Scientific Software International

Level-1 variance: homogeneous or heterogeneous?

Implicit in hierarchical linear models are assumptions concerning the distributions of the error or residual terms at the various levels of the hierarchy. In most cases, it is assumed that the errors in the level-1 model are normally distributed with expected mean zero and equal variance. Violation of this assumption is not without consequences: if the level-1 variances are assumed to be equal but are really unequal, the point estimation of the level-2 coefficients will not be biased. However, these estimates will be inefficient and the associated standard errors will be biased. As such, it is advisable to check the validity of the assumptions after model specification by performing a test for the homogeneity of the level-1 variances.

Causes of heterogeneous level-1 variance

Heterogeneity of the level-1 variances may have several causes:

- The omission of one or more important level-1 variables. Heterogeneity of variance would result if the excluded variable were distributed with unequal variance across groups.
- Fixing or omitting the effects of a level-1 predictor that is random or non- randomly varying.
- Bad data. Extreme data values due to, for example, bad coding, may inflate the variance for a group, and overall significant heterogeneity of variance may consequently be observed.
- The use of non-normal data with heavy tails. Parametric tests are sensitive to the presence of more extreme observations than expected, and heavy tails (kurtosis) can cause a significant test for heterogeneity of variance.

A full discussion of the process of testing for heterogeneity using HLM and the residual files produced by the program is given on pages 207-211 of the Bryk & Raudenbush text. In short, the test is based on the standardized measure of dispersion for each group:

$$d_j = \frac{\ln\left(S_j^2\right) - \left[\sum f_j \ln\left(S_j^2\right) / \sum f_j\right]}{\left(2/f_j\right)^{1/2}}$$

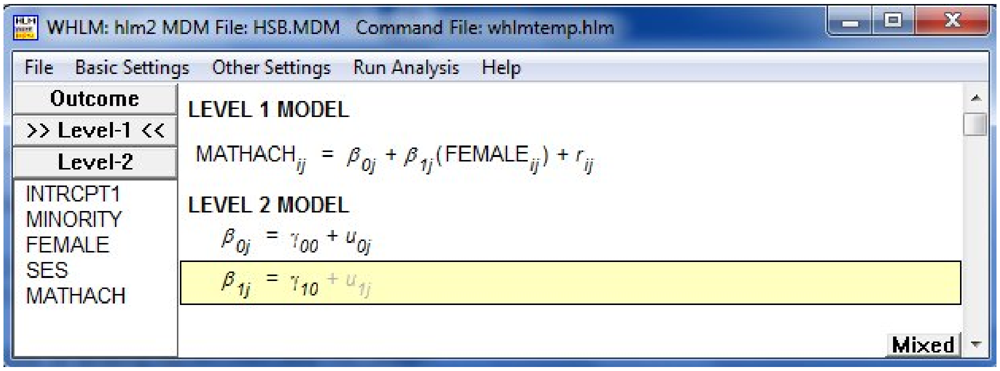
Use is made of the simple and commonly used test statistic for homogeneity

$$H = \sum d_j^2$$

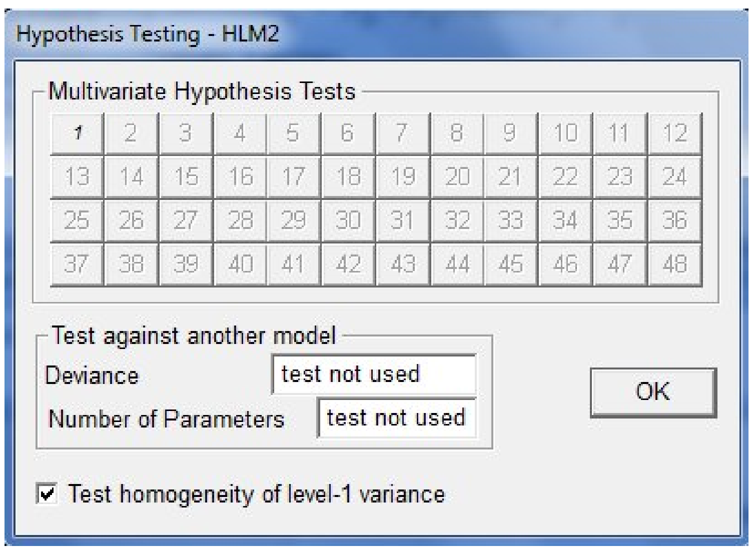
which has a large sample chi-square distribution with $J-1$ degrees of freedom under the homogeneity hypothesis. This test is appropriate when the data are normal and sample size per unit are 10 or more. The homogeneity test is a function of the OLS residuals (OLSRSVAR in the residual file). There is a $n_j - r$ factor (r being the number of random effects) in the denominator used to compute this. A such, the degrees of freedom reported by HLM will be based on the number of level-2 units where n_j exceeds 2.

