Scientific Software International

Four-level models (HLM4)

Level-1 Model

$$Y = A^{R}\pi^{R} + A^{F}\pi^{F} + e \tag{1}$$

Level-2 Model

$$\begin{aligned} \pi^{R} &= X^{RR}\beta^{RR} + X^{RF}\beta^{RF} + r \\ \pi^{F} &= X^{FR}\beta^{FR} + X^{FF}\beta^{FF} \end{aligned} \tag{2}$$

Level-3 Model

$$\begin{aligned} \beta^{RR} &= W^{RRR}\gamma^{RRR} + W^{RRF}\gamma^{RRF} + u^{RR} \\ \beta^{RF} &= W^{RFR}\gamma^{RFR} + W^{RFF}\gamma^{RFF} \\ \beta^{FR} &= W^{FRR}\gamma^{FRR} + W^{FRF}\gamma^{FRF} + u^{FR} \\ \beta^{FF} &= W^{FFR}\gamma^{FFR} + W^{FFF}\gamma^{FFF} \end{aligned} \tag{3}$$

Level-4 Model

$$\begin{aligned} \gamma^{RRR} &= G^{RRR}\delta^{RRR} + v^{RRR} \\ \gamma^{RRF} &= G^{RRF}\delta^{RRF} \\ \gamma^{RFR} &= G^{RFR}\delta^{RFR} + v^{RFR} \\ \gamma^{RFF} &= G^{RFF}\delta^{RFF} \\ \gamma^{RFR} &= G^{RFR}\delta^{RFR} + v^{RFR} \\ \gamma^{RFF} &= G^{RFF}\delta^{RFF} \\ \gamma^{FFR} &= G^{FFR}\delta^{FFR} + v^{FFR} \\ \gamma^{FFF} &= G^{FFF}\delta^{FFF} \end{aligned} \tag{4}$$

Revised representation

$$
\begin{aligned}
Y = {} & A^{R}\{X^{RR}[W^{RRR}(G^{RRR}\delta^{RRR} + v^{RRR}) + W^{RRF}(G^{RRF}\delta^{RRF}) + u^{RR}] \\
& + X^{RF}[W^{RFR}(G^{RFR}\delta^{RFR} + v^{RFR}) + W^{RFF}(G^{RFF}\delta^{RFF})] + r\} \\
& A^{F}\{X^{FR}[W^{FRR}(G^{FRR}\delta^{FRR} + v^{FRR}) + W^{FRF}(G^{FRF}\delta^{FRF})] + u^{FR}] \\
& + X^{FF}[W^{FFR}(G^{FFR}\delta^{FFR} + v^{FFR}) + W^{FFF}(G^{FFF}\delta^{FFF})]\} + e
\end{aligned}
\tag{5}
$$

Degrees of Freedom

1. For an element of $\delta^{RRR}, \delta^{RFR}, \delta^{FRR}, or\ \ \delta^{FFR}$, we have

$$
\begin{aligned}
DF(\delta^{RRR}) &= LS^{RRR} - f(specific\ equation\ within\ \ \delta^{RRR}) \\
DF(\delta^{RFR}) &= LS^{RFR} - f(specific\ equation\ within\ \ \delta^{RFR}) \\
DF(\delta^{FRR}) &= LS^{FRR} - f(specific\ equation\ within\ \ \delta^{FRR}) \\
DF(\delta^{FFR}) &= LS^{FFR} - f(specific\ equation\ within\ \ \delta^{FFR})
\end{aligned}
\tag{6}
$$

where L is the number of level-4 units and $S^{RRR}, S^{RFR}, S^{FRR}, S^{FFR}$ are the number of random effects, in $v^{RRR}, v^{RFR}, v^{FRR}, v^{FFR}$ per level-4 unit, respectively and "*f(specific equation)*" is the number of fixed effects in a specific scalar equation within one of the fixed effects vectors.

2. For an element of $\delta^{RRF} or\ \delta^{FRF}$, we have

$$
\begin{aligned}
DF(\delta^{RRF}) &= K - LS^{RRR} - f^{RRF} \\
DF(\delta^{FRF}) &= K - LS^{RRR} - f^{FRF}
\end{aligned}
\tag{7}
$$

where K is the total number of level-3 units, and f^{RRF}, f^{FRF} are, respectively, the total number of fixed effects in $\delta^{RRF}, \delta^{FRF}$ per level-3 unit.

3. For an element of δ^{RFF}, we have

$$
DF(\delta^{RFF}) = J - L(S^{RRR} + S^{RFR}) - KQ^{RR} - f^{RFF}, \tag{8}
$$

where J is the total number of level-2 units and Q^{RR} is the number of random effects per level-3 unit.

4. For an element of δ^{FFF},

$$
DF(\delta^{FFF}) = N - JP^{R} - L(S^{RRR} + S^{RFR}) - KQ^{RR} - f^{FFF} \tag{9}
$$

where N is the total number of level-1 units and P^R is the number of random effects per level-2 unit.