AUXAL:

Auxological Analysis of Longitudinal Measurements of Human Stature

Structural Analysis with the

- BTT model
- JPA2 model
- Jenss-Bayley model

Nonstructural Analysis by

- Kernel Smoothing
- Spline Smoothing
- Fourier Transform of Residuals from the Structural models

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1.Introduction to Growth Curve Analysis

This version of the AUXAL program is designed for the analysis of data from longitudinal studies of human growth in stature. It determines a growth curve for each case in the study and summarizes the results for all cases. It displays the curves for height and velocity of growth, along with the observed heights and empirical velocities. Maxima and minima of growth velocity are detected and described quantitatively. The program gives the user the choice of structural representation of the growth curves based on three different structural models, or nonstructural representations based on two different methods of smoothing the observed measurements. These procedures are applicable to a wide range of growth studies, including those in which the data are incomplete or irregular and growth is atypical. In this chapter we outline the program's analytical methods; an addendum describes their statistical basis in more detail.

Addendum: Structural and Nonstructural Analysis

The procedures for structural analysis in AUXAL include the options of fitting either the BTT model of Bock, Thissen, & du Toit (see Addendum), the JPA2 model of Jolicoeur, Pontier, & Abidi (1992), or in the age range birth to five years, the Jenss-Bayley (1937) model. To fit these models, the program employs a Bayesian method that is highly robust in the presence of irregular and incomplete data. It permits the analysis of mixed longitudinal or even fragmentary growth records of the individual cases. Details of this method are described below and in the Addendum.

The BTT and JPA2 models are each based on eight free parameters and are capable of representing accurately the course of growth over the range from six months to maturity. The BTT model assumes three phases of logistic growth; it is able to detect and locate growth velocity maxima in adolescence and midchildhood, and their preceding velocity minima. The JPA2 model, which is similar to the Preese-Baines model but extends to ages below one year (Preese & Baines, 1979), fits height measurements well over the entire range and detects the adolescent peak height velocity and the preceding prepubertal minimum. Neither the BTT nor the JPA2 model requires a measurement of height at maturity.

The Jenss-Bayley model, which has four parameters to be estimated, gives a good representation of growth up to five or six years of age. The program is restricted here to five years.

For nonstructural smoothing of longitudinal measurements, the program offers the optimal kernel method described in Gasser, *et al.* (1984a, 1984b), the smoothing splines of Heckman & Ramsay (1996), and the finite Fourier transform of residuals from any of the structural models (see the Addendum). These procedures assume nothing about the functional form of growth, except that it is

continuous in chronological age. In particular, they are capable of detecting and locating multiple cycles of growth, including minor minima and maxima of velocity prior to adolescence (see Butler, McKie, & Ratcliffe, 1990). As is true of all nonparametric methods of smoothing, these procedures require relatively complete and more or less uniformly spaced data over the age range analyzed. Nevertheless, they are implemented by numerical methods that do not require strictly complete and uniform measurements as usually assumed in textbook presentations. Details of these nonstructural methods are described below.

The Structural Models

The BTT, JPA2, and Jenss-Bayley models are continuous functions in time and have derivatives of all orders. (For an evaluation of the goodness-of-fit of the BTT and JPA2 models, see Bock, 1995.) Strictly increasing functions of age, they approach smoothly the horizontal asymptote that defines mature stature. The program arbitrarily assumes that mature height, as calculated from the model, is reached at 25 years of age.

Once any of these models for stature is fitted, the program computes the derivative of the fitted curve to obtain the curve for growth velocity. It then computes the derivative of the velocity curve to obtain the acceleration curve, the zeros of which locate maxima and minima of growth velocity. For height y and age x, the functional forms of the models are as follows:

The BTT model

$$y = \frac{a_1}{\left[1 + \exp\left(-b_1 x - c_1\right)\right]^{d_1}} + \frac{a_2}{\left[1 + \exp\left(-b_2 x - c_2\right)\right]^{d_2}} + \frac{a_3}{\left[1 + \exp\left(-b_3 x - c_3\right)\right]^{d_3}}$$

The c_1 term and the exponents d_1, d_2, and d_3 are not estimated but are constants chosen to give the generally best-fitting family of growth curves. In our analyses of the Fels and Berkeley data, we have found the respective values 0.25, 0.815,0.70,0.615 for males and 0.25,0.85,0.63,0.60 for females to be most satisfactory. These are the program defaults, but they may be altered by the user.

The JPA2 model

$$y = a\left\{1 - \frac{1}{1 + \left[b_1\left(x + e\right)\right]^{c_1} + \left[b_2\left(x + e\right)\right]^{c_2} + \left[b_3\left(x + e\right)\right]^{c_3}}\right\}$$

Jolicoeur, *et al.* (1992) use the reciprocals of b_1, b_2, and b_3.

The Jenss-Bayley model

$$y = a + bx - e^{c-dx}$$

The Jenss-Bayley (1937) model is restricted in the program to ages zero through five years.

Fitting the Structural Models

The conventional least squares method fitting of complex growth models is less than ideal in the present application because it requires a number of distinct observations equal to or greater than the number of parameters. Even when the number of observations is sufficient for least squares, the parameters may not all be identifiable if the observations are poorly positioned. A much better method for fitting growth models is Bayes modal estimation, which chooses, from among a specified population of growth curves, that curve which is most probable given the data. We refer to this estimation procedure as MAP (Maximum A Posteriori) to distinguish it from estimation of the mean, or expected value, of the posterior distribution. The latter is commonly called Bayes estimation but is referred to here as EAP (Expected A Posteriori). MAP has the advantage of being numerically tractable for eight-parameter nonlinear models, whereas EAP (although providing minimum expected mean square error) is computationally too burdensome to be practical in this application. MAP estimation as implemented in the program by the Fisher-scoring (Newton-Gauss) method is extremely fast, and nearly as robust as EAP estimation. The iterative calculations of the MAP solution almost always converge, the exception being when there is an extreme outlier, usually because of clerical error. In that case, removing the bad observation will correct the problem.

An advantage of Bayes methods in general, including MAP, is that information from collateral data - such as height of first-degree relatives, measures of skeletal age, signs of sexual maturation, etc. - can easily be introduced to improve the accuracy of the fit when the data are sparse. This is accomplished by using the collateral data to condition the specification of the population (or so-called "prior") distribution. In our implementation of MAP, we assume the population distribution to be multivariate normal, and we provide, with the program, estimates of the population mean and covariance matrix for each sex based on data from the Fels Longitudinal Study. The estimation of these quantities was carried out by the maximum marginal likelihood (MML) method for nonlinear models described in Bock & du Toit (1994) and du Toit (1992). The supplied values are the defaults; the user can conveniently substitute other, possibly conditioned, values.

Bayes estimation of the parameters of the structural models also requires a specification of the autocorrelation function of the residual variation. If the observations are uniformly spaced and nearly complete, it is possible to estimate the function concurrently with

the model parameters for a particular case. As we do not wish to assume data of this quality when employing the structural models, we have not included a procedure for this purpose in the program. Instead, we have supplied an *average* autocorrelation function for each sex estimated from a sample of cases in the Fels data that are complete and uniformly spaced over the range from two to 18 years. The estimated functions, which are continuous in real time, were obtained by averaging the spectra of the residuals for these cases and invoking the Wiener-Khinchin inversion theorem (see Bloomfield, 1976) to obtain the corresponding average autocorrelation functions. The functions go essentially to zero after a lag of five years. Details of this procedures are described in the Addendum.

The user also has the option of assuming the residuals to be uncorrelated, thus speeding the calculations, but this option is not a desirable alternative when the spacing of the data is irregular. The inverse of the autocorrelation matrix, which figures in the MAP procedure, optimally weights the observations in this situation. Without such weighting, clusters of closely-spaced points will bias the fitting of the growth curve relative to other parts of the data where the points are less dense. With weighting, the influence of the closely-spaced, and thus highly-correlated, measurements is made to match that of the more widely-spaced and less-correlated observations. In addition, autocorrelation of the residuals is then correctly taken into account in computing the standard errors of the estimated model parameters.

In the MAP procedure, we assume the residuals to be multivariate normal with mean zero and covariance matrix constructed from the autocorrelation function. The autocorrelation at lag zero is not 1.0, but is the intraclass correlation coefficient associated with estimation of the finite Fourier transform (which in this application is truncated at the fifth harmonic). The average residual covariance matrix is the scalar product of this matrix and the measurement error variance. The measurement error variance is not estimated by the program, but is a constant that the user supplies to the program (default value 1.0 cm"). The user can adjust this value to determine the relative weight given to the observations vs. that given to the prior distribution in the MAP estimation. In high quality longitudinal studies, the measurement error may be assumed as small as 0.3 or even 0.2 cm^2; in data obtained by untrained observers without the benefit of a stadiometer, an assumed standard deviation as large as 1.5 cm may be more realistic.

The function maximized in MAP estimation is the following (see Bock & Thissen, 1980):

$$F(\theta) = -\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda^{jk} \left[y_j - f(\theta, x_j) \right] \cdot \left[y_k - f(\theta, x_k) \right]$$

$$+ \sum_{l=1}^{8}\sum_{m=1}^{8} \sigma^{lm} (\theta_l - \mu_l)(\theta_m - \mu_m)$$

where

λ^{jk} is the j, k element of the sex-specific inverse average residual covariance matrix,

$f(\theta, x_i)$ is the growth model evaluated for parameter values θ at age x_j

σ^{lm} is the l, m element of the inverse covariance matrix of the model parameters for the male or female populations,

and μ_l is the sex-specific population mean for parameter l.

This function is maximized for each case by the Fisher scoring method using the posterior information matrix, the l, m element of which is

$$I(\theta_l, \theta_m) = \sum_{j=1}^{n} \sum_{k=1}^{n} \lambda^{jk} \left(\frac{\partial f(\theta, x_j)}{\partial \theta_l} \right) \cdot \left(\frac{\partial f(\theta, x_k)}{\partial \theta_k} \right) + \sigma^{lm}.$$

Nonstructural Smoothing

To smooth growth data nonstructurally, the user must make an essentially arbitrary decision about the amount of detail to be represented in the resulting curve. For the optimal kernel procedure, the extent of smoothing is controlled by the so-called "band-width" parameter, which is at the user's disposal. Our experience with the Fels and Berkeley databases (Tuddenham & Snyder, 1984) have led us to choose a bandwidth of 2.5 years as the default value. This value seems to work well in generating kernel weights that yield plausible nonstructural curves for height, velocity, and acceleration. In the program, the weights may be calculated for any age between one year and 18 years (or more at the option of the user), thus providing continuous interpolations for purposes of analysis and plotting. [1]

For the penalized smoothing spline procedure of Heckman & Ramsay (1996), the degree of smoothing is influenced by the order of derivative to be penalized and by a smoothing parameter. Both of these quantities can be set by the user if the default values are not satisfactory.

In the Fourier analysis of residuals from the BTT, JPA2, or Jenss-Bayley models, the degree of smoothing is controlled by the number of harmonics included in the transform. Again based on results from the Fels and Berkeley data, we found the transform through the fifth harmonic to be most satisfactory, but this choice is also at the disposal of the user. With the program defaults, the three smoothing methods produce very similar growth, velocity, and acceleration curves. A special feature of the Fourier analysis is that, by averaging the estimated parameters of the growth model used to de-trend the data and the estimated coefficients of the Fourier transform, we obtain a well-defined nonstructural average curve. In addition, as mentioned above, by obtaining and inverting the average spectrum of the sample Fourier analysis (i.e., mean squares of the coefficients for each harmonic), we have an estimate of a positive-definite autocorrelation function, in continuous time, for residuals of the model in question, as required in MAP maximization function

(see the Addendum).

1: The procedures for obtaining these weights were contributed by Professor James O. Ramsay, McGill University, based on results in Gasser, *et al*. (1984a).

2. The AUXAL Program

In addition to smoothing observed heights or fitting growth models, the AUXAL program analyzes the growth of each case to locate cycles of accelerated and decelerated growth including, most notably, the adolescent growth spurt. If a number of same-sex cases are analyzed together, the program summarizes and computes sample statistics for these and other features of the growth curves. These functions of the program are discussed in this chapter, along with general aspects of data preparation and use of the program.

Descriptive Features of the Growth Curves

AUXAL reports various descriptive features of the growth curves, including estimates of mature stature and the location and intensity of various maxima and minima in the growth velocity curve. As mentioned above, mature stature is the height of the case at age 25 estimated with the fitted BTT or JPA2 model. If kernel or spline smoothing has been selected, mature stature is estimated from a fitted BTT model that provides values for the growth curve outside the range of the kernel smooth. If the data records include the known mature stature of the case, the program computes the difference between the estimate based on the model and the known adult height and reports the size of the prediction error. If there are two or more cases in the analysis, the SUMMARY procedure reports the distribution of the prediction errors.

The age of adolescent peak height velocity (PHV), and height and velocity at PHV, are prominent features of auxological analysis. The program computes these values for each case, and it also reports similar information for other turning points of the velocity curve, depending on the procedure chosen. The BTT and JPA2 models provide a report of the adolescent peak height velocity, the preceding prepubertal minimum, plus the midchildhood maximum, or mid-spurt, and preceding early childhood minimum if the fitted curve is found to exhibit these features.

In principle, the nonstructural analysis could detect many more maxima and minima of the growth velocity; depending on the smoothing parameters for the highest harmonic of the Fourier analysis. We have simplified the program results in two ways: first, we do not report small variations at very low velocities as maturity is approached; second, we have provided a parameter value that the user may set to suppress the report of any maximum of growth velocity that does not exceed the preceding minimum by more than the specified value. The default is set rather small, at 0.25 cm/yr, and most users will probably prefer to substitute the higher value. Finally; we restrict the total number of turning points reported to six: three maxima and three minima.

If a still more detailed examination of the growth curves is required, the user has the option of printing out a table showing the

values of height, velocity, and acceleration from ages one to 25, or zero to five with the -Jenss-Bayley model, in any age interval. The default value of the interval is one year or one-fourth year, respectively, but it may be set to smaller values to reveal more detail. AB is true of mature height, the values in this table are extrapolations from the structural model when they are outside the age range of the data.

Summary of the Analysis of a Sample of Cases

When records from more than one case are analyzed in the same program execution, the user may request a summary of the individual results. Although such summaries will require perhaps 20 or 30 cases for acceptable statistical stability; the program permits summary of as few as two cases. The following are the main summary statistics produced by the program.

1. *Statistics of the cross-sectional observed heights and residuals from the fitted or smooth curve for height in the selected age stepsize (default =* 1.0 *yr; Jenss-Bayley, 0.25 yr).* They include the number of cases, mean, standard deviation, standard error of the mean, and coefficients of skewness and kurtosis.

2. *The cumulative distribution of fitted or smoothed heights and velocities at evenly spaced age points.* The relative frequencies are shown to the nearest 2.5 cm of height and 0.25 cm/yr for velocity. They are shown in their raw form and after kernel smoothing.

3. *Percentiles of the smoothed cumulative distributions of fitted or smoothed heights and velocities at the specified ages.* The 5, 10, 25, 50, 75, 90, and 95 percent points are given. The height and velocity percentiles are further smoothed over the age range.

4. *Statistics for the distributions of the estimated model parameters and Fourier coefficients (if any) in the sample.* These statistics include the mean, standard deviations, and variance-covariance matrix. The variance-covariance matrix is computed from the unbiased estimator for Bayes estimates of the individual cases. It is the sum of the variance-covariance matrix for the Bayes estimators plus the mean posterior variance-covariance matrix in the sample. Because the Bayes *modal* estimator is applied in the individual cases, the posterior variance-covariance matrix is approximated by the inverse of the posterior information matrix defined above. We caution the user that this method of estimating the population variance-covariance matrix of the model parameters, or the Fourier coefficients, is not fully efficient and not considered a substitute for the maximum marginal likelihood estimator described in Bock & du Toit (1994) and du Toit (1992). The matrix is reported by the program only to give a provisional estimate of the population variability in the model parameters, and the degree of their correlation (the correlation matrix is also shown).

5. *The average inverse posterior information matrix of the sample cases.* This matrix conveys the average error in estimating the model parameters and the degree of correlation among the parameter estimates.

6. *Statistics for the maximum and minima of the velocity curves of the cases and for mature stature.* These include the number of cases, the number of cases, mean, standard deviation, standard error of the mean, and coefficients of skewness and kurtosis. These values depend, of course, upon the criterion for reporting the maximum set by the user as discussed above.

7. *When the* BTT, JPA2, *or Jenss-Bayley model is selected, the program performs an analysis of the structural average curve, the parameters of which are the average for the sample.* When the Fourier method of smoothing is selected, the program produces a similar average curve represented by the mean of the model parameters in the sample plus the mean of the Fourier coefficients from the analysis of residuals.

