Scientific Software International

Effect sizes

1. Introduction

Fixed effects are dependent on the scale of the dependent or outcome variable. Comparison of these results over studies, or among multiple variables within a study, may be problematical as a result. A number of measures have been suggested over time to address this problem.

The simplest way of standardizing coefficients is to standardize them prior to analysis. This is done by rescaling them so that the mean of the variable in question is equal to zero, and the standard deviation equal to 1. Standardizing effects after an analysis using unstandardized variables is another alternative, suing the standard deviation of the variable in question and the standard deviation of the outcome variable to do so.

Another option is to use Cohen's d, which is an effect size used to indicate the standardized difference between two means. It can be used, for example, to accompany reporting of t-test and ANOVA results. It is also widely used in meta-analysis. Cohen's d is an appropriate effect size for the comparison between two means. It is defined as

$$d = \frac{m1 - m2}{\sqrt{\left[\left(s1^2 + s2^2\right)/2\right]}}$$

Where $m1$ is the mean of the first group, $m2$ the mean of the second group, and $\sqrt{\left[\left(s1^2 + s2^2\right)/2\right]}$ the pooled standard deviations for the two groups. However, if we should calculate this measure based on the level-1 model with no covariates, it may not be a good measure as it would not be controlling for level-2 unit membership and other associated covariates.

The ever popular R^2 is another option. However, interpreting that in a multilevel model is considerably more complex that in the case of a single level linear model, especially when random slopes are introduced into the model. Snijders and Bosker (2012) formulated an R^2 to be used in multilevel context as

$$R^2 = 1 - \frac{\sigma_F^2 + \tau_F^2}{\sigma_E^2 + \tau_E^2}$$

Where σ_F^2 represents the level-1 random error variance and τ_F^2 the ;eve;=2 random error variance for the full model containing the effect of interest. Similarly, σ_E^2 and τ_E^2 represent the error variances for the unconditional model. This statistic is based om the proportional reduction in prediction error at the individual level.

Aiken and West (1991) suggested using the effect size

$$f^2 = \frac{R_2^2 - R_1^2}{1 - R_2^2}$$

Where R_2^2 represents the variance explained for a model with the given effect, and R_1^2 the variance explained for the model without the effect. It is considered to be small at a value of 0.02, medium at a value of 0.15, and large at a value of 0.35 (Cohen, 1992).

We now take a look at how to obtain these effects for models based on the HSB data.

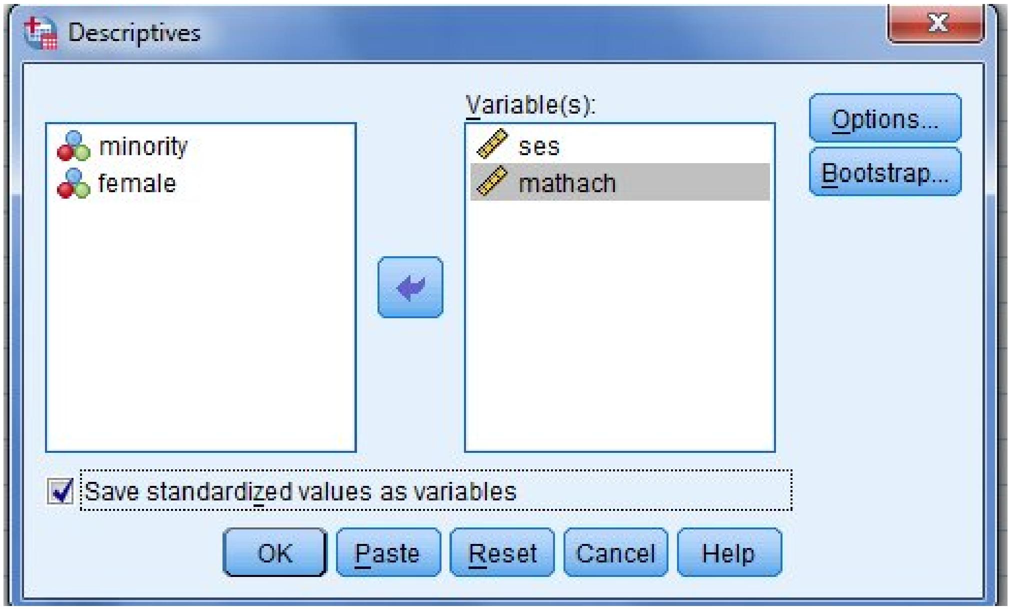
2. Standardizing variables prior to analysis

The variables of interest for the purposes of this illustration are the outcome variable MATHACH and the level-1 predictor variable SES. Descriptive statistics for these are provided below.

