Two-level models (HLM2)

Conventional Representation

At level 1, we have

$$Y_j = X_j^R \beta_j^R + X_j^F \beta_j^F + r_j . \qquad (1)$$

Here

- Y_j is the n_j by 1 outcome vector,
- X_j^R is the n_j by P^R matrix of predictors having random coefficients,
- β_j^R is the P^R by 1 vector of level-1 coefficients that will be random,
- X_j^F is the n_j by P^F matrix of predictors having fixed coefficients,
- β_j^F is the P^F by 1 vector of level-1 coefficients that will be fixed, and
- r_j is the n_j by 1 vector of level-1 random errors.

At level-2, the model is

$$\begin{aligned} \beta_j^R &= W_j^R \gamma^R + u_j \\ \beta_j^F &= W_j^F \gamma^F \end{aligned} \qquad (2)$$

where

- W_j^R is the P^R by Q^R matrix of level-2 predictors of the level-1 coefficients having random effects;
- γ^R is the Q^R by 1 vector of fixed effects in the model for the level-1 coefficients having random effects;
- u_j is the P^R by 1 vector of level-2 random effects;
- W_j^F is the P^F by Q^F matrix of level-2 predictors of the level-1 coefficients having fixed effects; and

- γ^F is the Q^F by 1 vector of fixed effects in the model for the level-1 coefficients having fixed effects.

Revised Representation

The logic of allocating degrees of freedom is easier to follow if we represent the model as follows:

$$Y_j = X_j^R(W_j^R\gamma^R + u_j) + X_j^F W_j^F \gamma^F + r_j \tag{3}$$

The elements of β_j^R are $\beta_{p^R j}^R$, for $p^R = 1,\ldots,P^R$. The elements of γ^R are $\gamma_{p^R q_{p^R}}$ for $q_{p^R} = 1,\ldots,Q_{p^R}$ where Q_{p^R} is the number of level-2 coefficients in the model for $\beta_{p^R j}^R$ and $\sum_{p^R=1}^{P^R} Q_{p^R} = Q^R$ is the dimension of γ^R.

The elements of β_j^F are $\beta_{p^F j}^F$, for $p^F = 1,\ldots,P^F$. The elements of γ^F are $\gamma_{p^F q_{p^F}}$ for $q_{p^F} = 1,\ldots,Q_{p^F}$ where Q_{p^F} is the number of level-2 coefficients in the model for $\beta_{p^F j}^F$ and $\sum_{p^F}^{P^F} Q_{p^F} = Q^F$ is the dimension of γ^F.

Degrees of Freedom

1. For any element $\gamma_{p^R q_{p^R}}$ of γ^R, the degrees of freedom are $J - Q_{p^R}$.
2. For any element $\gamma_{p^F q_{p^F}}$ of γ^F, the degrees of freedom are $N - JP^R - Q^F$.