A corresponding type of average curve can be derived for kernel or spline smoothing by the method of curve registration introduced by Gasser, *et al.,* (1984a). (See also Ramsay, Bock, & Gasser, 1995.) This procedure is not presently implemented in AUXAL.

Types of Data

For the convenience of the user, the program provides four types of data input referred to as Types 1, 2, 3, and 4. The data file may be prepared either in fixed-column format, in which the data fields such as case 10, sex, age, and height are in the same columns in each record line, or, alternatively, in free-column format, in which the data fields are in the same left-to-right order in each record but not necessarily in exactly the same columns. In free-column format there must be spaces or commas between data fields; in fixed-column format, the fields may be separated or not separated, at the user's convenience. Fixed-column format requires the user to supply a variable format statement describing the column assignments (see Variable Format); free-column format does not require any intervention from the user if the data fields are in the order assumed by the program. If they are in a different order or are intermixed with other space or comma delimited fields, the user must indicate their serial positions by means of the FIELDS keyword of the INPUT command.

When there are many cases in the sample, Type 1 input is perhaps most convenient: it requires the observations of each case to appear on successive data records, each containing the case identification (ID), sex code, age and height. The program detects the end of each case by the change of the 10 field. (See Chapter 6, Example 1.)

In Type 2 data, the number *n* of observations for the case, the sex code, and the case ID appear on a header record preceding the observed ages and heights of each case. The data appear in pairs

(e.g., age_1, hgt_1, age_2, hgt_2 …, age_n, hgt_n) on a following record or records. If data fields are specified in the INPUT command, each age-height pair must be in a separate record (line) of the data file. (See Chapter 6, Example 3.)

Type 3 data are similar to Type 2, except that the order of the observed ages or heights is (age_1, age_2,…, age_n, hgt_1, hgt_2, …hgt_n). They may appear sequentially in one or more records. (See Chapter 6, Example 5.)

Type 4 data is similar to Type 3, except that the ages are assumed to be the same for all cases and are entered separately, just once, following the AGES command. (See Chapter 6, Example 6.)

Type 3 and Type 4 data may be free-column *i.e.,* no format statement required) and the IGNORE keyword used to pass over irrelevant leading fields. Field selection is not available for Type 3 or Type 4 data.

- •AUXAL also has provisions for limiting the range of ages or heights that will be processed (VARIABLES command),
- • setting the age range over which nonstructural smoothing will occur (TECHNICAL command),
- •selecting or omitting cases by their IDs from a larger file (CASES command), and
- •excluding cases that have too few observations, an insufficient age range of observations, or too large an age gap for analysis (TECHNICAL command).

Instructions for preparing the data file, and the header records for data Types 2, 3, and 4, appear in the description of the INPUT command in Chapter 3. Examples of the various data layouts appear in the sample problems of Chapter 6.

Using the AUXAL Program

Once AUXAL is installed, there are three steps to using the program:

1. *Preparing the data file in one of the types of data layout described above.* This must be a plain ASCII file and may have any convenient name, with or without extension (for example, GIRLS. DAT).

2. *Preparing. the program command file. This file controls the various program functions and options.* Instructions for preparing the command file are presented in Chapter 3. Examples of typical command files appear in Chapter 6. The command file must also be plain ASCII and must have the extension AXL in its name (for example, GIRLS .AXL).

3. *Executing the program.* To execute the program, open an existing command file from the **File** menu of the AUXAL program window or create a new command file with the program editor. When satisfied that all specifications are correct, click on the **Run** menu and execute the command file. See Chapter 4 for more details.

The amount of detail in the listed output is controlled by the LEVEL keyword and NOPRINT option of the LIST command.

To inspect or print the growth curve displays, use the plotting facility described in Chapter 4.

3.The AUXAL Commands

Command Syntax

The AUXAL commands employ the general syntax:

> >NAME $\mathrm{KEYWORD}_1 = a,$
>
> $\mathrm{KEYWORD}_2 = (b_1, b_2, \ldots, b_n), \ldots,$
> $\mathrm{OPTION}_m, \ \mathrm{OPTION}_n;$

The following rules apply.

- A greater-than sign (> must be entered in column 1 of the first line of a command and followed without a space by the command name.

- Command names, keywords, and options may be entered in full or abbreviated to the first three (command names) or two (keywords and options) characters.

- At least one space must separate the command name from any keywords or options.

- All keywords and options must be separated by commas, blank characters, or both.

- The equals sign is used to set a keyword equal to a value, which may be integer, real, or character. A real value must contain a decimal point. A character value must be enclosed in single quotes if:

   - it contains more than eight characters
   - it begins with a numeral
   - it contains embedded blanks, commas, semicolons, or slashes

  Example: `OMIT='EXAMPL02.0MT'.`

A keyword may be vector valued; *i.e.,* set equal to a list of integer, real, or character constants, separated by commas or spaces, and enclosed in left and right parentheses.

Example: `FIELDS=(1, 8, 4, 6)`

- The *italic* elements in the command format description are variables that the user needs to replace.

- Command lines may not exceed 80 columns. Continuation on one or more lines is permitted.

- Each command terminates with a semicolon (;). The semicolon functions as the command delimiter; it signals the end
of the command.

The following table gives the AUXAL command summary. Detailed explanations of the commands follow.

SUMMARY OF COMMANDS

COMMANDS	KEYWORDS	OPTIONS
TITLE		
COMMENT		
PROCEDURE	EVALUATE=, STEPSIZE=	BTT/JPA2/JENSS-BAYLEY, KERNEL,SPLINE,FOURIER, DESCRI BE/DISPLAY, SUMMARY, PERCENTILE
VARIABLES	INDVAR=, DEPVAR=	
MEAN	FORMAT=, FILE=	MALE/FEMALE
COVARIANCE	FORMAT=, FILE=	MALE/FEMALE
ERROR	DS=	MALE/FEMALE
AGES	NUMBER=	YEARS/MONTHS
CASES	SELECT=,OMIT=	
LIST	LEVEL=	NOPRINT
TECHNICAL	MAXITER=, CRITERIA=, PERCENTILE=, RANK=, TRIM=, NSMIN=, STMIN= LOWER=, UPPER=, BANDWIDTH=, UAGE=, LAGE=,SPURT=,MSHAPE= FHSAPE=	UNCOR
SAVE	MASTER=, PARM=, SE=, COVER=, AGES=, OBSERVED=, EXPECTED=, VELOCITY=,DESCRIPT=, MEAN=, EVAL=, FOURIER=, CROSSECTION=, RESIDUAL=, PERCENTILE=	
INPUT	TYPE=, HEADER=, FORMAT=,IGNORE=, SEXCODE=, IDCHAR=, LENGTH=, DATA/MASTER= MISSING=, FIELDS=, RECORDS=	CM/IN, TOP
CONTINUE		
STOP		

KEYWORDS   require a value after the equals sign.
OPTIONS   invoked if present; a forward slash (j) indicates a choice
            between two options, the first one being the default.

TITLE

(Required)

PURPOSE

A label that will be used throughout the output to identify the problem run.

FORMAT

>TITle

    text...
    text...

NOTES

Following the **TITLE** command, insert one or two title lines. The text will be printed verbatim at the head of each of the three program phases.

EXAMPLE

```
>TITLE;
FIVE CONSECUTIVE FEMALE CASES FROM THE FELS DATA FILE OF
OCTOBER, 1962.
FOURIER SMOOTHING FROM THE BTT MODEL. TYPE 1 DATA LAYOUT.
>COMMENT The procedure in this example prepares a smooth
nonstructural
```

COMMENT

(Optional command)

PURPOSE

To enter one or more lines of explanatory remarks into the program output stream.

FORMAT

>COMment

      ... text ...

      ... text ...

NOTES

This line and all subsequent lines preceding the **PROCEDURE** command will be printed verbatim in the initial output stream. The maximum length of each line is 80 characters. A semicolon to signal the end of the command is not needed. Comments are optional.

EXAMPLE

```
>TITLE;
EXAMPLE 4. INCOMPLETE DATA FROM CASE 602
   JPA2 MODEL. TYPE 2 FREE-COLUMN DATA LAYOUT
>COMMENT
   This is the same problem as the first case in Example 3a,
except that the JPA2 model is used in the structural analysis.
In addition, the header and data are read in the free-column mode.

>PROCEDURE JPA2,DESCRIBE,DISPLAY,EVALUATEi
```

DEFAULT

No comments.

PROCEDURE

(Required command)

PURPOSE

To give the model and analysis specifications.

FORMAT

$$> \text{PROCEDURE} \left[\left[\begin{array}{c} \text{BTt} \\ \text{JPa2} \\ \text{JEnss} \\ \text{KErnel} \\ \text{SPline} \end{array} \right], \text{FOurier} \right], \text{DEscribe, DIsplay,}$$

$$\text{EValuate}=a, \text{STepsize}=b, \text{SUmmary, PErcentile;}$$

OPTIONS

BTT

Fits the BTT model to observed heights and evaluates growth velocity and acceleration.

JPA2

Fits the JPA2 model to the observed heights and evaluates growth velocity and acceleration.

JENSS

Fits the Jenss-Bayley model to the observed heights and evaluates growth velocity and acceleration.

KERNEL

Selects kernel smoothing of the observed heights and kernel estimation of growth velocity and acceleration.

SPLINE

Selects spline smoothing of observed heights by the method of Heckman & Ramsay (1996) and evaluates growth velocity and acceleration.

FOURIER

Smoothes the observed heights by finite Fourier analysis of the residuals from the BTT, JPA2 or Jenss-Bayley curve, whichever is selected. The smoothed curve is the sum of the structural curve and the smoothed residuals. Evaluated growth velocity and acceleration are the sums of the respective derivatives of the structural curve and the fitted Fourier series.

DESCRIBE

Selects analysis of turning points of the velocity curve for each case and for the structural average case. If KERNEL, SPLINE, or FOURIER is selected, the analysis is performed on the nonstructural curve; otherwise, it is performed on the BTT or JPA2 structural curve. There are no turning points for the Jenss-Bayley curve.

Default: no description

DISPLAY

Causes files to be written in *command filename.* DS1 and *command filename.* DS 2 containing data for the graphics plot of the growth curve for each case and for the structural average case (except with KERNEL and SPLINE, which do not provide structural averages).

DISPLAY forces the DESCRIBE option.

SUMMARY

Summarizes the analytical results when two or more cases are analyzed in the same program execution. Computes age cross-sections of observed heights and velocities.

PERCENTILE

Computes the cross-sectional distribution of the *fitted* or *smoothed* heights and velocities and estimates the 5, 10, 25, 50, 75, 90, and 95 percent points for height and velocity.

KEYWORDS

EVALUATE = *a*

Evaluates the fitted height, velocity, and acceleration of the structural or nonstructural curve over the span from one to 25 years in intervals of *a*.

Default: 1. 0

STEP SIZE = *b*

Sets the age interval in years for the cross-sectional summaries of observed heights and residuals. Minimum step size is 0.25 years.

Default: 1.0 year.

VARIABLES

(Required command)

PURPOSE

To give the minimum and maximum values for the variables.

FORMAT

>VARiables INdvar = *(a, b),* DEpvar = *(c,* d);

KEYWORDS

INDVAR = *(a,b)*

a and *b* are, respectively, the minimum and maximum values of the independent variable (in this case, AGE) that the program will treat as data. Observations with ages outside this range will be ignored during input.

Defaults: 1.0 *(a,* years)
          30.0 (b, years)

DEPVAR = *(c, d)*

c and *d* are similar values for the dependent variable (in this case, HEIGHT).

Defaults: 70.0 (c, cm)
         200.0 (d,cm)

MEAN

(Optional command)

PURPOSE

This command replaces the default values of the population means of the structural model parameters. Once entered, the new values continue in force for all remaining problems in the command file.

FORMAT

$$> \text{MEAn} \quad \begin{bmatrix} \text{MALE} \\ \text{FEMALE} \end{bmatrix}, \text{FORMAT} = a, \text{FILE} = b;$$

OPTIONS
MALE

The means assumed for the male population will be replaced by means supplied by the user.

FEMALE

The means for the female population will be replaced by means supplied by the user.

NOTE

If both male and female population means are to be replaced, the MEAN command must appear twice in the problem setup.

KEYWORDS

FORMAT=*a*

If FORMAT appears, the command record must be followed by *a* eighty-column record(s) containing the variable format (in parentheses) describing data records containing the values of mean growth parameters for the population.

The BTT parameters must appear in the order:

$$a_1, b_1, a_2, c_2, b_2, a_3, c_3, b_3$$

The JPA2 parameters must appear in the order:

$$a_1, b_1, b_2, b_2, c_1, c_2, c_3, e$$

The JENSS parameters must appear in the order:

$$a, b, c, d$$

FILE = *b*

The data records containing the means may appear following

the variable format in the command file, or if b is set equal to a filename, the data exist as ASCII characters in that file. If the filename begins with a number, is more than 8 characters in length (including path and extension), or contains embedded blanks or punctuation, it should be enclosed in single quotes.

NOTES

If FORMAT is *not* selected, the means records are assumed *to* be in free-column format, *i.e.,* the values include decimal points and are delimited by spaces or commas. A variable format record is not then required. The free-column means may be in the command file following the MEAN command, or in records of length 80 in file b if FILE $=$ b appears.

COVARIANCE

(Optional command)

PURPOSE

This command replaces the default male or female population covariance matrix according as MALE or FEMALE is selected. Once the new values are entered, they continue in force for all remaining problems in the command file.

FORMAT

$$> \text{COVariance} \begin{bmatrix} \text{MAle} \\ \text{FEmale} \end{bmatrix}, \text{FOrmat} = a, \text{FIle} = b;$$

OPTIONS

MALE

The covariances assumed for the male population will be replaced by covariances supplied by the user.

FEMALE

The covariances for the female population will be replaced by covariances supplied by the user.

NOTE

If both male and female population means are to be replaced, the COVARIANCE command must appear twice in the problem setup.

KEYWORDS

FORMAT = *a*

If FORMAT appears, the command record must be followed by *a* eighty-column record(s) containing the variable format (in parentheses) describing data records containing the covariances for the population.

FILE=*b*

The data records containing the covariances may appear following the variable format in the command file, or if *b* is set equal to a filename, the data may exist as ASCII characters in that file. If the filename begins with a number, is more than 8 characters in length (including path and extension), or contains embedded blanks or punctuation, it should be enclosed in single quotes.

NOTES

If **FORMAT** is *not* selected, the covariance records are assumed to

be in free-column format, *i.e.,* the values include decimal points and are delimited by spaces or commas. A variable format record is not then required. The free-column covariances may be in the command file following the COVARIANCE command, or in records of length 80 in file b if **FILE =** b appears.

Rows and columns of the matrix correspond to the growth parameters in the same order as that of the means (see MEAN command above). Only the 36 unique elements in the diagonal and below the diagonal of the covariance matrix are entered. Order of entry of the elements is left to right in successive rows of the lower triangular array, *i.e.,*

$$
\begin{array}{lll}
c_{(1,1)} & & \\
c_{(2,1)} & c_{(2,2)} & \\
c_{(3,1)} & c_{(3,2)} & c_{(3,3)} \\
\end{array}
$$

$$\cdots$$

$$\cdots$$

ERROR

(Optional command)

PURPOSE

The program normally assumes a measurement error standard deviation of 1.0cm for both male and female cases. This command sets the error standard deviation for males or females to the value *a* according as MALE or FEMALE is selected.

The error standard deviation controls the relative weights given the data and the prior distribution in the MAP estimation of the model parameters. Smaller SD's give more weight to the data; longer, to the prior.

FORMAT

$$>\text{ERRor} \begin{bmatrix} \text{MAle} \\ \text{FEmale} \end{bmatrix}, \text{SD} = a;$$

OPTIONS

MALE

The standard deviation assumed for male cases will be replaced by a standard deviation supplied by the user.

FEMALE

The standard deviation assumed for female cases will be replaced by a standard deviation supplied by the user.

NOTE

If both male and female standard deviations are to be replaced, the ERROR command must appear twice in the problem setup.

KEYWORDS

SD=*a*

This keyword sets the error standard deviation to the value *a*. Once the new value is entered it continues in force for all remaining problems in the command file.

Default: 1. 0

AGES

(Optional command)

PURPOSE

To specify the age levels (in months or years) for all cases. If all cases are measured at the same age points, it is not necessary to input the ages with the data of each case (see INPUT command below). They may be input once and for all with the AGES command.

FORMAT

$$> \text{AGEs} \qquad \text{NUmber} = a, \begin{bmatrix} \text{YEars} \\ \text{MOnths} \end{bmatrix};$$

KEYWORD
NUMBER = *a*

Specifies the number of age levels for all cases.

OPTIONS

YEARS/MONTHS

According as YEARS or MONTHS is selected, values may be expressed in years and decimal years or months and decimal months.
Default: YEARS

NOTES

Immediately after the AGES command, one or more 80-column records containing the list of ages in free-column format should follow. The age values must contain a decimal point and be separated by a space or comma, and terminated with a semicolon (;) on the right.

