



Multiple imputation analyses

The program conducts a separate HLM analysis for each plausible value. For each HLM model, the program runs each of the number of specified plausible values internally and produces their average value and the correct standard errors. There will seem to be one estimate, but the five HLM estimates from the plausible values are produced and their average and measurement error calculated correctly, thus ensuring an accurate treatment of plausible value data.

The output of the separate HLM analyses is written to files with consecutive numbers, for example, OUT.1, OUT.2, OUT.3, etc. Then, HLM calculates the average of the parameter estimates from the separate analyses and computes the standard errors. The output of the average HLM parameter estimates and their standard errors is found in the output file with the extension AVG, which is the file opened after completion of the analysis or from the File, View Output option. The individual output files OUT.1, OUT.2, OUT.3, etc. can be found in the same folder as the MDM and MLM files.

Note that the AVG file does not produce a deviance statistic. This is because the contents are calculated, and not estimated. A deviance statistic will be produced for each individual run and can be found in the output files OUT.1, OUT.2, OUT.3, etc., depending on the method of estimation used.

The following parameter estimates are averaged by HLM:

- The fixed effects (gammas)
- The reliabilities
- The parameter variances (tau) and its correlations
- The chi-square values to test whether the parameter variance is zero
- The standard errors for the variance-covariance components (full maximum likelihood estimates)
- Multivariate hypothesis testing for fixed effects

The standard error of the averaged fixed effects (gammas) is estimated as described below. The Student's t -value is calculated by dividing the average gamma by its standard error, and the probability of the t -value is estimated from a standard t -distribution table.

The standard error of the gammas consists of two components – sampling error and measurement error. The following routine provided in the *NAEP Data Files User Guide* (Rogers, *et al.*, 1992) is used to approximate the component of error variance due to the error in imputations and to add it to the sampling error.

Let θ_m ($m = 1, \dots, M$) represent the m -th plausible value. Let \hat{t}_m represent the parameter estimate based on the m -th plausible value. Let U_m represent the estimated variance of \hat{t}_m .

- Five HLM runs were conducted based on each plausible value θ_m . The parameter estimates from these runs were averaged:

$$\hat{t}^* = \frac{\sum_{m=1}^M \hat{t}_m}{M}$$

- The variances of the parameters from these runs were averaged:

$$U^* = \frac{\sum_{m=1}^M U_m}{M}$$

- The variance of the m estimates, \hat{t}_m , was estimated:

$$B_m = \frac{\sum_{m=1}^M (\hat{t}_m - \hat{t}^*)^2}{(M - 1)}$$

- The final estimate of the variance of the parameter estimate is the sum of the two components:

$$V = U^* + (1 + M^{-1})B_m$$

where the degrees of freedom is computed as:

$$d.f. = (M - 1)(1 + r)^2,$$

where

$$r = \frac{U^*}{B \left(1 + \frac{1}{M} \right)}.$$

The square root of this variance is the standard error of the gamma, and it is used in a standard Student's t formula to evaluate the statistical significance of each gamma.