Example

Using the HSB data provided with the program, we now illustrate testing for homogeneity and specifying a heterogeneous level-1 variance. Consider the model



in which the outcome MATHACH represents a measure of mathematics achievement and FEMALE the gender of participant. In order to check that the two genders have the same model and variance of mathematics scores, a test for the homogeneity of the level-1 variance is performed using the **Test homogeneity of level-1 variance** option on the **Hypothesis Testing** dialog box accessed via the **Other Settings** menu.

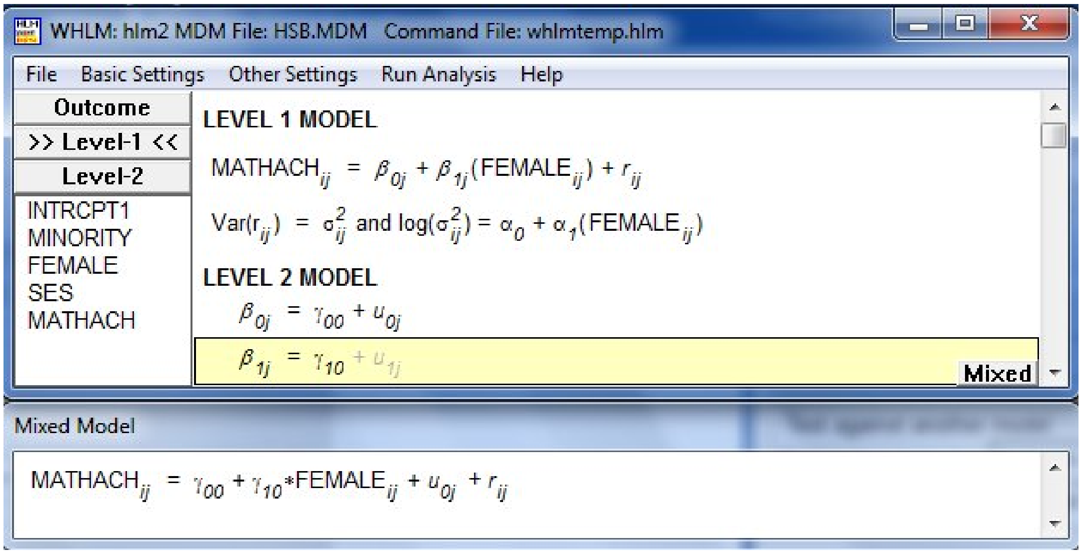


Test of homogeneity of level-1 variance

χ^2 statistic = 291.17435
degrees of freedom = 159
p-value = 0.000

If the chi-square statistic is significant it indicates that the null hypothesis of homogeneity of the level-1 variance is rejected. Note that this option is only available for two level models analyzed with HLM2.

Results obtained for this model indicates that the assumption of homogeneity of the level-1 variance is unrealistic. In order to check whether the level-1 variance may differ between the two gender groups, the **Heterogeneous sigma^2** option on the **Estimation Settings** dialog box is used to model the level-1 variance as a function of gender:



At the end of the output file, additional results for the heterogeneous model are printed, in addition to the standard results for the homogeneous model. A test for statistical significance based on the deviances for the two models is also automatically performed by HLM, with results as shown below.

RESULTS FOR HETEROGENEOUS SIGMA-SQUARED
(macro iteration 12)

Var(R) = sigma^2 and
log(sigma^2) = alpha0 + alpha1(AGE)

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	13.347069	0.260465	51.243	159	<0.001
For FEMALE slope, β_1					
INTRCPT2, γ_{10}	-1.363968	0.185227	-7.364	7024	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	2.84439	8.09056	159	1597.58939	<0.001

Model for level-1 variance

Parameter	Coefficient	Standard Error	*Z*-ratio	*p*-value
INTRCPT1 ,α_0	3.70771	0.024645	150.444	0.000
FEMALE ,α_1	-0.09307	0.034023	-2.736	0.007

40.760515 37.138124

Summary of Model Fit

Model	Number of Parameters	Deviance
1. Homogeneous σ^2	4	47051.48309
2. Heterogeneous σ^2	5	47044.02705

Model Comparison	χ^2	d.f.	*p*-value
Model 1 vs Model 2	7.45604	1	0.006

The t-ratio for γ_{10} of -7.364 and the Z-ratio of -2.736 for α_1 for FEMALE indicate that the math achievement scores of males (FEMALE = 0) are on average higher and more variable than those of the female students (FEMALE = 1). Both are significant. From the **Summary of Model Fit** table it also follows that a model with heterogeneous within-school variance is more appropriate.