To input ages in months when using the other forms of data entry specified in the INPUT command below, select MONTHS, but
omit the NUMBER keyword.

CASES

(Optional command)

PURPOSE

To select or omit cases.

FORMAT

$$> \text{CAses} \quad \begin{bmatrix} \text{SElect} = a \\ \text{OMit} = b \end{bmatrix};$$

KEYWORDS

SELECT = a

If a is a filename, the ID's of cases to be selected for fitting should appear in the leading characters of each record of that file. The ID field must begin in column 1. The numbers of columns in the field must equal the IDCHAR value of the INPUT command.

OMIT = b

If b is a filename, the ID's of cases to be omitted from fitting should appear in the leading characters of each record of that file.

LIST

(Optional command)

PURPOSE

To control the amount of detail in the printed output.

FORMAT

> LISt   LEvel = a, NOprint;

KEYWORD

LEVEL = *a*

Specifies the amount of detail in the printed output of the program ($1 \leq a \leq 5$). Level 5 prints copious diagnostic information intended only for error tracing.

Default: 2

OPTION

NOPRINT

This option suppresses printing of individual case reports when only the summary results are desired.

TECHNICAL

(Optional command)

PURPOSE

This command changes various technical constants in the program. The changed values apply to all cases in the problem set.

FORMAT

> TEChnical MAxiter = *a,* CRiteria = *b,* PErcentile = c,

RAnk = *d,* TRim = *e,* UNcor, NSmin = *f,* STmin =

g,

LOwer = *h,* UPper = *i,* BAndwidth = *j,*

UAge = *k,* LAge = *l,* SPurt = *In,* SHape = *(n, 0, p)*:

KEYWORDS

MAXITER = *a*

Sets the maximum number of iterations performed by the Newton-Gauss algorithm when solving the MAP equations for each case.

Default: 80

CRITERIA = *b*

Sets the convergence criterion for the Newton-Gauss algorithm. *b* is the maximum correction for any parameter at convergence.

Default: 0.0001

PERCENTILE = *c*

The user may replace the default percentile limits of the program plots with the values in file c. In this file, the column fields from left to right correspond to the 5, 25, 50, 75, and 95 percent points. The rows for the height and velocity percentiles correspond to the ages 1.8 year to 21.8 years in steps of 0.4 years *(i.e.,* 51 rows). The order of these arrays in the file is as follows:

> Male velocity percentiles
> Male height percentiles
> Female velocity percentiles
> Female height percentiles

Entries must be separated by spaces or commas. The PER-CENTILE keyword of the SAVE command produces a file in this form.

Default: The default values for height were obtained by structural fitting of the 4, 10, and 18 year points of the 5, 25, 50, 75 and 95 percent cross-sectional limits found in Hamill, *et al.* (1979). The default bounds for velocity were adapted from Tanner & Davis (1985).

RANK = *d*

Sets the highest harmonic of the Fourier transform to the integer d ($0 \leq d \leq 13$.

Default: 5

TRIM = *e*

Sets the trimming constant of the Tukey biweight in the Fourier analysis to the real-value e, $e > 0.0$. Trimmed observations are signaled in the program output.

Default: 4.0

NSMIN = *f*

Sets the minimum number of observations required for non-structural analysis.

Default: 18

STMIN = *g*

Sets the minimum number of observations for structural analysis.

Default: 0

LOWER = *h*

Sets the value h as the lower age point of the interval in which nonstructural analysis is performed.

Default: 1.5 (years)

UPPER = *i*

Sets the value i as the upper age point of the interval in which
nonstructural analysis is performed.

Default: 18.0 (years) (JENSS, 5.0)

BANDWIDm = *j*

The bandwidth of the optimal kernels.

Default: 2.5 (years); (JENSS, 1.0)

Penalty parameter of the spline-smooth.

Default: 5.0; (JENSS, 0.0001)

UAGE = *k*

The program requires at least one observation at age k or older for either structural or nonstructural analysis of the case.

Default: 1.0 (years)

LAGE = *l*

The program requires at least one observation at age l or younger for either structural or nonstructural analysis of the case.

Default: 25.0 (years)

SPURT = *m*

Sets the smallest difference in cm/yr between a maximum of velocity and the preceding minimum that will be identified as a growth spurt in nonstructural analysis.

Default: 0.25 (cm/year)

MSHAPE = (*n*, 0, *p*)

Sets the shape constants (exponents) of the respective terms of the BTT model for males.

Default: 0.815, 0.70,0.615

FSHAPE = (*q,r,s*)

Sets the shape constants (exponents) of the respective terms of the BTT model for females.

Default: 0.85,0.63,0.60

OPTION
UNCOR

If this option is selected, the residuals in fitting the structural model are treated as uncorrelated.

SAVE

(Optional command)

PURPOSE

To save certain intermediate results for each case.

FORMAT

>SAVe   MAster = *a,* PArm = *b,* SE = *c*, COvar = *d,* AGes = *e,*
OBserved = *f,* EXpected = *g,* VElocity = *h,*
DEscript = *i*, MEan = *j* EVal = *k,* FOurier = *l,*
CRossection = *m*, REsidual = *n,* PErcentile = *o*;

KEYWORDS

MASTER = *a*

The binary system file containing the data for all cases.

PARM = *b*

The estimated growth parameters in the order specified on
the MEAN command above.

SE = *c*

Standard errors (posterior standard deviations) of the esti-
mates.

COVAR = *d*

The posterior covariance matrix of the parameter estimates.
The matrix appears in lower triangular form as described on
the COVARIANCE command above.

AGES = *e*

The ages in decimal years at which heights for the case are
observed.

OBSERVED = *f*

The observed heights for the case.

EXPECTED = *s*

The fitted heights for the case at the above ages.

VELOCITY = *h*

The velocities computed from the fitted model at the above
ages.

DESCRIPT = *i*

The descriptive parameters for the case.

MEAN = *j*

The estimated population mean and covariance matrix.

EVAL = *k*

Expected heights, velocities, and accelerations at specified ages for each case.

FOURIER = 1

Results from the Fourier analysis of each case.

CROSSECTION = *m*

Cross-sectional summaries.

RESIDUAL = *n*

Residuals from structural model for each case.

PERCENTILE = *o*

The 5, 25, 50, 75, and 95 percent points of the fitted or smoothed height and velocity cross-sections in steps of age specified by the STEPSIZE keyword of the PROCEDURE command. This file is in a form suitable for the PERCENTILE keyword of the TECHNICAL command.

NOTES

The first record of each file contains the keyword name of the result, the serial case number, the number of age levels for the case, the sex, and the case identification. The remaining records contain numerical results in fixed-column formats.

If two or more files are assigned the same name, the results will be concatenated in the order shown above.

The results of the SUMMARY procedure may be saved with MEAN,
CROSSSECTION, and PERCENTILE.

INPUT

(Required command)

PURPOSE

To describe the input data specifications. See also Chapter 2, Types of Data.

FORMAT

$$>\text{INPUT} \quad \text{TYPE} = a, \begin{bmatrix} \text{CM} \\ \text{IN} \end{bmatrix}, \text{HEader} = b, \text{FOrmat} = c,$$

$$\text{IGnore} = d, \text{SExcode} = (e, f), \text{IDchar} = g,$$

$$\text{LEngth} = h, \begin{bmatrix} \text{DAta} = i \\ \text{MAster} = j \end{bmatrix}, \text{MIssing} = k,$$

$$\text{FIelds} = (l, m, n, o), \text{REcords} = p, \text{TOp};$$

KEYWORDS

TYPE = a

(See also the examples in Chapter 6.)

$a = 1$: The data consist of a separate record for each observation. Each record must include fixed columns or fields for the case ID, for age in decimal years or months (see the AGES command), and for height in centimeters or inches according as CM or IN is selected. Records for successive cases follow one another without separators.

$a = 2$: Data for each case consist of a header record containing:
- N number of age levels for case ($0 < n < 51$, integer),
- S sex of subject coded by an arbitrary letter or number designated in SEXCODE
- MS mature stature (cm) of case if available; if not available, set to 0.0
- ID up to f characters of case identification (fA1);
- default is 8, maximum is 40

In type 2 input, the heFigure Ader record is followed by a data record or records consisting of N pairs of numbers, the first giving an age for the case and the second, the height for the case at that age. Both values must be decimal numbers. Unless MONTHS has been selected on the AGES command, the ages are in decimal years. Unless the IN option has been selected on the INPUT command, the heights are in centimeters.

a = 3: Data for each case consist of a header record as above, followed by a data record or records containing the *N* age levels for the case, followed by the *N* heights at these ages. Ages and heights must be decimal numbers.

a = 4: All cases are measured at the same age levels, which have been entered by the AGES command. Data for each case consist of a header record as above, followed by a record or records containing the heights at the corresponding ages. Heights must be decimal numbers.

HEADER = *b*

The case header record is in fixed-column format described by a variable format statement spanning *b* records (with parentheses) immediately following the INPUT command in the command file (1 $<$ *b* $<$ 3).

Default: If this keyword is *not* specified, the header record is in free-column format. A variable format statement is not required; entries in the header record must be delimited by spaces or commas on the right.
Note:
Because the 10 *field will often be of varying length, the free-column header record should end with a forward slash (f). If the free-column* 10 *begins with a number, contains punctuation or slashes, or more than eight characters, it must be placed in single quotes.*

FORMAT = *c*

The data record (file line) for each observation is in fixed-column format described by c (0 $<$ c $<$ 4) variable format records immediately following the header format records (if any). If the header is in free-column format, the data variable format record immediately follows the INPUT command record.

Data records contain N ages and heights as described for TYPE= 1,2, or 3. Data records for TYPE = 4 do not contain ages. Each data record for TYPE = 1 must also contain the case ID and sex code. If the data are in the command file, the end of data for TYPE=2, 3, or 4 is signaled by one completely blank header card following the data of the last case. If the data are in an external file, the blank header record is not required.

Default: If FORMAT is not selected, the data records are in free-column format (values contain decimal points and are delimited on the right by a space or a comma).

IGNORE = *d*

When the data records are in free-column format, it may happen that other fields precede the ages and heights. Setting *d* greater than zero causes the program to ignore *d* such leading fields. A 'field' is defined as a consecutive string of char-

acters terminated on the right by a space or comma.

Default: 0

SEXCODE = (e, f)

The codes for sex and case that appear on the header cards.
The codes may consist of integer numbers or any ASCII (plain
text) readable characters. If the codes are numbers, or char-
acters other than letters, and are designated alphanumeric
in the header or format record (A1), they should be enclosed
in single quotes.

    e the sexcode for males (default is M)
    f the sexcode for females (default is F)

IDeBAR = g

The number of consecutive identification characters to be
read from the case header or data cards *(i.e.,* the number of
columns in the ID field) ($0 < f < 41$).

Default: 8

LENGTH = h

The record length: the maximum number of characters in
the header and data records *($g < 512$).*

Default: 80

DATA = i

Header records and/or data appear in the file named by *i*
rather than in the command file. If the filename begins with
a number, is more than eight characters in length (including
path and extension), contains embedded blanks or punctua-
tion, it should be enclosed in single quotes.

MASTER = j

The data appear in an unformatted file, named *j*, prepared
by the SAVE command.

MISSING = k

If the measurement is missing at some age, either age or
height may be set to *k,* which may be any decimal number
outside the range of the data. The observation will then be ig-
nored and the number of observations for that case adjusted
downward accordingly.

Default: 0.0

FIELDS = (l, m, n, 0)

Sequence numbers of data fields, separated by blanks or a
comma, to be input to the program:

TYPE1 data: *1* = ID, *m* = sex, *n* = age, *o* = height
TYPE2 data: *l* = age, m = height (In Type 2 data, the ID and

sex are read from the header record.)

FIELDS may not be invoked for type 3 or 4 data. Free-column data input and the IGNORE keyword may be used with all data types.

RECORDS = *p*

The number of records (file lines) of data for each case (must be the same for all cases).

Default: 1

OPTION

CM/IN

If the cm option is specified, heights are in centimeters.
If the IN option is specified, heights are in inches.

Default: cm

TOP

Sets access to the data back to the top of the file for another analysis of the same data.

CONTINUE

(Optional command)

PURPOSE

To indicate that another analysis will follow in the command file.

FORMAT

>CONtinue;

If there is more than one sample of cases in the command file, the CONTINUE command marks the end of the commands and data of the current sample. If the data are in the command file, the CONTINUE command must follow the data.

STOP

(Required command)

PURPOSE

To indicate the end of the command file.

FORMAT

>STOp;

The **STOP** command marks the end of the command file. However, it may be followed by the **PLOT** command (see next page).

PLOT

(Optional command)

PURPOSE

To display plots on screen after the program finishes an analysis.

FORMAT

> PLOT

The PLOT command without any further specifications will result
in all plots being sequentially displayed on screen after a
successful
analysis. The user may select to display only a subset by adding
plot
numbers after the PLOT command.

EXAMPLES

```
)STOP;
        )PLOT 1 3 5
```

The program will display the first, third, and fifth plot.

```
)STOP;
)PLOT 4 3 4 3
```

The program will display the fourth plot, followed by the third
plot,
then repeats the sequence.

Variable Format Statements

Variable format statements describe the column assignments of fields in the data records. These statements have the following general form:

```
(aAl,nX,bAl,Fc.d,Tw,Fc.d)
```

where:
a	the number of columns in an ID field (default: $a = 8$)
b	the number of columns in a sex code (usually $b = 1$)
c	the number of columns in age or height field
d	the number of columns to the right of the decimal place in an age or height field
nX	columns skipped between fields, where n is the number of columns to be passed over
Tw	tab designator inserted in any of the fields, if the fields in the data records are not in the above order, or in place of nX (w = the position of the first column of the field, counting from column one)

The forward slash (J) means "read a new record." For example,

```
(SA1,SX,Al/T20,F6.2,TlO,F6.3)
```

would read the case ID and sex from record 1; next, skip to the second Record and tab to column 20 to read age from columns 20-25; and finally, tab to column 10 on the same record to read height from columns 10-15. See also the examples in Chapter 6.

4. The AUXAL User Interface

Using the AUXAL User Interface

Start AUXAL by double clicking on the **axlwin.exe** program file or select the AUXAL program from the Windows Start, Programs menu.
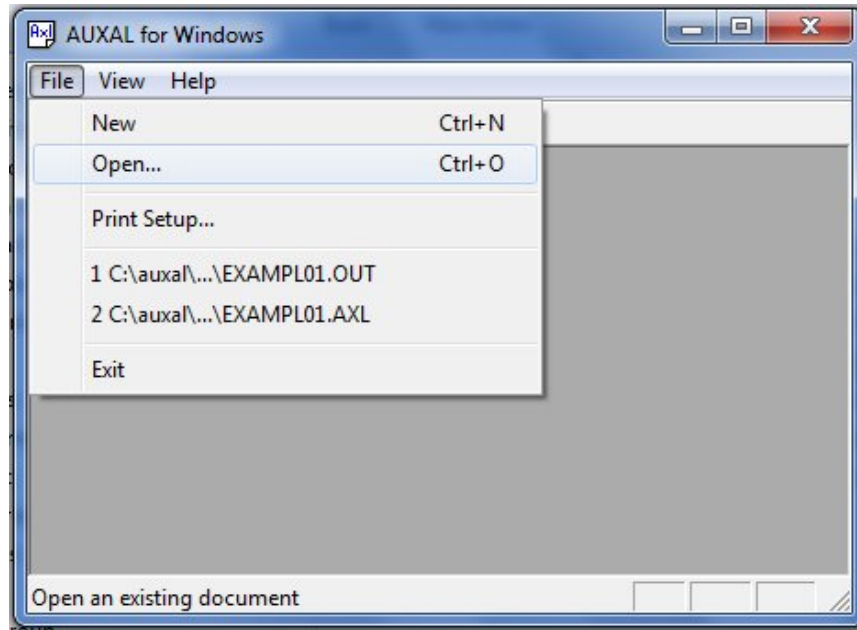
Figure 4.1   **Select "Open" from the AUXAL File Menu**

From the main menu bar (see Fig. 4.1), select the File, Open option and browse for the folder where examples were installed to. In this guide, it is assumed that the default example folder is c:\program files\auxal3\examples). Note that all AUXAL command files have the file extension axl.