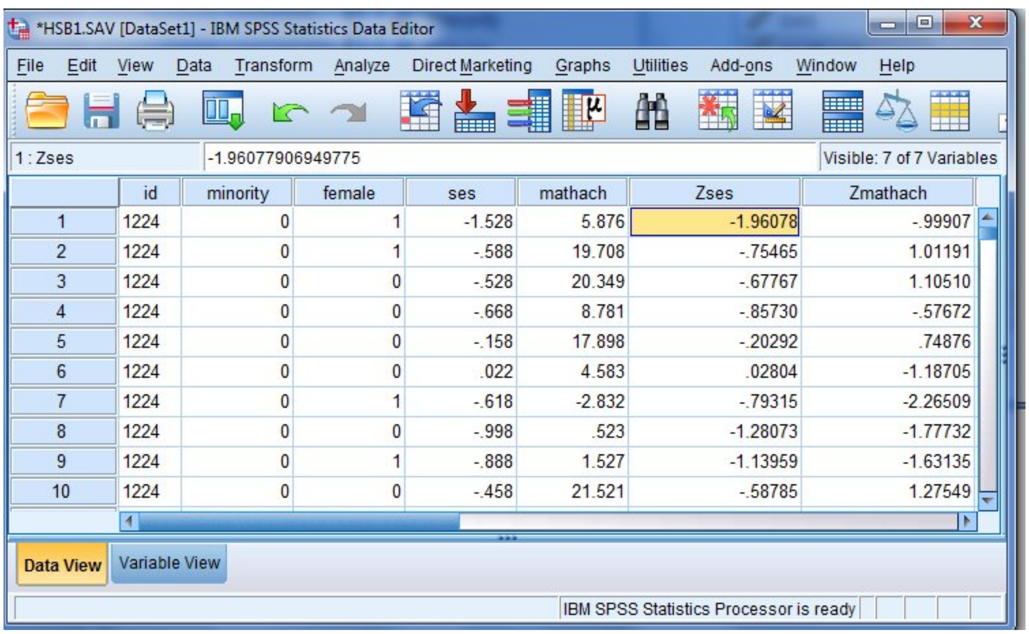
Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
ses	7185	-3.758	2.692	.00014	.779355
mathach	7185	-2.832	24.993	12.74785	6.878246
Valid N (listwise)	7185				

Standardizing these variables prior to creating the MDM is done in the stat package of choice. Here we use SPSS to do it. Request descriptive statistics for the variables to be used and check the **Save standardized values as variables** check box.



The standardized variables, with a z" prefix, are shown as two new additional variables in the spreadsheet.

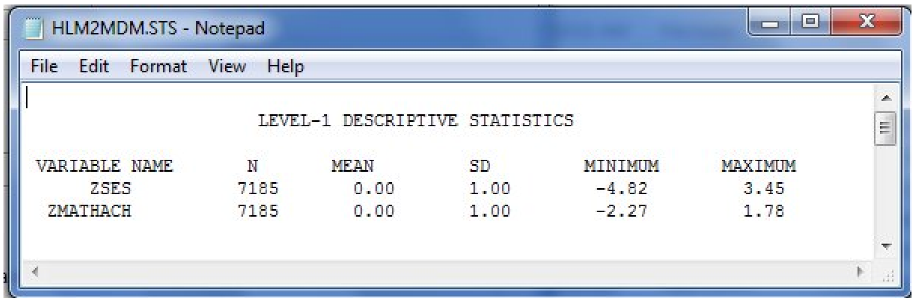


Verify their means and standard deviations by again running descriptive statistics on the new variables (remember to uncheck the **Save standardized values as variables** check box). Descriptive statistics for the standardized variables are shown below.

