Reliability of random coefficients

The reliabilities of random coefficients can be estimated at both levels 2 and 3 of a HLM3 model. In essence, reliabilities can be used to evaluate questions concerning the average reliability of random intercepts and slope(s), had these been based on OLS regressions for separate groups. These reliabilities depend on the degree to which the true underlying parameters vary between groups, and the precision with which each group's regression equation is estimated.

Consider the case where repeated measurements are the level-1 units. These measurements are nested within students (level-2 units), which in turn are nested within schools (level-3 units). For the children (level-2 units), the estimated reliability is defined as the ratio between the level-2 variance component and the sum of the level-2 and level-1 components, with the latter divided by the number of observations within that particular cluster, i.e.

$$\lambda_{0jk} = \frac{\tau_{00}}{\tau_{00} + \sigma^2 / n_{jk}}.$$

Given the general definition for the reliability of a measurement (see pg. 25 of *Multilevel Analysis*[1]) as shown below, σ^2 / n_{jk} is the measurement error for the level-2 variance τ_{00} in the above formula:

$$reliability = \frac{variance\ of\ true\ scores}{variance\ of\ observed\ scores}$$

While τ_{00} is constant over level-2 units, σ^2 / n_{jk} is dependent on the level-1 sample size n_{jk} and varies from level-2 unit to level-2 unit. The reliability of aggregated variables increases as the number of level-1 units nested within a level-2 unit increases. If n_{jk} is very large, λ_{0jk} will get closer to a value of 1.

Similarly, when the group means vary substantially across the groups (holding constant the sample size per group), λ_{0jk} will tend to approach a value of 1.

The reliability of a level-3's sample mean as an estimate of its true mean can be obtained similarly by using the equation

$$\lambda_{00k} = \frac{\tau_{000}}{\tau_{000} + \left\{ \sum \left[\tau_{00} + \sigma^2 / n_{jk} \right]^{-1} \right\}^{-1}}.$$

The equation can be rearranged in the form shown below by substituting the reliability of level-2, λ_{0jk}, in it:

$$\lambda_{00k} = \frac{\tau_{000}}{\tau_{000} + \tau_{00} / \sum \lambda_{0jk}}.$$

This formula is analogous to the level-2 reliability, except that we are using the sum of the reliabilities over the level-2, $\sum \lambda_{0jk}$, instead of level-3 sample size, n_{jk}, as the divider of the level-2 variance component τ_{00}, and thus $\tau_{00} / \sum \lambda_{0jk}$ becomes the measurement error for the level-3 variance τ_{000}. The reliability of a certain level-3 unit increases as the overall level-2 reliability increases. When τ_{00} is small with respect to τ_{000} while n_{jk} is very large, then λ_{00k} will tend to approach 1 when the reliabilities on the level-2 are getting closer to a number of 1. When groups are homogeneous with respect to a random slope, the slope estimation will have poor precision.

The two expressions given above, representing average reliabilities over children and schools, may be used as summary statistics of the children and school mean, respectively.

See also:

- Raudenbush, S.W. & Bryk, A.S. (2002). *Hierarchical Linear Models. Applications and Data Analysis Methods*. Second edition, pp. 50-1, 79. Sage Publications.
- Snijders, T. & Bosker, R. (1999) *Multilevel Analysis*. Sage Publications.