Use the Open dialog box (see Fig. 4.2 to select, for example, the command file exampl06.axl and click the Open button. The command file is displayed as shown in Fig. 4.3. Note that the main menu bar has changed too.
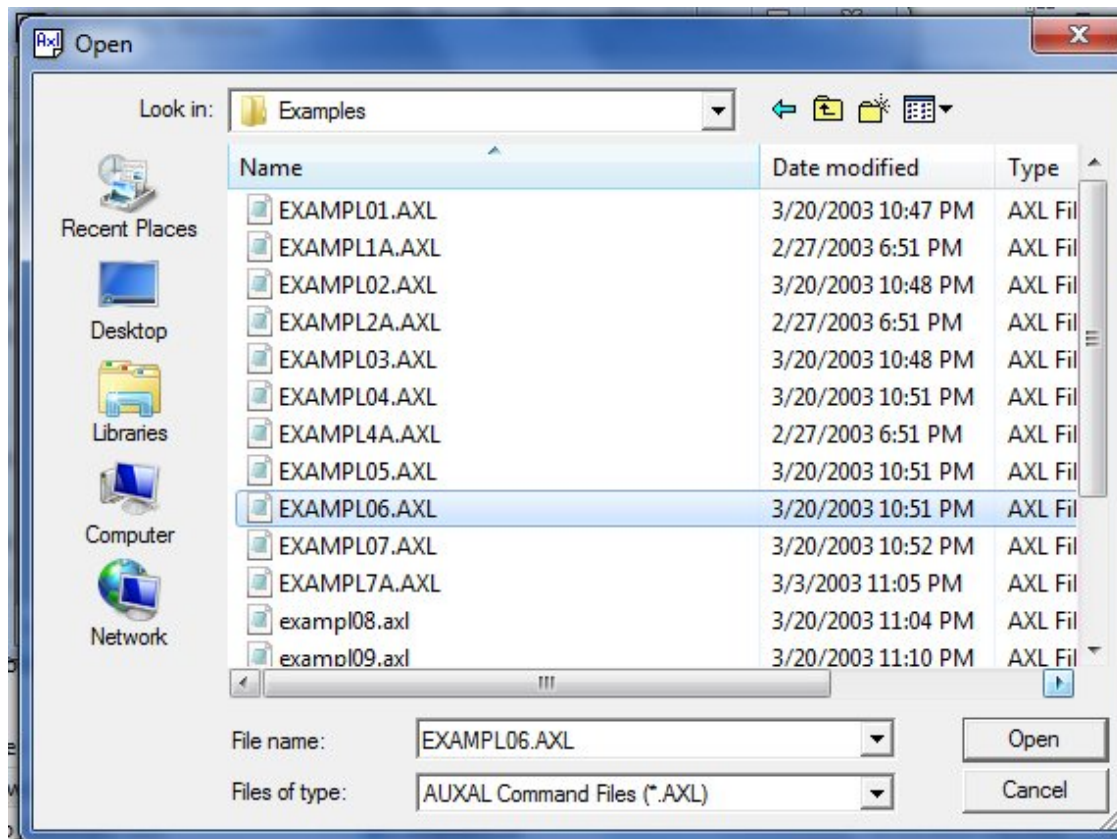
Figure 4.2   The "Open" Dialog Box
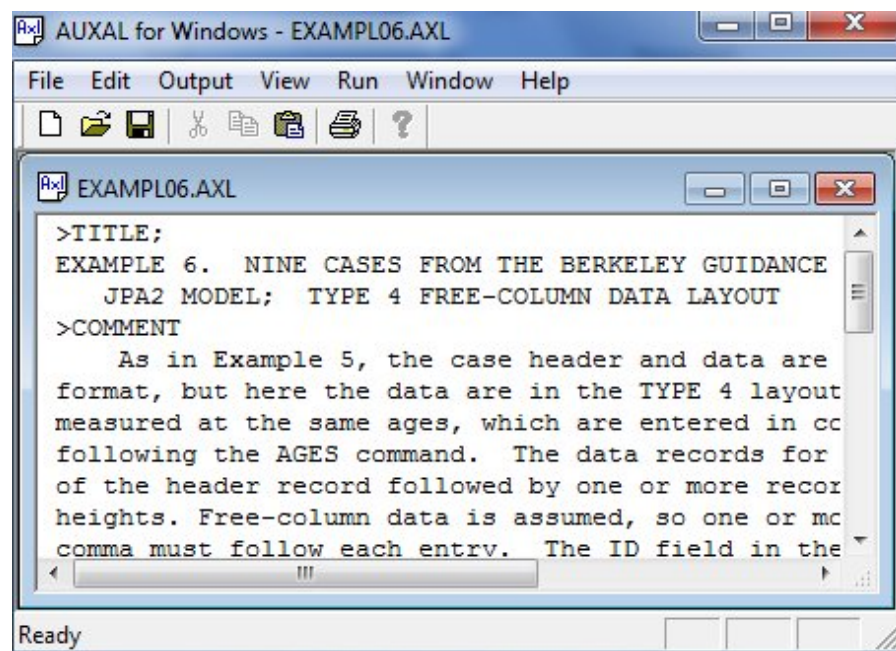
Figure 4.3   The Command File Example 6

To run Example 6, click on the Run button and select the Run
EXAMPL06 option as shown in Fig. 4.4.
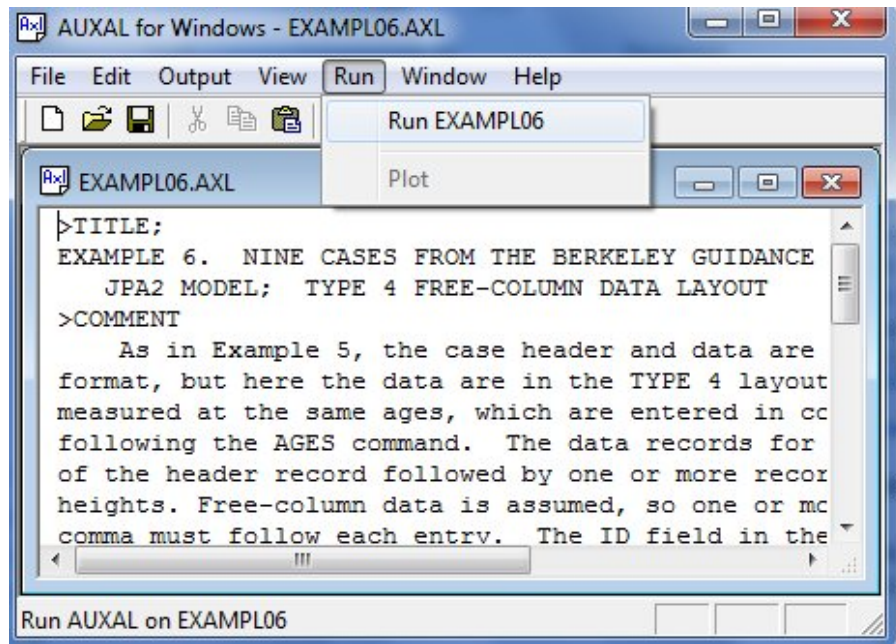
Figure 4.4   Select "Run EXAMPLOO" from the Run Menu

A portion of the resulting list output file is displayed in Fig. 4.5. Browse the output file by using the horizontal and vertical scroll bars or other standard Windows functions.

Growth Curve Displays

The AUXAL companion program AUXALPLOT produces high quality graphs of each case analyzed by the program, and - when the SUMMARY procedure is invoked - of the structural and nonstructural average curves. The plots include observed heights, empirical annual velocities, fitted height and velocity curves, and indicators of velocity maxima and minima.

When the user selects the DISPLAY option of the PROCEDURE command, the program prepares two files that pass plotting information cases to the plotting program. The program displays the height and velocity curves for each case successively on the computer monitor, and allows the user to print selected cases or save the plot as a Windows metafile (WMF or EMF format).

The plot displays also include percentile bands at the 5, 25, 50, 75, and 95 cross-sectional percent points for height and velocity. The
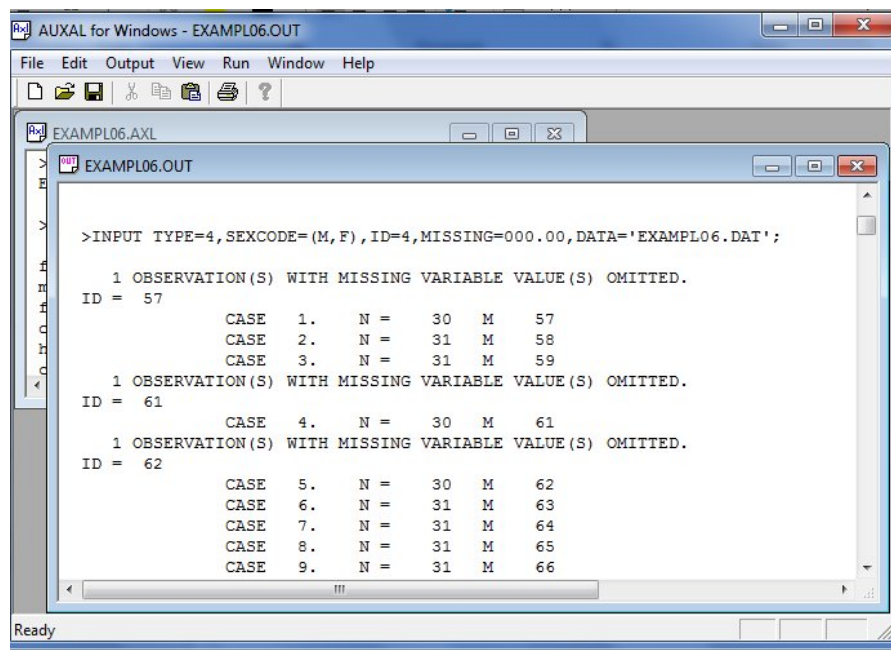
Figure 4.5   The EXAMPL06.0UT list output file

default values for the height percentiles supplied by the program are from Hamill, *et al.* (1979). Those for velocity are from Tanner & Davis (1985). A provision ofthe TECHNICAL command of the program allows the user to replace the default percentiles with other values.

The AUXAL program prints a listing of values represented in the growth curve displays. These include the ages and observed heights and velocities. In order to avoid seasonal effects on growth, the empirical velocities are calculated from differences in observed heights only for those pairs of age points that differ by approximately one year (± 0.15 yrs). The result is printed on the line of the older age. These are also the values shown in the displayed curve at the mid-point of the age interval This listing also includes the fitted or smoothed heights and velocities at the observed ages and at integral years. The residuals calculated as observed height minus fitted or smoothed heights are also listed.

Each plot is accompanied by a box (see Fig. 4.6) containing the legends for the growth curve display.

- OBS HGT
FIT HGT
○ OBS VEL
FIT VEL
▲ INFLX PTS
— ·· HGT 95%
— · HGT 75%
— HGT 50%
— · HGT 25%
— ·· HGT 5%
— ·· VEL 95%
— · VEL 75%
— VEL 50%
— · VEL 25%
— ·· VEL 5%
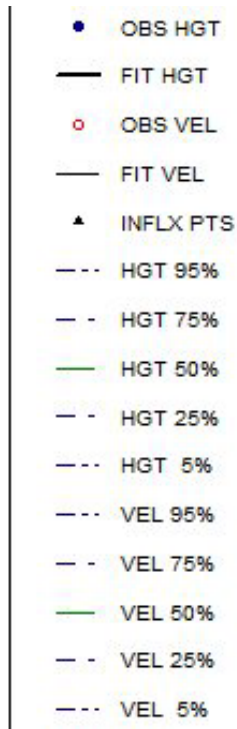
Figure 4.6   Legend box included with the growth curve displays

Using the Plot Option

For a graphical representation of the fitted and observed data, select the Plot option from the Run menu as shown in Fig. 4.4. Note that the Plot option will not be enabled if the analysis did not complete successfully.

Several files may be open simultaneously as a result of running a number of different analyses in one session. To view the correct plot, either close all irrelevant files, or click on the top bar of the axl file of interest to make this the active window.

By clicking on the Plot option, the graphics module is activated and the screen shown in Fig. 4.7 is displayed. This screen contains options to plot curves with or without percentiles and to exit the program.

Growth Curves with Percentiles   Plot Curves

Growth Curves without Percentiles   No Percentiles

Exit Plot Program   Exit
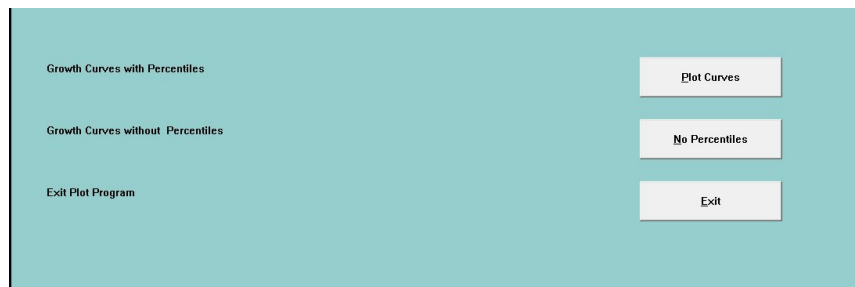
Figure 4.7 The AUXAL Graphics Main Window

To obtain curves with percentiles, click the Plot Curves button. The

graphics display in Fig. 4.9 shows, for case number 1, the observed curves for height and velocity plotted against age (in years). One can use the Next button to go to a similar display for case 2, case 3, etc., and the Prev button to return, for example, from case 6 to case 5.
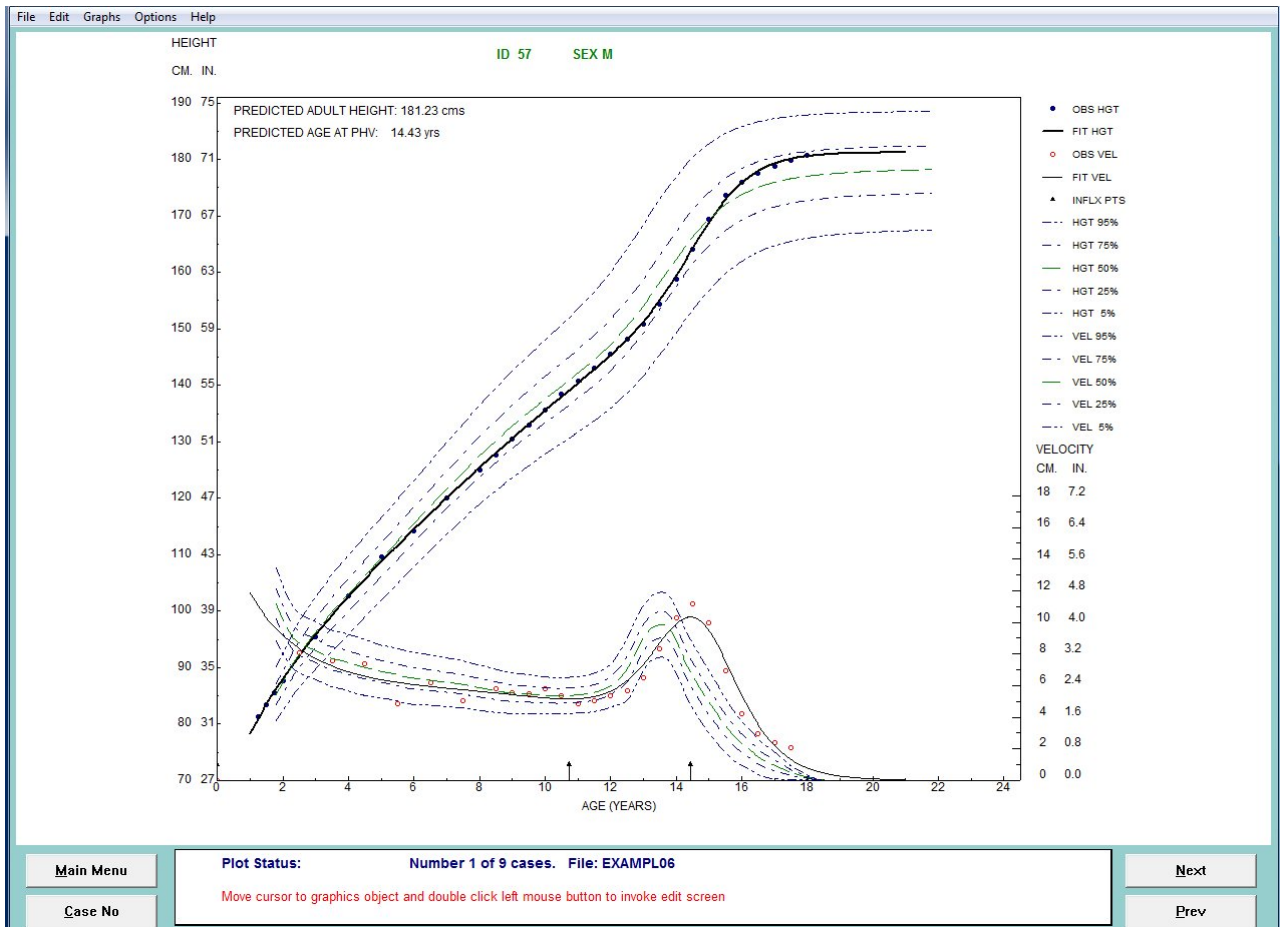
4.9 Growth Curve for Case 1 of Example 6 without Percentiles

To advance to a specific case number, one can also click on the Case No button to open the Enter Case Number dialog box. Type the desired case number and click the OK button to produce the growth curve for that specific case. Note that each display also contains the 5, 25, 50, 75, and 95 percentiles for both height and velocity.