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Zscore(ses)	7185	-4.82212	3.45395	.0000000	1.00000000
Zscore(mathach)	7185	-2.26509	1.78027	.0000000	1.00000000
Valid N (listwise)	7185				

Note that the descriptive statistics obtained in SPSS duplicate the results for the same variables in the HLM2MDM.STS file obtained for the MDM file made with the 2 standardized variables.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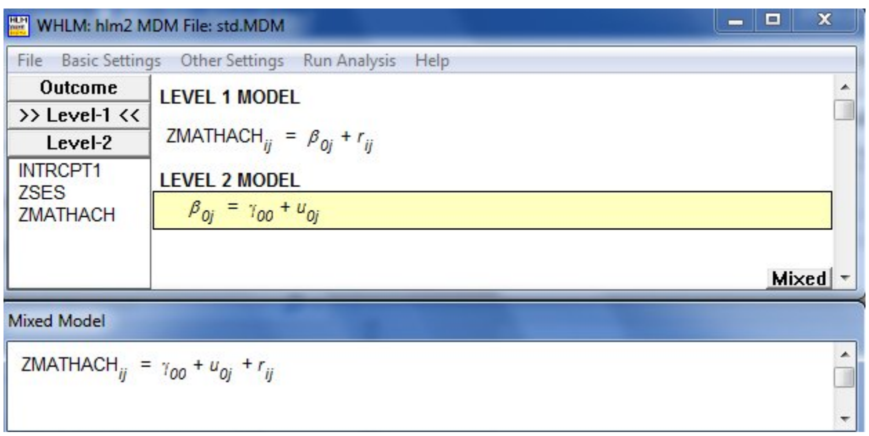
LEVEL-1 DESCRIPTIVE STATISTICS

VARIABLE NAME	N	MEAN	SD	MINIMUM	MAXIMUM
ZSES	7185	0.00	1.00	-4.82	3.45
ZMATHACH	7185	0.00	1.00	-2.27	1.78

An unconditional model is fitted first:



Final results for this model are shown below:

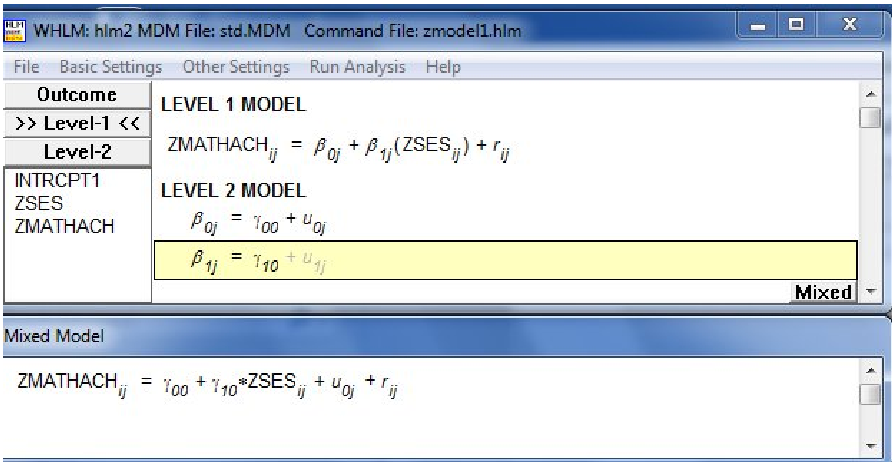
Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	-0.016121	0.035420	-0.455	159	0.650

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	0.42671	0.18208	159	1660.23259	<0.001
level-1, r	0.90966	0.82748			

The second model fitted is a random intercept only model



for which the following results were obtained:

