained to be zero. Then df= $K - LG^r - S_p - 1$.
1. All other cases: df= $N - LG^r - JQ^r - KS^r - F^*$, where G^r is the number random cluster effects, Q^r is the number of random row effects, S^r is the number of random column effects, and F^* is the total number of fixed effects not covered by cases 1-3.