To return to the main menu, click the Main Menu button. The graphics display shown in Fig. 4.10 was produced by clicking the No Percentiles button on the AUXAL Graphics main menu. Note that the graph can be saved as a Windows metafile or printed from the File menu. Select the Help button to learn how to change the colors, fonts, and text of displays.
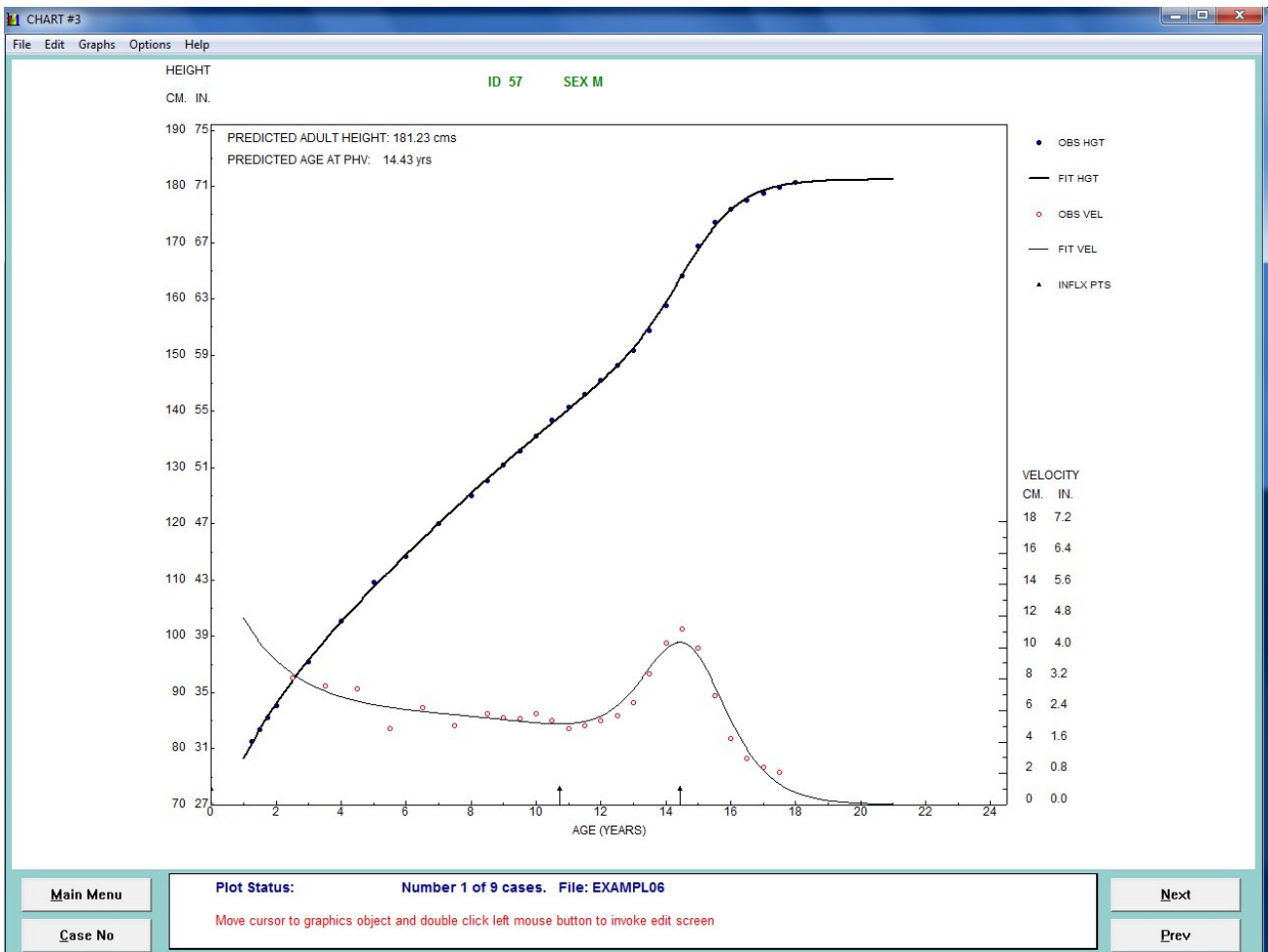
Figure 4.10   Growth Curve for Case 1 of Example 6 without Percentiles.

5. Using Priors

Adjusting Priors for Different Populations

If measurements of a large sample of N cases from a suitable longitudinal growth study are available, the population mean covariance matrix of the model parameters can be estimated from the MAP estimate and posterior covariance matrix for each case.

Because the population mean and covariance matrix are required in the prior distribution for estimating the parameter means and covariances, a "boot strap" procedure is required in their use. Initially, one starts with the existing AUXAL priors for the BIT, JPA2, or Jenss-Bayley models. These priors are based on USA data. Provided the number of well-spaced data points per case exceeds, say, 20, a pass through the cases with this provisional prior will give a good approximation to the population quantities. The revised prior can be saved to an external file via the SAVE command.

```
>SAVE means='priors.par';
```

The file priors. par contains the estimated population mean and population covariance matrix of the model parameters.

A second or third pass, each time substituting the resulting provisional prior will yield a sufficiently accurate estimation of the population mean and covariance matrix for practical use in MAP estimates of the model parameters. The prior obtained from the previous run is read into the program using the commands:

```
>MEAN MALE FILE='male.mea';

>COVARIANCE MALE FILE='male.cov';
```

See examples exampl10.axl and exampl11.axl.

6.Sample Command and Data Files

Some of the sample problems that are included with the program are used in this chapter to illustrate the various options and modes of data input that AUXAL provides. Each example shows the files - command, data, and cases files - that the user supplies to the program. Chapter 6 shows the corresponding program output for the first example. The command file of each example includes comments describing the analysis and the relevant commands with options and keywords. The optional input and output files of each example are designated in various commands. Data for these examples are drawn from the Fels Longitudinal Study and from the Berkeley Guidance Study as reported by Tuddenham & Snyder (1954).

Example 1

Five consecutive female cases from the Fels data. Fourier smoothing of residuals from the BTT model. Type 1 data layout.
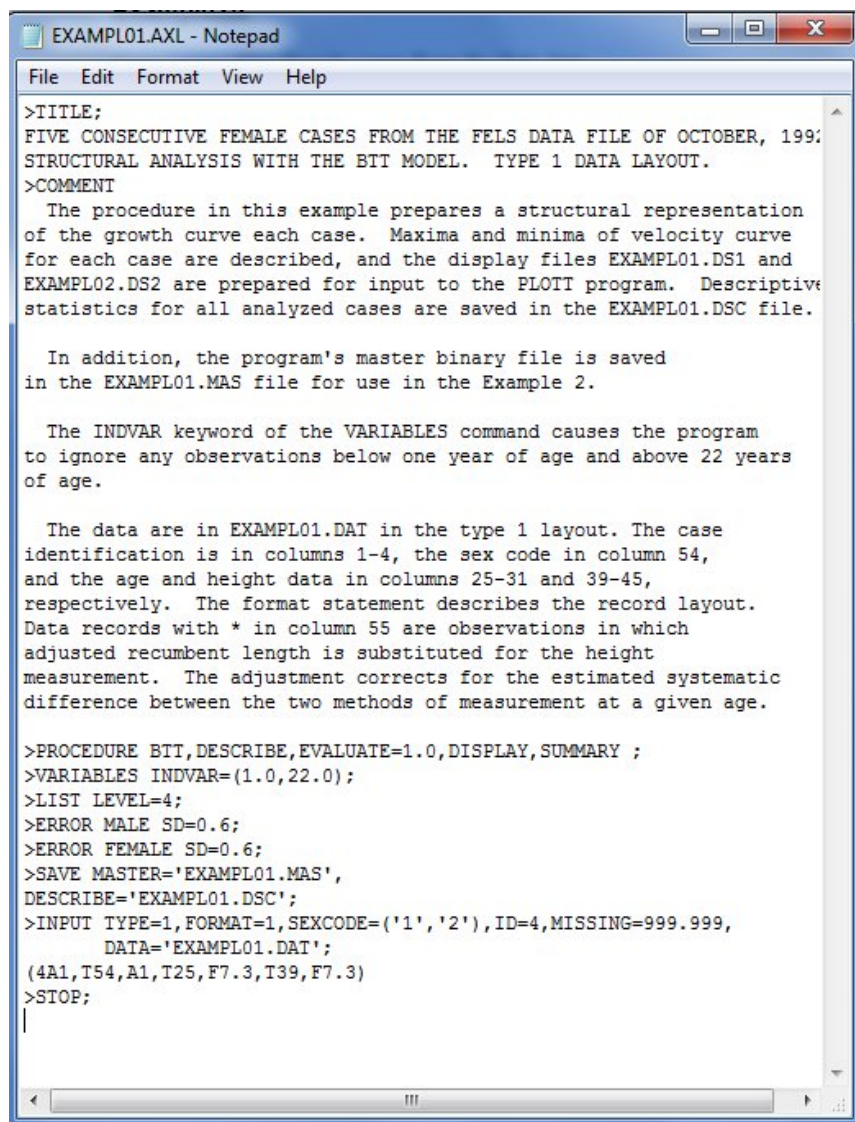
Data File: EXAMPL01.DAT

```
5872281 47621001621001 0.000999.999999.999 3.18092
5872281 47621001621103 0.089     53.400      4.21092*
5872281 47621001630107 0.266 60.400 59.400  6.15092*
5872281 47621001630406 0.514     65.200      7.26092*
```

Similar lines

```
5962104 114630407780620 15.202162.250161.600 54.20092
5962104 114630407790620 16.202163.150162.250 54.70092
5962104 114630407800412 17.014162.700162.300 55.40092
5962104 114630407810414 18.019163.600162.250 56.20002
```

Command File: EXAMPL01.AXL

```
EXAMPL01.AXL - Notepad

File   Edit   Format   View   Help

>TITLE;
FIVE CONSECUTIVE FEMALE CASES FROM THE FELS DATA FILE OF OCTOBER, 1992
STRUCTURAL ANALYSIS WITH THE BTT MODEL.  TYPE 1 DATA LAYOUT.
>COMMENT
  The procedure in this example prepares a structural representation
of the growth curve each case.  Maxima and minima of velocity curve
for each case are described, and the display files EXAMPL01.DS1 and
EXAMPL02.DS2 are prepared for input to the PLOTT program.  Descriptive
statistics for all analyzed cases are saved in the EXAMPL01.DSC file.

  In addition, the program's master binary file is saved
in the EXAMPL01.MAS file for use in the Example 2.

  The INDVAR keyword of the VARIABLES command causes the program
to ignore any observations below one year of age and above 22 years
of age.

  The data are in EXAMPL01.DAT in the type 1 layout. The case
identification is in columns 1-4, the sex code in column 54,
and the age and height data in columns 25-31 and 39-45,
respectively.  The format statement describes the record layout.
Data records with * in column 55 are observations in which
adjusted recumbent length is substituted for the height
measurement.  The adjustment corrects for the estimated systematic
difference between the two methods of measurement at a given age.

>PROCEDURE BTT,DESCRIBE,EVALUATE=1.0,DISPLAY,SUMMARY ;
>VARIABLES INDVAR=(1.0,22.0);
>LIST LEVEL=4;
>ERROR MALE SD=0.6;
>ERROR FEMALE SD=0.6;
>SAVE MASTER='EXAMPL01.MAS',
DESCRIBE='EXAMPL01.DSC';
>INPUT TYPE=1,FORMAT=1,SEXCODE=('1','2'),ID=4,MISSING=999.999,
      DATA='EXAMPL01.DAT';
(4A1,T54,A1,T25,F7.3,T39,F7.3)
>STOP;
```

Example 1A

Five consecutive female cases from the Fels data. Fourier smoothing
of residuals from the BTT model. Type 1 input from data fields.

Command File: EXAMPL1A.AXL

```
EXAMPL1A.AXL - Notepad

File  Edit  Format  View  Help

FIVE CONSECUTIVE FEMALE CASES FROM THE FELS DATA FILE OF OCTOBER, 199:
  STRUCTURAL ANALYSIS WITH THE JPA2 MODEL.   TYPE 1 INPUT FROM DATA F:
>COMMENT
  This is analysis with the data of Example 1, but the data records
have been altered so that blank spaces separate the various
data fields.  Data in this form can be input by selecting the
ID, sex, age, and height fields, in that order, using the
FIELDS keyword of the INPUT command.  The number of records that
make up each observation must also be specified in the RECORDS
keyword of the command.  A data format statement is not required.

    The data layout is illustrated by the following two records from
the EXAMPL1A.DAT file.

 587 2281   47621001621001  0.000 999.999 999.999  3.1809 2
 587 2281   47621001621103  0.089  53.400  52.400  4.2109 2 *

    The Jolicoeur, Pontier, and Abidi Model 2 is fitted.

>PROCEDURE JPA2,DESCRIBE,DISPLAY,SUMMARY;
>VARIABLES INDVAR=(1.0,22.0);
>LIST LEVEL=4;
>ERROR MALE SD=0.6;
>ERROR FEMALE SD=0.6;
>TECHNICAL LOWER=1.0,UPPER=18.0;
>INPUT TYPE=1,SEXCODE=('1','2'),ID=4,MISSING=999.999,
       DATA='EXAMPL1A.DAT',RECORDS=1,FIELDS=(1,8,4,6);
>STOP;
```

Data File: EXAMPL1A.DAT
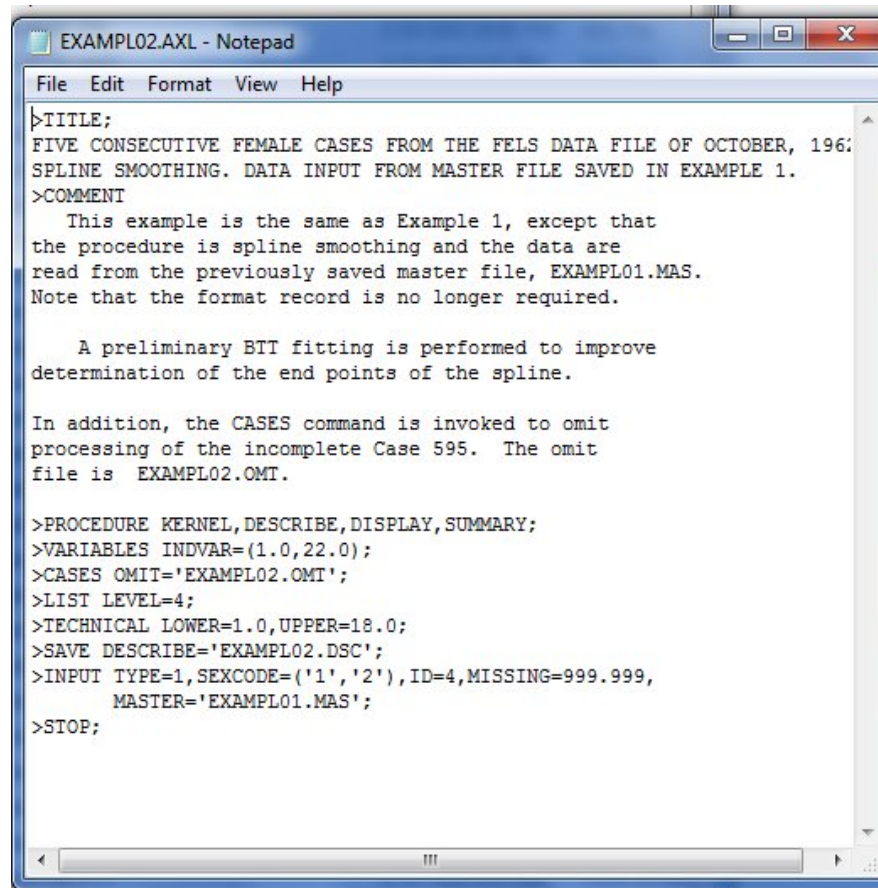
```
596 21041 14630407780620 15.202 162.250 161.600 54.2009 2
596 21041 14630407790620 16.202 163.150 162.250 54.7009 2
596 21041 14630407800412 17.014 162.700 162.300 55.4009 2
596 21041 14630407810414 18.019 163.600 162.250 56.2000 2
```

Similar lines omitted.

```
596 21041 14630407780620 15.202 162.250 161.600 54.2009 2
596 21041 14630407790620 16.202 163.150 162.250 54.7009 2
596 21041 14630407800412 17.014 162.700 162.300 55.4009 2
596 21041 14630407810414 18.019 163.600 162.250 56.2000 2
```