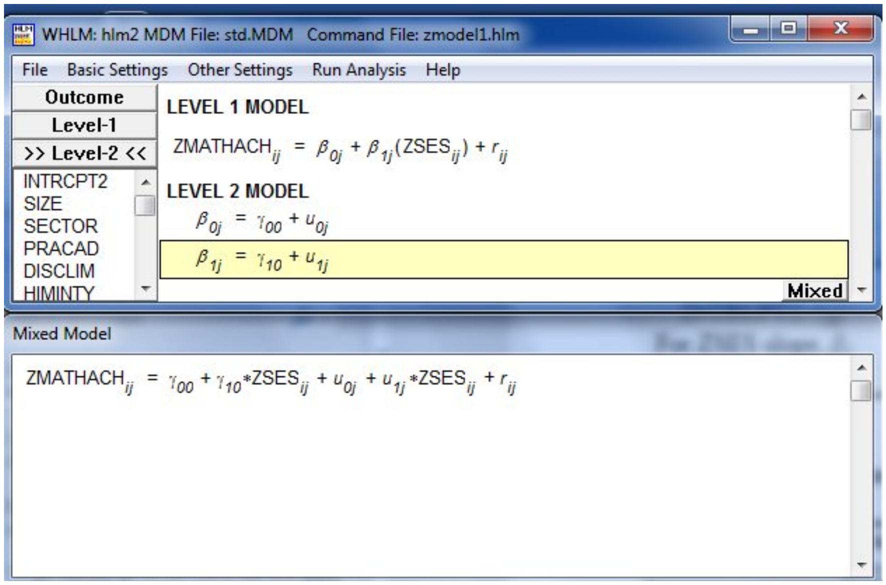
Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	-0.013089	0.027235	-0.481	159	0.631
For ZSES slope, β_1					
INTRCPT2, γ_{10}	0.270827	0.013519	20.034	7024	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	0.31747	0.10078	159	1037.09077	<0.001
level-1, r	0.88476	0.78280			

Finally, a random intercept-and-slope model is fitted to these data.



Output for this model was as follows:

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	-0.012005	0.027514	-0.436	159	0.663
For ZSES slope, β_1					
INTRCPT2, γ_{10}	0.271244	0.013336	20.339	159	<0.001

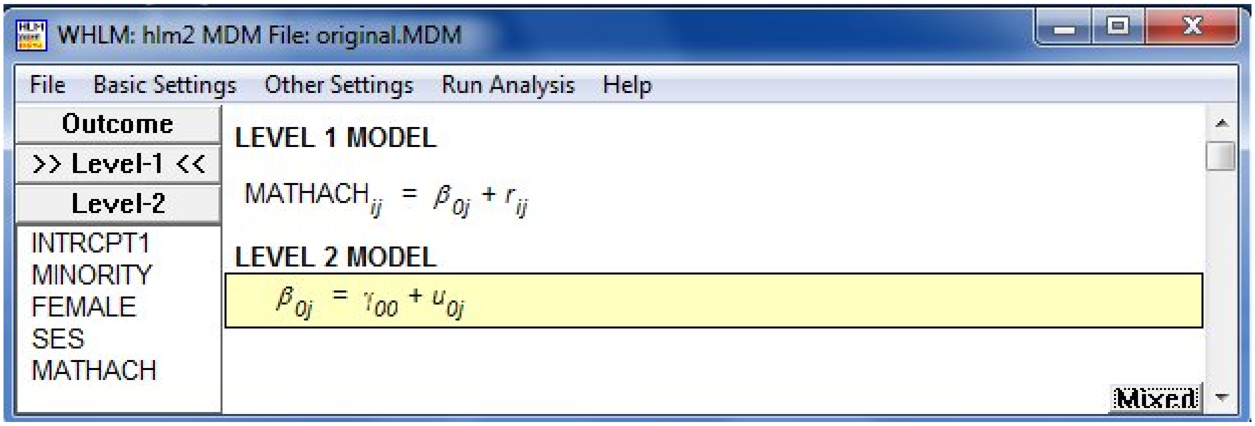
Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	0.31951	0.10209	159	905.25396	<0.001
ZSES slope, u_1	0.07328	0.00537	159	216.21176	0.002
level-1, r	0.88229	0.77844			

As these models are based on standardized variables, all effects are already standardized as well.

3. Standardizing variables after analysis

The same three models are now fitted to the unstandardized data. An unconditional model is fitted first.



