Cross-classified random effects model (HLMHCM)

Within-Cell Model: Level-1

$$Y_{mijk} = \sum_{g=0}^{G-1} f_{gmijk} \psi_{gijk} + e_{mijk} \quad (1)$$

Typically, $\psi_{0ijk} = 1$ for all *i*,*j*,*k*.

Within-Cell Model: Level-2

$$\psi_{gijk} = \sum_{p_g=0}^{P_g-1} a_{p_g ijk} \pi_{p_g jk} + e_{gijk} \quad (2)$$

Between-Cell Model

$$\pi_{p_g jk} = \theta_{p_g} + \sum_{q_{p_g}=1}^{Q_{p_g}} (\gamma_{p_g q_{p_g}} + c_{p_g q_{p_g} k}) W_{pq_{p_g} j} + \sum_{s_{p_g}=1}^{S_{p_g}} (\beta_{p_g s_{p_g}} + b_{p_g s_{p_g} j}) X_{p_g s_{p_g} k}$$
$$+ \sum_{p_g=1}^{P_g} \sum_{r_{p_g}=1}^{R_{p_g}} \delta_{p_g r_{p_g}} H_{p_g r_{p_g} jk} + b_{p_g 0j} + c_{p_g 0k}. \quad (3)$$

Note there are $P = \sum_{g=1}^{G} P_g$ equations in the between-cell model. Any random term $c_{p_g q_{p_g} k}, b_{p_g s_{p_g} j}$ may be constrained to be zero.

- The number of row-level predictors across all equations having fixed row intercepts in the case where e_{gijk} is not constrained to zero is Q^{RF}. The number of row-level predictors across all equations having fixed row intercepts in the case where e_{gijk} is constrained to zero is Q^{FF} .

- The number of column-level predictors across all equations having fixed column intercepts in the case where e_{gijk} is not constrained to zero is S^{RF}. The number of column-level predictors across all equations having fixed row intercepts in the case where e_{gijk} is constrained to zero is S^{FF} .
- The total number of row-by-column predictors in H is $R = \sum_{p=1}^{P} \sum_{R_{p_g}=1}^{R_p} R_{p_g}$.
- The total number of random row effects (including intercepts) is JQ^r.
- The total number of random column effects (including intercepts) is KS^r.
- The total number of level-2 random effects (within cells) is NP^r .

Degrees of Freedom

1. For any $\gamma_{p_g q_{p_g}}$ in an equation having a random row intercept $b_{p_g 0j}$,df= $J - Q_{p_g} - 1$.

2. For any $\beta_{p_g s_{p_g}}$ in an equation having a random row intercept $c_{p_g 0k}$, df=$K - S_{p_g} - 1$.

3. For any $\gamma_{p_g q_{p_g}}$ in an equation for which the row level intercept $b_{p_g 0j}$ is constrained to be zero, but where the level-2 random effect e_{gijk} is not constrained to be zero, df= $N - JQ^r - KS^r - Q^{RF} - S^{RF} - R$.

4. For any $\beta_{p_g q_{p_g}}$ in an equation for which the column-level level intercept $c_{p_g 0k}$ is constrained to be zero, but where the level-2 random effect e_{gijk} is not constrained to be zero, the same result holds as in (4): df=$N - JQ^r - KS^r - Q^{RF} - S^{RF} - R$.

5. For all other fixed coefficients df=$M - NP^r - JQ^r - KS^r - F^*$, where F^* is the total number of fixed effects not covered by cases 1-4 and M is the grand total number of level-1 observations.