Example 2

Five consecutive female cases from the Fels data. Kernel smoothing of heights, velocities, and accelerations. Data are input from a master file saved in Example 1.

```
EXAMPL02.AXL - Notepad

File  Edit  Format  View  Help

>TITLE;
FIVE CONSECUTIVE FEMALE CASES FROM THE FELS DATA FILE OF OCTOBER, 196:
SPLINE SMOOTHING. DATA INPUT FROM MASTER FILE SAVED IN EXAMPLE 1.
>COMMENT
    This example is the same as Example 1, except that
the procedure is spline smoothing and the data are
read from the previously saved master file, EXAMPL01.MAS.
Note that the format record is no longer required.

    A preliminary BTT fitting is performed to improve
determination of the end points of the spline.

In addition, the CASES command is invoked to omit
processing of the incomplete Case 595.  The omit
file is  EXAMPL02.OMT.

>PROCEDURE KERNEL,DESCRIBE,DISPLAY,SUMMARY;
>VARIABLES INDVAR=(1.0,22.0);
>CASES OMIT='EXAMPL02.OMT';
>LIST LEVEL=4;
>TECHNICAL LOWER=1.0,UPPER=18.0;
>SAVE DESCRIBE='EXAMPL02.DSC';
>INPUT TYPE=1,SEXCODE=('1','2'),ID=4,MISSING=999.999,
       MASTER='EXAMPL01.MAS';
>STOP;
```
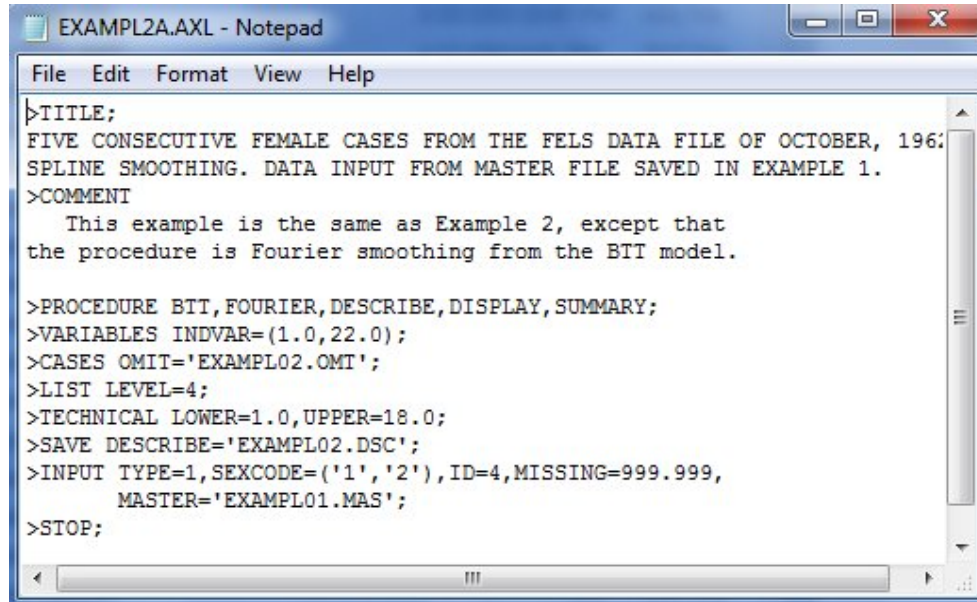
Omit file: EXAMPL02.0MT

595

Example2A

Five consecutive female cases from the Fels data.

Command File: EXAMPL2A.AXL
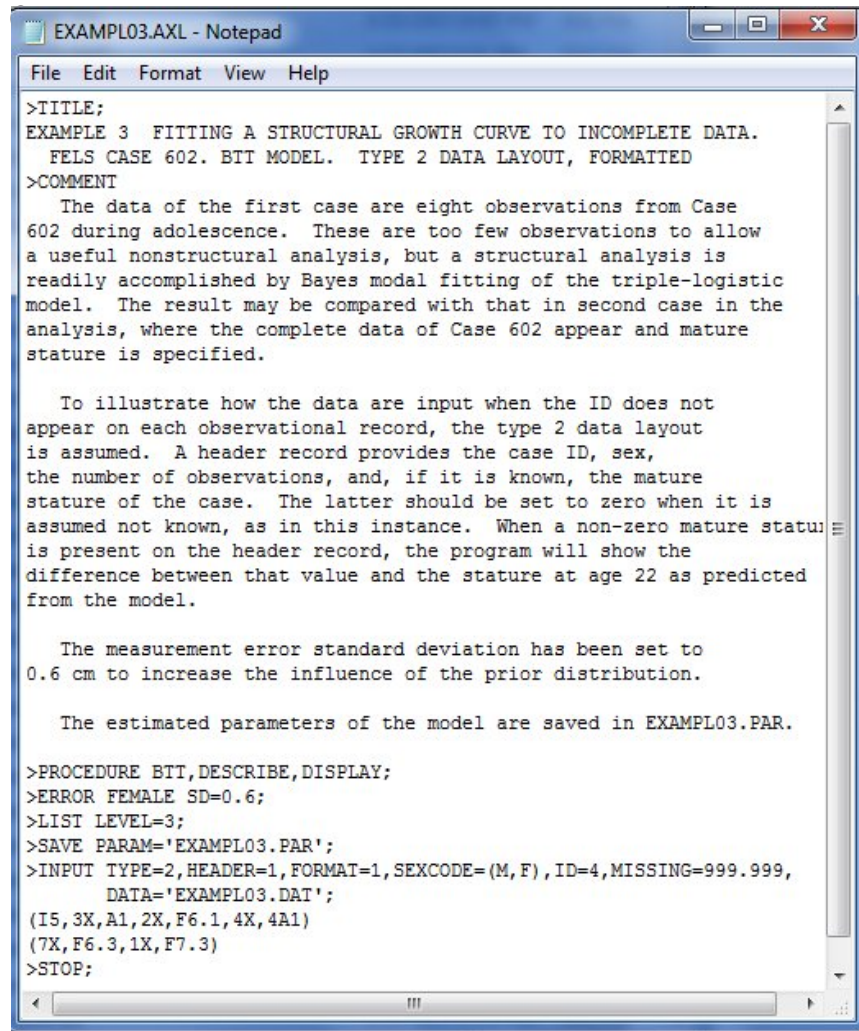
```
EXAMPL2A.AXL - Notepad

File   Edit   Format   View   Help

>TITLE;
FIVE CONSECUTIVE FEMALE CASES FROM THE FELS DATA FILE OF OCTOBER, 1962
SPLINE SMOOTHING. DATA INPUT FROM MASTER FILE SAVED IN EXAMPLE 1.
>COMMENT
    This example is the same as Example 2, except that
the procedure is Fourier smoothing from the BTT model.

>PROCEDURE BTT,FOURIER,DESCRIBE,DISPLAY,SUMMARY;
>VARIABLES INDVAR=(1.0,22.0);
>CASES OMIT='EXAMPL02.OMT';
>LIST LEVEL=4;
>TECHNICAL LOWER=1.0,UPPER=18.0;
>SAVE DESCRIBE='EXAMPL02.DSC';
>INPUT TYPE=1,SEXCODE=('1','2'),ID=4,MISSING=999.999,
       MASTER='EXAMPL01.MAS';
>STOP;
```

Example 3

Fitting a structural growth curve to partial data of Fels Case 602.
BTT model; type 2 data layout, formatted.

```
>TITLE;
EXAMPLE 3  FITTING A STRUCTURAL GROWTH CURVE TO INCOMPLETE DATA.
  FELS CASE 602. BTT MODEL.  TYPE 2 DATA LAYOUT, FORMATTED
>COMMENT
   The data of the first case are eight observations from Case
602 during adolescence.  These are too few observations to allow
a useful nonstructural analysis, but a structural analysis is
readily accomplished by Bayes modal fitting of the triple-logistic
model.  The result may be compared with that in second case in the
analysis, where the complete data of Case 602 appear and mature
stature is specified.

   To illustrate how the data are input when the ID does not
appear on each observational record, the type 2 data layout
is assumed.  A header record provides the case ID, sex,
the number of observations, and, if it is known, the mature
stature of the case.  The latter should be set to zero when it is
assumed not known, as in this instance.  When a non-zero mature statur
is present on the header record, the program will show the
difference between that value and the stature at age 22 as predicted
from the model.

   The measurement error standard deviation has been set to
0.6 cm to increase the influence of the prior distribution.

   The estimated parameters of the model are saved in EXAMPL03.PAR.

>PROCEDURE BTT,DESCRIBE,DISPLAY;
>ERROR FEMALE SD=0.6;
>LIST LEVEL=3;
>SAVE PARAM='EXAMPL03.PAR';
>INPUT TYPE=2,HEADER=1,FORMAT=1,SEXCODE=(M,F),ID=4,MISSING=999.999,
       DATA='EXAMPL03.DAT';
(I5,3X,A1,2X,F6.1,4X,4A1)
(7X,F6.3,1X,F7.3)
>STOP;
```

Data File: EXAMPL03.DAT

```
   8 F      0.0     602
4740427 10.536133.100133.500 29.40092
4741012 10.995136.500136.000 30.90092
4750412 11.495139.200137.800 32.20092
4751011 11.992142.300141.100 34.70092
```
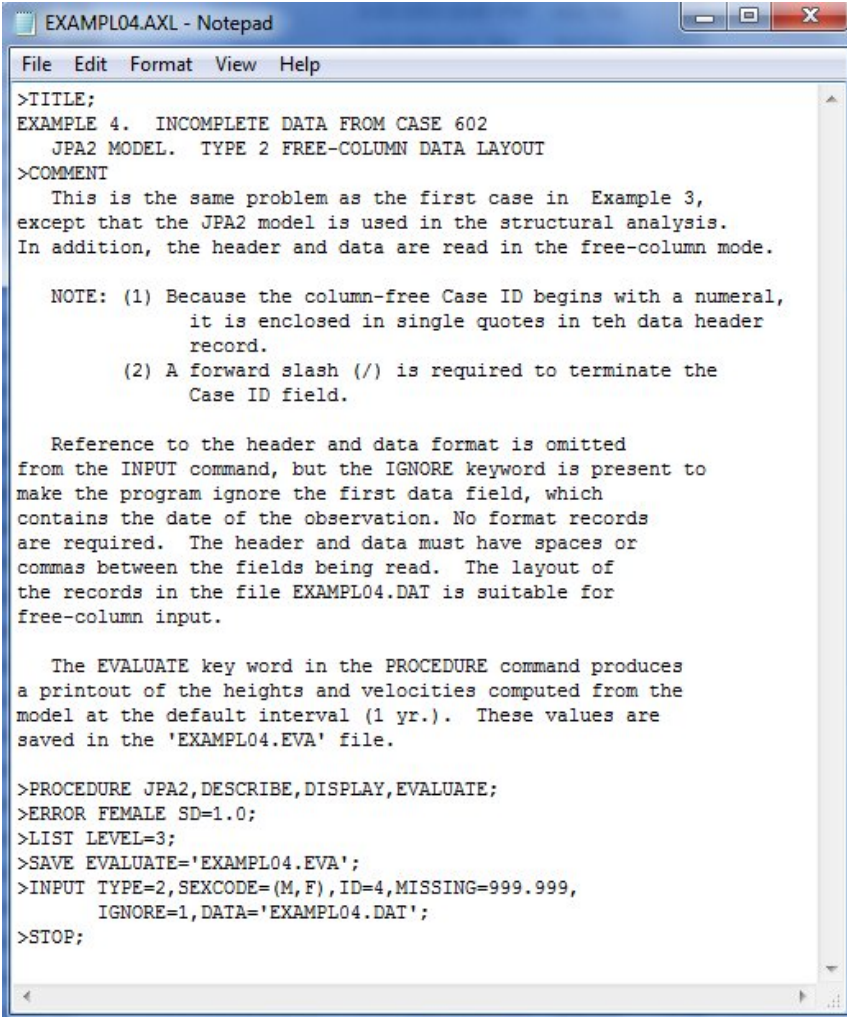
Similar lines omitted

```
4791013 15.997161.050160.150 57.00092
4801011 16.992162.100161.050 60.15092
4811229 18.208163.000161.450 56.90002
4831108 20.067162.850162.150 54.75002
```

Example 4

Analysis of Fels Case 602 with the JPA2 model. Type 2, free-column data.

Command file: EXAMPL04.AXL

```
EXAMPL04.AXL - Notepad
File  Edit  Format  View  Help
>TITLE;
EXAMPLE 4.   INCOMPLETE DATA FROM CASE 602
   JPA2 MODEL.   TYPE 2 FREE-COLUMN DATA LAYOUT
>COMMENT
   This is the same problem as the first case in  Example 3,
except that the JPA2 model is used in the structural analysis.
In addition, the header and data are read in the free-column mode.

   NOTE: (1) Because the column-free Case ID begins with a numeral,
             it is enclosed in single quotes in teh data header
             record.
         (2) A forward slash (/) is required to terminate the
             Case ID field.

   Reference to the header and data format is omitted
from the INPUT command, but the IGNORE keyword is present to
make the program ignore the first data field, which
contains the date of the observation. No format records
are required.  The header and data must have spaces or
commas between the fields being read.  The layout of
the records in the file EXAMPL04.DAT is suitable for
free-column input.

   The EVALUATE key word in the PROCEDURE command produces
a printout of the heights and velocities computed from the
model at the default interval (1 yr.).  These values are
saved in the 'EXAMPL04.EVA' file.

>PROCEDURE JPA2,DESCRIBE,DISPLAY,EVALUATE;
>ERROR FEMALE SD=1.0;
>LIST LEVEL=3;
>SAVE EVALUATE='EXAMPL04.EVA';
>INPUT TYPE=2,SEXCODE=(M,F),ID=4,MISSING=999.999,
       IGNORE=1,DATA='EXAMPL04.DAT';
>STOP;
```
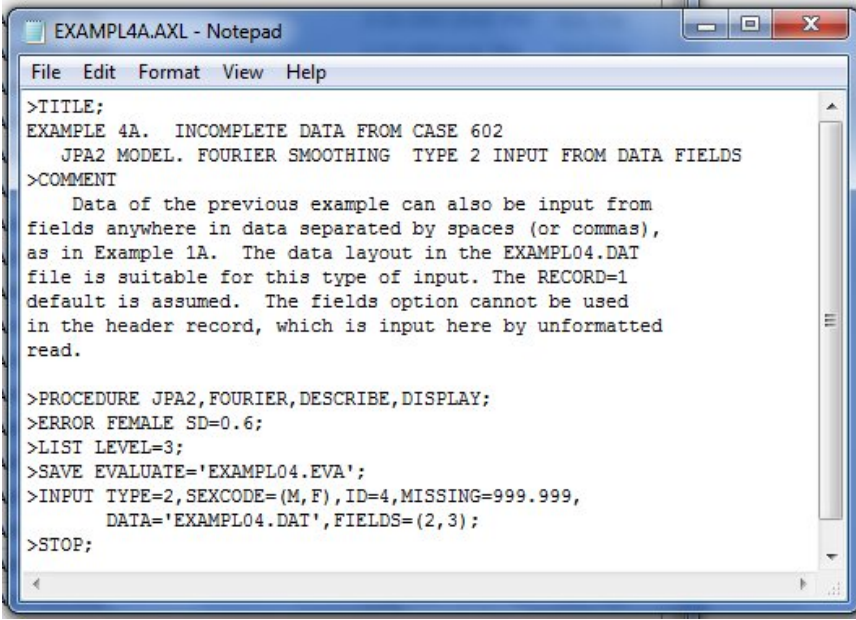
Data file: EXAMPL04.DAT
```
   8   F    162.3    '602'/
4740427 10.536    133.500
4741012 10.995   136.000
4750412 11.495   137.800
4751011 11. 992    141.100
4760403 12.470    145.100
4761008 12.984  148.000
4770405 13.475   151.400
4771024 14.027    155.150
```

Example4A

Analysis of Case 602 with the JPA2 model. Type 2 input from data fields.