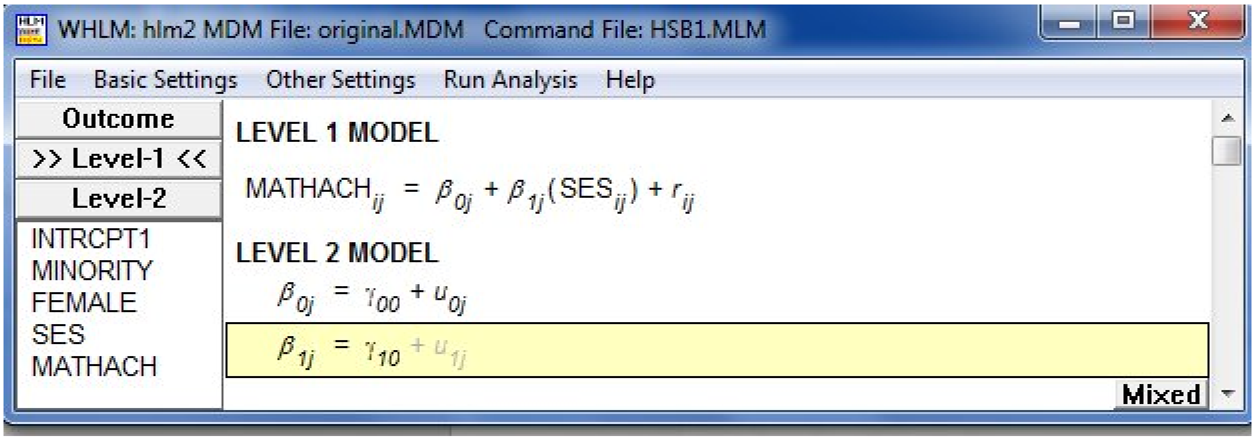
Final estimation of fixed effects (with robust standard errors)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. $d.f.$	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.636972	0.243628	51.870	159	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	$d.f.$	χ^2	p-value
INTRCPT1, u_0	2.93501	8.61431	159	1660.23259	<0.001
level-1, r	6.25686	39.14831			

This is followed by the fitting of a random slopes model



Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.657481	0.187330	67.568	159	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.390199	0.119309	20.034	7024	<0.001

Final estimation of variance components

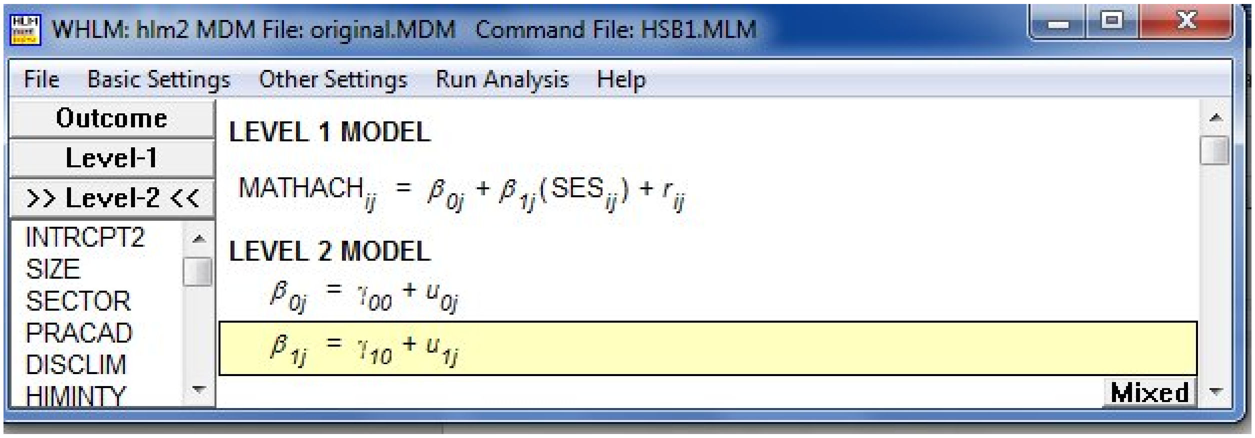
Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	2.18361	4.76815	159	1037.09077	<0.001
level-1, r	6.08559	37.03440			

The standardized effect of the variable SES is obtained by multiplying the estimated coefficient by the standard deviation of the outcome variable MATHACH and dividing it by the standard deviation of the variable itself:

$$\hat{\gamma}^{*}_{10} = \frac{\hat{\gamma}_{10} \times 0.779355}{6.878246} = 0.2708$$

The standardized SES effect obtained is identical to that obtained in the similar model using the pre-analysis standardized variables.

