



Proportion variance explained: two-level models

This document covers the following topics:

- Calculating the intraclass correlation coefficient ($\hat{\rho}$) for an unconditional model (Model 1)
- Proportion variance explained at level-1 after addition of a level-2 predictor (Model 3)
- Proportion variance explained at level-2 after addition of a level-2 predictor (Model 4)
- Conditional $\hat{\rho}$ after addition of a level-2 predictor (Model 4)
- Proportion variance explained and conditional $\hat{\rho}$ for level-2 model (Model 2)

1. Discussion of models specified

Four models were fitted, as described below. Results are shown in Table 1.

Table 1: Variance components for models

Variance components for 4 two-level models				
Coefficient	Model 1 (unconditional)	Model 2 (MEANSES added to level-2)	Model 3 (Level-1 variable SES added)	Model 4 (Final)
r_{ij}	39.15	39.16	36.71	36.71
u_{0j}	8.62	2.64	8.68	0.66
u_{1j}			0.68	0.66

Model 1: A fully unconditional model.

$$y_{ij} = \beta_{0j} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

Model 2: A model with no level-1 predictors and a single level-2 predictor, which may be useful in evaluating the difference between the unconditional variance in outcome over schools and the variance in outcome over schools after each school's MEANSES has been taken into account.

$$y_{ij} = \beta_{0j} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01} (MEANSES)_j + u_{0j}$$

Model 3: Random-intercept-and-slope model with a group-mean centered level-1 predictor.

$$y_{ij} = \beta_{0j} + \beta_{1j} (SES)_{ij} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

Model 4: The final model, containing both level-1 predictor and conditional random slope and intercept coefficient at level-2.

$$y_{ij} = \beta_{0j} + \beta_{1j} (SES)_{ij} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01} (MEANSES)_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11} (MEANSES)_j + u_{1j}$$

2. Calculating the intraclass correlation coefficient $\hat{\rho}$ for an unconditional model (Model 1)

In the unconditional model, the total variation in outcome can be clearly divided between variation over students and variance on a school level. σ^2 represents the total variance in outcomes within school that can be explained by a level-1 model, while τ_{00} is the total explainable variation at level-2 (schools). As such, this model serves as baseline for comparison with subsequent, more complex models.

The intraclass correlation coefficient represents the proportion of the variance in outcome between the schools:

$$\hat{\rho} = \frac{\hat{\tau}_{00}}{\hat{\tau}_{00} + \hat{\sigma}^2} = \frac{8.62}{8.62 + 39.15} = 0.1804.$$

18% of the variance is thus over schools, and the remaining 82% at a student level.

3. Proportion variance explained at level-1 after addition of a level-2 predictor (Model 3)

In the third model, the level-1 model is extended to include the group-mean centered predictor SES, representing the socioeconomic status of each student:

$$y_{ij} = \beta_{0j} + \beta_{1j} (SES_{ij} - SES_{.j}) + r_{ij}$$

and the level-2 model now contains both a random intercept and a random SES slope:

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

From Table 1, we see a relatively small reduction in the level-1 residual variance. The proportion variance explained at level-1 can be calculated as

$$\frac{\hat{\sigma}^2(\text{Unconditional}) - \hat{\sigma}^2(\text{Model 3})}{\hat{\sigma}^2(\text{Unconditional})} = \frac{39.15 - 36.71}{39.15} = 0.0623$$

or 6%. Thus 6% of the level-1 variance in outcome is accounted for by the predictor added at level-1.

4. Proportion variance explained at level-2 after addition of a level-2 predictor (Model 4)

In model 4, the intercept-and-slope-as-outcomes model is considered. The extension of the level-2 model to

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{MEANSES})_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{MEANSES})_j + u_{1j}$$

implies that the variance components at this level are now conditional variances - τ_{00} and τ_{11} are the variances in intercept and slope over schools, after controlling for each school's MEANSES.

When this model is compared to Model 3, we see a notable decrease in $\hat{\tau}_{00}$. The τ_{00} estimates can be used to develop an index of the proportion reduction in intercept variance at level-2:

$$\frac{\hat{\tau}_{00}(\text{Model 3}) - \hat{\tau}_{00}(\text{Model 4})}{\hat{\tau}_{00}(\text{Model 3})} = \frac{8.68 - 2.65}{8.68} = 0.6947$$

Inclusion of the level-2 predictor MEANSES reduced the intercept variance at school level by 69%.

In contrast, the variance in SES slope was only reduced by $\left(\frac{0.68 - 0.66}{0.68}\right) \times 100 = 2.94\%$. The

school's MEANSES helps explain differences between school intercepts, but is not helpful in explaining the differences between the slopes.

5. Conditional $\hat{\rho}$ after addition of a level-2 predictor (Model 4)

A conditional $\hat{\rho}$, measuring the degree of dependence among observations within schools that are of the same MEANSES, can be calculated as

$$\hat{\rho}_{\text{meanses}} = \frac{\hat{\tau}_{00}(\text{Model 4})}{\hat{\tau}_{00}(\text{Model 3}) + \hat{\sigma}^2(\text{Model 4})} = \frac{2.65}{2.65 + 36.71} = 0.0673.$$

We see that the conditional $\hat{\rho}$ is about a third the value of the unconditional, indicating that the addition of the SES effect(s) helped to explain a lot of the variation in outcome over schools.

6. Proportion variance explained and conditional $\hat{\rho}$ for level-2 model (Model 2)

The final model to be considered is model 2. In this model, only the level-2 predictor MEANSES was introduced. $\hat{\tau}_{00}$ is now a conditional variance, representing variance in outcome over schools after controlling for MEANSES.

Given that models 1 and 2 share the same level-1 model, model 1 is the appropriate choice as a baseline for comparisons.

As in Section 4 above, the proportion of variance explained in β_{0j} can be calculated as

$$\frac{\hat{\tau}_{00}(\text{Model 1}) - \hat{\tau}_{00}(\text{Model 2})}{\hat{\tau}_{00}(\text{Model 1})} = \frac{8.62 - 2.64}{8.62} = 0.6937.$$

with conditional $\hat{\rho}$ of

$$\frac{\hat{\tau}_{00}(\text{Model 2})}{\hat{\tau}_{00}(\text{Model 2}) + \hat{\sigma}^2(\text{Model 2})} = \frac{2.64}{2.64 + 39.16} = 0.0632.$$

This is similar to the $\hat{\rho}$ obtained in Section 5 after the addition of the level-1 predictor SES. It can be concluded that the addition of the predictor SES had little impact on $\hat{\rho}$.