Command File: EXAMPL4A.AXL
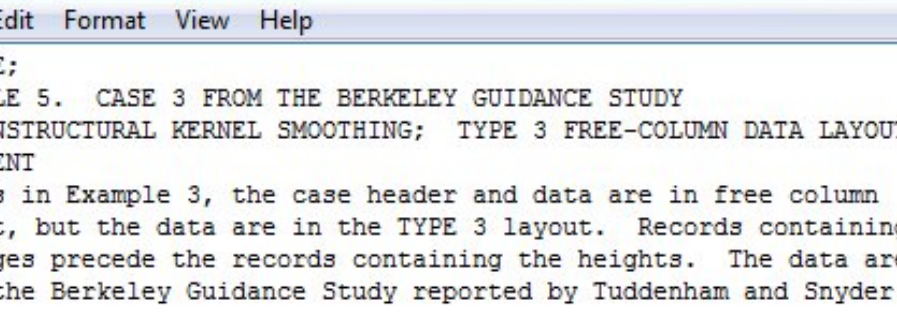
```
EXAMPL4A.AXL - Notepad
File  Edit  Format  View  Help

>TITLE;
EXAMPLE 4A.  INCOMPLETE DATA FROM CASE 602
   JPA2 MODEL. FOURIER SMOOTHING  TYPE 2 INPUT FROM DATA FIELDS
>COMMENT
    Data of the previous example can also be input from
fields anywhere in data separated by spaces (or commas),
as in Example 1A.  The data layout in the EXAMPL04.DAT
file is suitable for this type of input. The RECORD=1
default is assumed.  The fields option cannot be used
in the header record, which is input here by unformatted
read.

>PROCEDURE JPA2,FOURIER,DESCRIBE,DISPLAY;
>ERROR FEMALE SD=0.6;
>LIST LEVEL=3;
>SAVE EVALUATE='EXAMPL04.EVA';
>INPUT TYPE=2,SEXCODE=(M,F),ID=4,MISSING=999.999,
       DATA='EXAMPL04.DAT',FIELDS=(2,3);
>STOP;
```

Example5

A case from the Berkeley Guidance Study analyzed by Fourier smoothing of residuals from the JPA2 model. Type 3, free-column data.

Command File: EXAMPLO5.AXL

```
EXAMPLO5.AXL - Notepad

File   Edit   Format   View   Help

>TITLE;
EXAMPLE 5.   CASE 3 FROM THE BERKELEY GUIDANCE STUDY
   NONSTRUCTURAL KERNEL SMOOTHING;  TYPE 3 FREE-COLUMN DATA LAYOUT
>COMMENT
     As in Example 3, the case header and data are in free column
format, but the data are in the TYPE 3 layout.  Records containing
the ages precede the records containing the heights.  The data are
from the Berkeley Guidance Study reported by Tuddenham and Snyder.

     Kernel smoothing nonstructural analysis is performed. A
preliminary BTT fitting is performed to improve determination
of the end points of the kernel smooth.

>PROCEDURE   KERNEL,DESCRIBE,DISPLAY;
>ERROR MALE SD=1.0;
>LIST LEVEL=4;
>INPUT TYPE=3,SEXCODE=(M,F),ID=4,MISSING=000.00,
DATA='exampl05.dat';
>STOP;
```
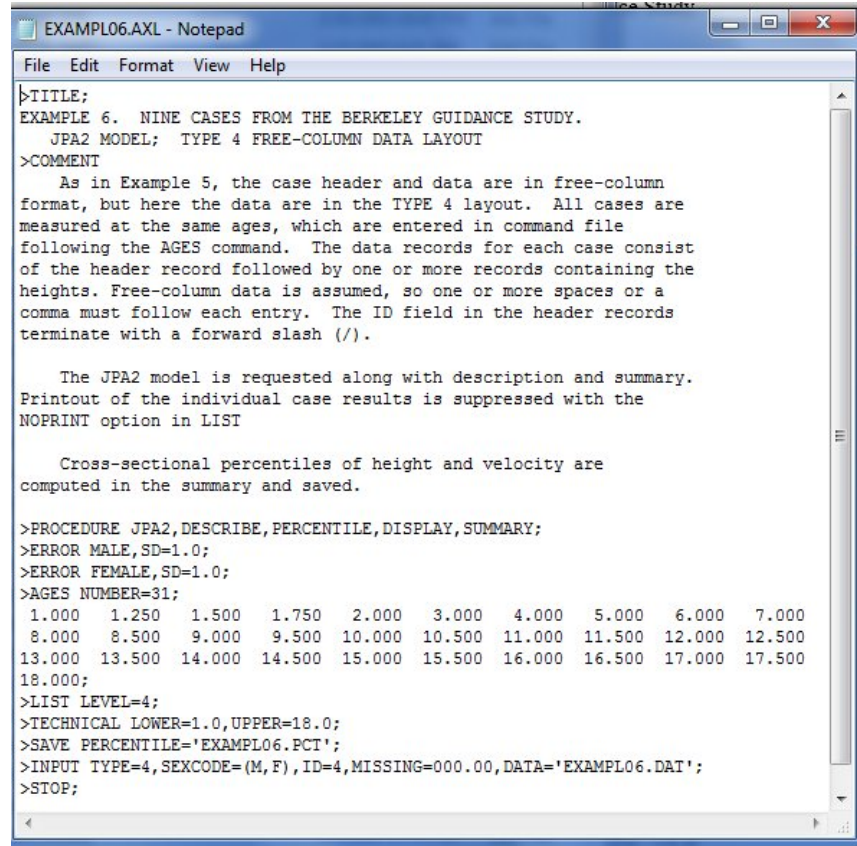
Data File: Exampl05.DAT

```
   29  M   0.0      ' 3'/
  1.500  1. 750   2.000   3.000   4.000   5.000   6.000   7.000   8.000   8.500
  9.000   9.500  10.000  10.500  11.000  11.500  12.000  12.500  13.000  13.500
 14.000  14.500  15.000  15.500  16.000  16.500  17.000 17 .500  18.000
  80.80   83.60   86.40   96.70  104.10 111. 40  11S.40  126.00 131. 40  134.20
 136.50  139.00  142.10  145.20  147.90  150.30  152.30  154.50  156.20  157.90
 160.40  163.10  165.90  168.70  172.00  175.10  17S.40 181. 20  183.70
```

Example 6

Kernel smoothing of nine cases from the Berkeley Guidance Study.
Type 4, free-column data.

Command File: EXAMPL06.AXL

```
EXAMPL06.AXL - Notepad
File  Edit  Format  View  Help
>TITLE;
EXAMPLE 6.  NINE CASES FROM THE BERKELEY GUIDANCE STUDY.
   JPA2 MODEL;  TYPE 4 FREE-COLUMN DATA LAYOUT
>COMMENT
    As in Example 5, the case header and data are in free-column
format, but here the data are in the TYPE 4 layout.  All cases are
measured at the same ages, which are entered in command file
following the AGES command.  The data records for each case consist
of the header record followed by one or more records containing the
heights. Free-column data is assumed, so one or more spaces or a
comma must follow each entry.  The ID field in the header records
terminate with a forward slash (/).

    The JPA2 model is requested along with description and summary.
Printout of the individual case results is suppressed with the
NOPRINT option in LIST

    Cross-sectional percentiles of height and velocity are
computed in the summary and saved.

>PROCEDURE JPA2,DESCRIBE,PERCENTILE,DISPLAY,SUMMARY;
>ERROR MALE,SD=1.0;
>ERROR FEMALE,SD=1.0;
>AGES NUMBER=31;
 1.000   1.250   1.500   1.750   2.000   3.000   4.000   5.000   6.000   7.000
 8.000   8.500   9.000   9.500  10.000  10.500  11.000  11.500  12.000  12.500
13.000  13.500  14.000  14.500  15.000  15.500  16.000  16.500  17.000  17.500
18.000;
>LIST LEVEL=4;
>TECHNICAL LOWER=1.0,UPPER=18.0;
>SAVE PERCENTILE='EXAMPL06.PCT';
>INPUT TYPE=4,SEXCODE=(M,F),ID=4,MISSING=000.00,DATA='EXAMPL06.DAT';
>STOP;
```

Data File:EXAMPL06.DAT

```
 31  M 0.0     ' 57'/
000.00    S1.30    83.50    85.60    87.70    95.40   102.60   109.60   114.20   120.10
124.90   127.60   130.40   132.90   135.60   13S .40   140.70   14 3.00   145.50   14S.10
150.90   154.30   158.S0   164.10   169.40   173.60   176.00   177.60   178.80   179.90
1S0.S0
```
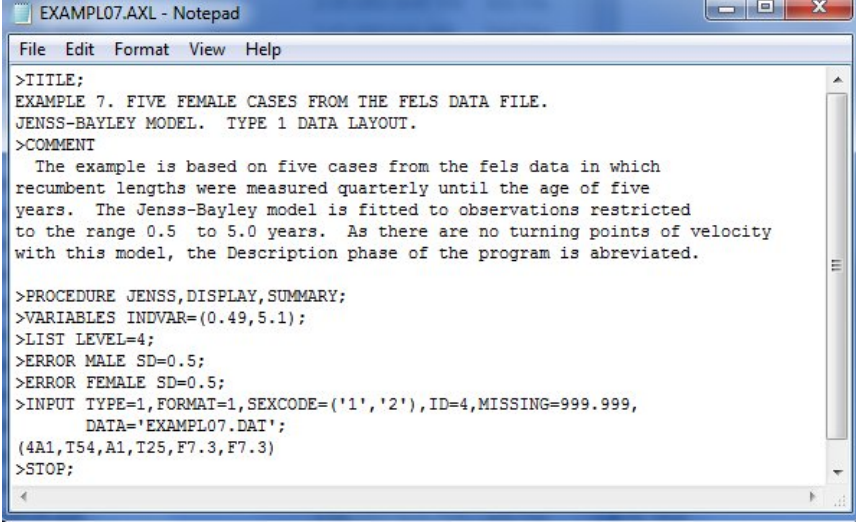
Similar lines omitted

```
 31  M 0.0     ' 66'/
 75.00    79.20    82.20    84.60    87.00    95.80   103.10   110.20   116.10   122.20
127.70   130.70   133.70   136.S0   139.80   142.60   145.10   147.30   150.20   153.30
156.70   160.50   164.10   167.70  171. 00   173.10   174.70   175.60   176.10   176.30
176.40
```

Example 7

Five female cases from the Fels data file.

Command File: EXAMPL07.AXL

```
EXAMPL07.AXL - Notepad
File  Edit  Format  View  Help
>TITLE;
EXAMPLE 7. FIVE FEMALE CASES FROM THE FELS DATA FILE.
JENSS-BAYLEY MODEL.   TYPE 1 DATA LAYOUT.
>COMMENT
  The example is based on five cases from the fels data in which
recumbent lengths were measured quarterly until the age of five
years.  The Jenss-Bayley model is fitted to observations restricted
to the range 0.5 to 5.0 years.  As there are no turning points of velocity
with this model, the Description phase of the program is abreviated.

>PROCEDURE JENSS,DISPLAY,SUMMARY;
>VARIABLES INDVAR=(0.49,5.1);
>LIST LEVEL=4;
>ERROR MALE SD=0.5;
>ERROR FEMALE SD=0.5;
>INPUT TYPE=1,FORMAT=1,SEXCODE=('1','2'),ID=4,MISSING=999.999,
      DATA='EXAMPL07.DAT';
(4A1,T54,A1,T25,F7.3,F7.3)
>STOP;
```

Data File: EXAMPL07.DAT
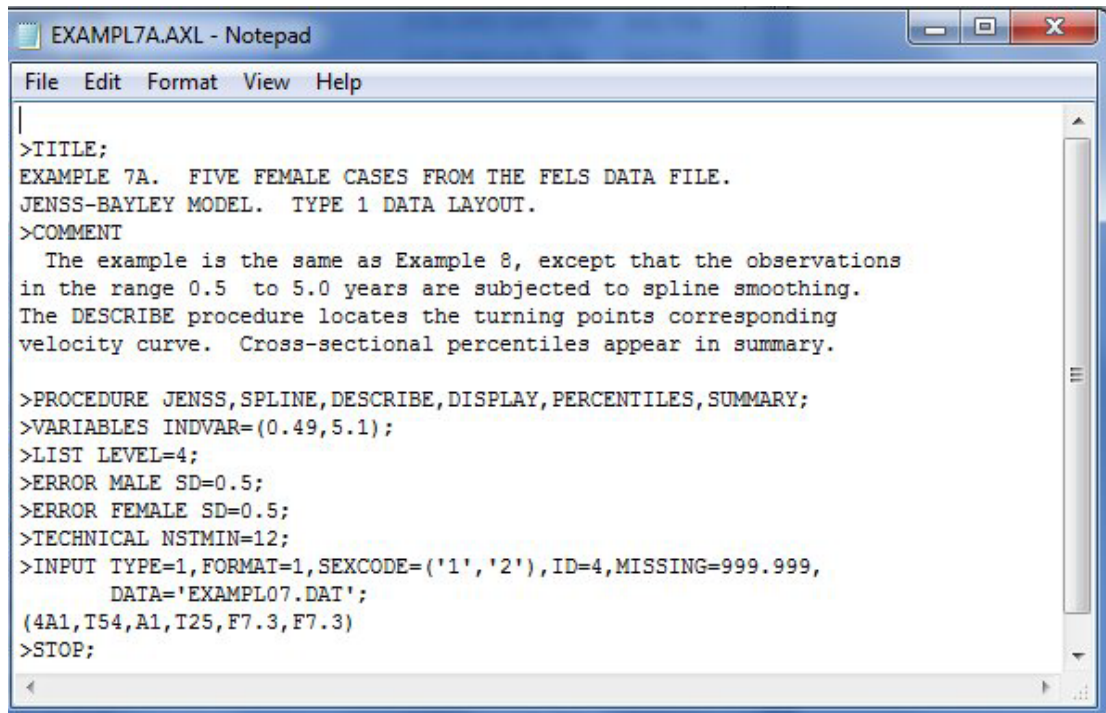
```
129472946300411300411 0.000999.999999.999 3.85992
129472946300411300417 0.016999.999999.999 3.27092
129472946300411300512 0.086999.999999.999 3.88092
129472946300411300618 0.186999.999999.999 3.56092
```

Similar lines omitted

```
133179317830040638100T   8.503137.400135.500 28.80092
133179317830040639040T   9.003140.500140.100 31.01092
133179317830040639101T   9.530999.999142.500 32.43092
133179317830040640040T   9.995999.999145.500 34.02092
```

Example7A

Command File: EXAMPL7A.AXL

```
EXAMPL7A.AXL - Notepad

File  Edit  Format  View  Help

|
>TITLE;
EXAMPLE 7A.  FIVE FEMALE CASES FROM THE FELS DATA FILE.
JENSS-BAYLEY MODEL.  TYPE 1 DATA LAYOUT.
>COMMENT
  The example is the same as Example 8, except that the observations
in the range 0.5  to 5.0 years are subjected to spline smoothing.
The DESCRIBE procedure locates the turning points corresponding
velocity curve.  Cross-sectional percentiles appear in summary.

>PROCEDURE JENSS,SPLINE,DESCRIBE,DISPLAY,PERCENTILES,SUMMARY;
>VARIABLES INDVAR=(0.49,5.1);
>LIST LEVEL=4;
>ERROR MALE SD=0.5;
>ERROR FEMALE SD=0.5;
>TECHNICAL NSTMIN=12;
>INPUT TYPE=1,FORMAT=1,SEXCODE=('1','2'),ID=4,MISSING=999.999,
       DATA='EXAMPL07.DAT';
(4A1,T54,A1,T25,F7.3,F7.3)
>STOP;
```

7.Output of Sample Problem

This chapter contains the printed and graphical output from Example 1 of Chapter 6. Three of the graphical displays produced by the AUXALPLOT program follow the printed output.

Example 1: Listing of EXAMPL01.OUT

```
-------------------------------------------------------------------
 FIVE CONSECUTIVE FEMALE CASES FROM THE FELS DATA FILE OF OCTOBER,
1992.
 STRUCTURAL ANALYSIS WITH THE BTT MODEL.  TYPE 1 DATA LAYOUT.

 -------------------------------------------------------------------
```

PHASE 1: COMMAND PROCESSING AND DATA INPUT

```
          >TITLE;
          >COMMENT


>PROCEDURE BTT,DESCRIBE,EVALUATE=1.0,DISPLAY,SUMMARY ;



>VARIABLES INDVAR=(1.0,22.0);

>LIST LEVEL=4;

>ERROR MALE SD=0.6;

>ERROR FEMALE SD=0.6;



>SAVE MASTER='EXAMPL01.MAS',

DESCRIBE='EXAMPL01.DSC';



>INPUT TYPE=1,FORMAT=1,SEXCODE=('1','2'),ID=4,MISSING=999.999,

       DATA='EXAMPL01.DAT';

   6 OBSERVATION(S) OUT OF VARIABLE RANGE OMITTED.
ID =  587
   1 OBSERVATION(S) WITH MISSING VARIABLE VALUE(S) OMITTED.
ID =  587
             CASE   1.   N =    32   F    587
   6 OBSERVATION(S) OUT OF VARIABLE RANGE OMITTED.
ID =  593
   1 OBSERVATION(S) WITH MISSING VARIABLE VALUE(S) OMITTED.
ID =  593
             CASE   2.   N =    30   F    593
   5 OBSERVATION(S) OUT OF VARIABLE RANGE OMITTED.
ID =  594
   1 OBSERVATION(S) WITH MISSING VARIABLE VALUE(S) OMITTED.
ID =  594
             CASE   3.   N =    31   F    594
   3 OBSERVATION(S) OUT OF VARIABLE RANGE OMITTED.
ID =  595
   1 OBSERVATION(S) WITH MISSING VARIABLE VALUE(S) OMITTED.
ID =  595
NO OBSERVATIONS.  CASE    4 OMITTED.
ID =  595
   4 OBSERVATION(S) OUT OF VARIABLE RANGE OMITTED.
ID =  596
   1 OBSERVATION(S) WITH MISSING VARIABLE VALUE(S) OMITTED.
ID =  596
```

```
                    CASE   5.    N =    32   F     596
          ----------------------------------------------------------

    FIVE CONSECUTIVE FEMALE CASES FROM THE FELS DATA FILE OF OCTOBER,
   1992.
    STRUCTURAL ANALYSIS WITH THE BTT MODEL.   TYPE 1 DATA LAYOUT.

          ----------------------------------------------------------
```

PHASE 2: PROLOG

    COMMENT:

The procedure in this example prepares a structural representation
of the growth curve each case. Maxima and minima of velocity curve
for each case are described, and the display files EXAMPL01.DS1 and
EXAMPL02.DS2 are prepared for input to the PLOTT program.
Descriptive statistics for all analyzed cases are saved in the
EXAMPL01.DSC file.