Finally, a random-intercept-and-slope model is fitted.



Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. $d.f.$	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.664935	0.189251	66.921	159	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.393878	0.117697	20.339	159	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	$d.f.$	χ^2	p-value
INTRCPT1, u_0	2.19768	4.82978	159	905.26472	<0.001
SES slope, u_1	0.64675	0.41828	159	216.21178	0.002
level-1, r	6.06864	36.82835			

The standardized effect of the variable SES is obtained by multiplying the estimated coefficient by the standard deviation of the outcome variable MATHACH and dividing it by the standard deviation of the variable itself:

$$\hat{\gamma}^{*}_{10} = \frac{\hat{\gamma}_{10} \times 0.779355}{6.878246} = 0.271244$$

Again, the result corresponds to the estimated SES effect in the similar model based on standardized data.

4. Calculating R^2 and f^2

Final results for the variance components under the models fitted to the unstandardized data in the previous section are summarized in the table below:

Effect	Unconditional model	Random intercept model	Random intercept and slope model
σ^2	39.14831	37.03440	36.82835
$\text{var}(u_{0j}) = \tau_0^2$	8.61431	4.76815	4.2978
$\text{var}(u_{1j}) = \tau_1^2$			0.41828
$\text{cov}(u_{0j}, u_{1j})$			-0.15399

We start by calculating R^2 as

$$
\begin{aligned}
R^2 &= 1 - \frac{\sigma_F^2 + \tau_F^2}{\sigma_E^2 + \tau_E^2} \\
&= 1 - \frac{37.03440 + 4.76815}{39.14831 + 8.61431} \\
&= 1 - 0.8752 \\
&= 0.1248
\end{aligned}
$$

The proportional reduction in prediction error at level-1 due to the inclusion of the variable SES is thus estimated at approximately 12.5%.

To calculate f^2, we need the values of R^2 for the model with the effect, and the model without the effect.

$$
\begin{aligned}
f^2 &= \frac{R^2}{1 - R^2} \\
&= \frac{0.1248}{1 - 0.1248} \\
&= 0.1425
\end{aligned}
$$

Based on this, we conclude that the model that includes the predictor SES explains 14.25% of the variance in MATHACH relative to the unexplained variance in MATHACH. According to Cohen (1992), a small effect is 0.02, a medium effect is 0.15, and a large effect is 0.35. The present effect is medium in size.

5. Conclusion

It should be noted that the addition of a random slope, as is present in the third of the models fitted above, complicates calculation of effects such as these. While the total variation at level-2 in the first two models can be expressed as

$$\textbf{total variation}(level2) = \text{var}\left(u_{0j}\right) = +\tau_0^{\ 2}$$

the similar expression for the third model would be

$$\begin{aligned}\textbf{total variation}(level-2) &= \text{var}\left(u_{0j}\right) + \text{var}\left(u_{1j}\right) + 2SES_{ij}\,\text{cov}\left(u_{0j}, u_{1j}\right)\\ &= \sigma^2 + \tau_0^2 + \tau_1^2 + 2SES_{ij}\,\text{cov}\left(u_{0j}, u_{1j}\right)\\ &= 36.82835 + 4.82978 + 0.41828 + 2(-0.15399)SES_{ij}\\ &= 42.0764 - \left(0.30798\right)SES_{ij}\end{aligned}$$

The expression is dependent on the value of the level-1 predictor SES and it is not longer simple to obtain a general expression for the total variation in the outcome. In the case of an indicator variable, two solutions exist; for a continuous predictor such as SES solutions exist for each possible value of SES. The graph below shows the values of total level-2 variation (var2) as a function of the SES values observed in the data.