In addition, the program's master binary file is saved
     in the EXAMPL01.MAS file for use in the Example 2.

The INDVAR keyword of the VARIABLES command causes the program to
ignore any observations below one year of age and above 22 years of
age.

The data are in EXAMPL01.DAT in the type 1 layout. The case
identification is in columns 1-4, the sex code in column 54,
and the age and height data in columns 25-31 and 39-45,
respectively. The format statement describes the record layout.
Data records with * in column 55 are observations in which
adjusted recumbent length is substituted for the height
measurement. The adjustment corrects for the estimated systematic
difference between the two methods of measurement at a given age.

    OPTIONS:

         PROCEDURES
             BTT MODEL
             DESCRIBE
             DISPLAY
             SUMMARY

    VARIABLES
             DEPENDENT   HEIGHT
               UNIT   CM
               MINIMUM   1.00
               MAXIMUM   200.00
             INDEPENDENT   AGE
               UNIT   YRS
               MINIMUM   1.00
               MAXIMUM   22.00

         NUMBER OF CASES   5
             MALE   0
             FEMALE   5

         TYPE OF DATA LAYOUT   1
         PRINT LEVEL   4
         CONVERGENCE CRITERION   0.00010
         MAXIMUM ITERATIONS   80
         RANGE OF SMOOTH
           LOWER AGE LIMIT   0.00
           UPPER AGE LIMIT   24.00
         MINIMUM NUMBER OF OBSERVATIONS:
           STRUCTURAL   0
           NONSTRUCTURAL   15
         MISSING DATA CODE   999.9990
          CORRELATED RESIDUALS   YES

  COMMAND FILE   = EXAMPL01.AXL
  OUTPUT FILE   = EXAMPL01.OUT
  DATA FILE   = EXAMPL01.DAT
  DISPLAY FILE 1   = EXAMPL01.DS1
  DISPLAY FILE 2   = EXAMPL01.DS2
```

```
DATA FORMAT =
    (4A1,T54,A1,T25,F7.3,T39,F7.3)

    RESULTS SAVED               FILE
      DESCRIPTORS                 EXAMPL01.DSC
      MASTER FILE                 EXAMPL01.MAS


 STRUCTURAL ANALYSIS:

 ASSUMED INDIVIDUAL GROWTH MODEL (Y = STATURE; X = AGE)

  Y = A1/{1 + EXP[-(B1*X + C1)]}**D1 + A2/{1 + EXP[-(B2*X +
C2)]}**D2
     + A3/{1 + EXP[-(B3*X + C3)]}**D3

     SEE BOCK (1993).

            FIXED CONSTANTS:
                    C1 =  0.2500
                  MALE:   D1 =  0.8150  D2 =  0.7000  D3 =  0.6150
                  FEMALE: D1 =  0.8500  D2 =  0.6300  D3 =  0.6000

 DATA FOR FIRST AND LAST CASES
    FIRST CASE (SEX = F)              LAST CASE (SEX =F)
  587                                 596

    AGE (YRS)    HEIGHT (CM)          AGE (YRS)       HEIGHT (CM)
   ----------------------------------------------------------------
    1.  1.01      71.0             1.   1.00       70.9
    2.  1.51      76.1             2.   1.49       77.2
    3.  2.00      81.9             3.   1.99       83.0
    4.  2.52      85.4             4.   2.52       87.8
    5.  3.02      89.1             5.   3.00       91.0
    6.  3.52      92.0             6.   3.53       95.2
    7.  4.03      96.4             7.   4.00       99.4
    8.  4.52      98.8             8.   4.52      102.2
    9.  5.01     101.9             9.   4.97      105.4
   10.  5.53     105.6            10.   5.52      109.6
   11.  5.98     107.9            11.   5.97      113.1
   12.  6.55     110.4            12.   6.51      116.8
   13.  7.03     113.5            13.   7.00      120.0
   14.  7.53     115.9            14.   7.50      122.9
   15.  8.03     119.4            15.   7.98      125.8
   16.  8.52     122.5            16.   8.50      128.7
   17.  9.05     125.0            17.   8.99      131.7
   18.  9.53     127.1            18.   9.47      134.2
   19. 10.01     129.4            19.   9.97      137.5
   20. 10.49     132.0            20.  10.51      139.8
   21. 11.06     134.0            21.  10.99      142.8
   22. 11.47     136.4            22.  11.50      145.9
   23. 11.98     138.0            23.  12.02      149.3
   24. 12.48     143.0            24.  12.44      152.9
   25. 13.02     146.8            25.  12.96      155.3
   26. 13.48     150.4            26.  13.50      158.1
   27. 14.01     153.6            27.  14.05      159.2
   28. 14.97     158.1            28.  14.58      160.6
   29. 16.06     158.2            29.  15.20      161.6
   30. 17.00     158.6            30.  16.20      162.3
   31. 17.98     158.3            31.  17.01      162.3
   32. 19.94     159.5            32.  18.02      162.3
   ----------------------------------------------------------------

 MALE MEASUREMENT ERROR STANDARD DEVIATION   =   0.6000

 FEMALE MEASUREMENT ERROR STANDARD DEVIATION =   0.6000



 DISPLAY  1.   MALE POPULATION MEAN


            1       2       3       4       5       6       7       8
            A1      B1      A2      C2      B2      A3      C3      B3

 1     82.428   0.840  75.460  -2.917   0.377  21.876 -18.766   1.382
```

```
DISPLAY   2.   MALE POPULATION COVARIANCE MATRIX

          1         2        3        4        5        6        7        8
          A1        B1       A2       C2       B2       A3       C3
B3

1   A1    17.009
2   B1    -0.287    0.011
3   A2     6.681   -0.113   23.732
4   C2     0.029    0.000    0.017    0.026
5   B2    -0.071    0.002   -0.011   -0.001    0.001
6   A3    -4.945    0.083  -10.780   -0.010    0.013   16.975
7   C3    -0.728    0.034   -0.306    0.001    0.008   -0.770    1.379
8   B3     0.002    0.001   -0.091   -0.007    0.000    0.047   -0.044
0.009


 DISPLAY   3.   FEMALE POPULATION MEAN

          1         2        3        4        5        6        7        8
          A1        B1       A2       C2       B2       A3       C3       B3

1    71.005    1.153   74.043   -2.381    0.405   19.724  -15.183    1.309


 DISPLAY   4.   FEMALE POPULATION COVARIANCE MATRIX

          1         2        3        4        5        6        7        8
          A1        B1       A2       C2       B2       A3       C3       B3

1   A1    11.107
2   B1    -0.173    0.012
3   A2     1.781   -0.028   17.805
4   C2     0.131   -0.008    0.060    0.006
5   B2    -0.027    0.002   -0.035   -0.002    0.001
6   A3    -2.472    0.038   -5.505   -0.041    0.015    9.780
7   C3     0.421    0.006    0.076   -0.003    0.001    1.389    2.449
8   B3     0.028   -0.002   -0.104    0.001    0.000   -0.140   -0.200
0.027


 PHASE 3: BAYES MODAL FITTING OF THE BTT MODEL


--------------------------------------------------------------------
*  C A S E   1  N = 32  ID =  587
--------------------------------------------------------------------

********************************************************************

 GENERALIZED LOGISTIC COMPONENTS OF CASE   1:   587

 SEX = F

INFANTILE COMPONENT    MIDCHILDHOOD COMPONENT   ADOLESCENT COMPONENT

ASYMPTOTE =  61.73     ASYMPTOTE =  73.57       ASYMPTOTE =  24.27
SLOPE     =  18.937    SLOPE     =   6.222      SLOPE     =   8.855
LOCATION  =  -0.32     LOCATION  =   5.07       LOCATION  =  12.78

 DISPLAY   5.   BAYES MODAL ESTIMATES OF THE STRUCTURAL MODEL
PARAMETERS

          1         2        3        4        5        6        7        8
          A1        B1       A2       C2       B2       A3       C3       B3

1    61.731    1.293   73.571   -2.482    0.398   24.271  -16.316    1.237


 DISPLAY   6.   STANDARD ERRORS OF THE ESTIMATED PARAMETERS

          1         2        3        4        5        6        7        8
          A1        B1       A2       C2       B2       A3       C3       B3

1    0.9537   0.4236   1.8606   0.1100   0.0258   0.9790   0.0658   0.0012
```

STRUCTURAL MODEL:

 ROOT MEAN SQUARE ERROR (CM) = 0.6191; MAXIMUM ERROR = -1.7 AT AGE
12.0 YRS.

 GENERALIZED DISTANCE FROM POPULATION CENTROID =   3.366

 NUMBER OF NEWTON-RAPHSON ITERATIONS IN SOLUTION OF THE MAP
EQUATIONS = 10

 STRUCTURAL MODEL:
  ANALYSIS OF VELOCITY CURVE OF CASE   1: 587

 SEX = F

 STRUCTURAL MODEL:
  ANALYSIS OF VELOCITY CURVE OF CASE   1: 587

 SEX = F

 EARLY CHILDHOOD MINIMUM OF GROWTH VELOCITY

       AGE   (YRS)   3.98   (MOS)   47.71
       HEIGHT (CM)   95.20   (IN)   37.48
       VELOCITY (CM/YR)   6.332 (IN/YR)   2.493

 MIDCHILDHOOD MAXIMIM OF GROWTH VELOCITY

       AGE   (YRS)   4.75   (MOS)   57.00
       HEIGHT (CM)   100.11   (IN)   39.41
       VELOCITY (CM/YR)   6.350 (IN/YR)   2.500

 PREPUBERTAL MINIMUM OF GROWTH VELOCITY

       AGE   (YRS)   9.58   (MOS)   114.97
       HEIGHT (CM)   126.88   (IN)   49.95
       VELOCITY (CM/YR)   4.537 (IN/YR)   1.786

 ADOLESCENT MAXIMUM OF GROWTH VELOCITY

       AGE   (YRS)   12.65   (MOS)   151.79
       HEIGHT (CM)   144.59   (IN)   56.93
       VELOCITY (CM/YR)   7.490 (IN/YR)   2.949

 ADOLESCENT INCREMENT   (CM)   32.70 (IN) 12.87
 PREDICTED MATURE STATURE   (CM) 159.57 (IN) 62.82

 EVALUATION OF MODEL IN STEPS OF 1.00   CASE: 1 ID: 587
SEX = F

      AGE   HEIGHT   VELOCITY   ACC.   THIRD DER.
     (YRS)   (CM)   (CM/YR)   (CM/YR/YR) (CM/YR/YR/YR)

      1.000   70.750   14.239   -8.100   3.184
      2.000   81.675   8.424   -3.430   4.190
      3.000   88.954   6.583   -0.715   1.450
      4.000   95.354   6.332   0.005   0.198
      5.000   101.697   6.344   -0.048   -0.220
      6.000   107.975   6.166   -0.320   -0.284
      7.000   113.936   5.718   -0.553   -0.156
      8.000   119.361   5.125   -0.590   0.098
      9.000   124.221   4.638   -0.326   0.445
     10.000   128.787   4.602   0.324   0.857
     11.000   133.707   5.400   1.288   0.918
     12.000   139.858   6.939   1.457   -1.097
     13.000   147.196   7.281   -1.211   -3.486
     14.000   153.394   4.789   -3.117   0.165
     15.000   156.766   2.168   -1.911   1.513
     16.000   158.202   0.896   -0.766   0.751
     17.000   158.814   0.407   -0.291   0.270
     18.000   159.111   0.214   -0.123   0.096
     19.000   159.277   0.127   -0.061   0.038
     20.000   159.378   0.080   -0.035   0.018
     21.000   159.443   0.052   -0.022   0.010
     22.000   159.486   0.035   -0.014   0.006
     23.000   159.515   0.023   -0.009   0.004
     24.000   159.534   0.016   -0.006   0.003

```
      25.000     159.547      0.010      -0.004      0.002
1STRUCTURAL GROWTH CURVE


  VALUES FROM PLOT FOR CASE   1:  587

  SEX = F
-----------------------------------------------------------------
AGE         OBSERVED HEIGHT  FITTED HEIGHT  RESIDUAL  VELOCITY
OBS.
(YRS)  (MOS)   (CM)   (IN)    (CM)   (IN)    (CM)    (CM/YR)
OBS
VELOCITY
-----------------------------------------------------------------
1.01    12.1   71.0   28.0    70.9   27.9    0.09     0.00
1.51    18.1   76.1   30.0    77.0   30.3   -0.93     0.00
10.97
2.00    24.1   81.9   32.2    81.7   32.2    0.18     0.00
2.52    30.3   85.4   33.6    85.7   33.7   -0.32     0.00
7.08
3.02    36.3   89.1   35.1    89.1   35.1    0.00     0.00
3.52    42.2   92.0   36.2    92.3   36.3   -0.28     0.00
7.24
4.03    48.4   96.4   38.0    95.5   37.6    0.86     0.00
4.52    54.2   98.8   38.9    98.6   38.8    0.16     0.00
5.61
5.01    60.1  101.9   40.1   101.8   40.1    0.13     0.00
5.50    66.0                 104.8   41.3             6.15
6.18
5.53    66.3  105.6   41.6   105.0   41.3    0.57     0.00
5.98    71.8  107.9   42.5   107.9   42.5    0.04     0.00
6.04    72.4                 108.2   42.6             5.70
4.71
6.55    78.6  110.4   43.5   111.3   43.8   -0.89     0.00
7.03    84.3  113.5   44.7   114.1   44.9   -0.59     0.00
5.62
7.53    90.3  115.9   45.6   116.9   46.0   -0.96     0.00
8.03    96.3  119.4   47.0   119.5   47.0   -0.09     0.00
6.64
8.52   102.2  122.5   48.2   121.9   48.0    0.55     0.00
9.05   108.7  125.0   49.2   124.5   49.0    0.52     0.00
4.54
9.53   114.4  127.1   50.0   126.7   49.9    0.44     0.00
10.01  120.1  129.4   50.9   128.8   50.7    0.58     0.00
5.11
10.49  125.9  132.0   52.0   131.1   51.6    0.90     0.00
10.98  131.8                 133.6   52.6             6.17
4.50
11.06  132.7  134.0   52.8   134.0   52.8   -0.02     0.00
11.47  137.6  136.4   53.7   136.4   53.7    0.00     0.00
11.52  138.2                 136.7   53.8             7.45
4.32
11.98  143.8  138.0   54.3   139.7   55.0   -1.73     0.00
12.48  149.8  143.0   56.3   143.3   56.4   -0.34     0.00
8.51
13.02  156.2  146.8   57.8   147.3   58.0   -0.51     0.00
13.48  161.8  150.4   59.2   150.5   59.3   -0.10     0.00
13.51  162.2                 150.7   59.3             3.34
6.83
14.01  168.1  153.6   60.5   153.4   60.4    0.15     0.00
14.49  173.9                 155.4   61.2             1.37
4.69
14.97  179.7  158.1   62.2   156.7   61.7    1.40     0.00
15.52  186.2                 157.7   62.1             0.58
0.05
16.06  192.7  158.2   62.3   158.3   62.3   -0.10     0.00
16.53  198.4                 158.6   62.4             0.29
0.48
17.00  204.0  158.6   62.4   158.8   62.5   -0.21     0.00
17.98  215.8  158.3   62.3   159.1   62.6   -0.81     0.00
19.94  239.3  159.5   62.8   159.4   62.7    0.13     0.00
-----------------------------------------------------------------



        -------------------------------------------------------
         * C A S E   2  N =  30  ID =  593
        -------------------------------------------------------
```

```
    ********************************************************************

     GENERALIZED LOGISTIC COMPONENTS OF CASE    2:   593

     SEX = F

          INFANTILE COMPONENT   MIDCHILDHOOD COMPONENT   ADOLESCENT
    COMPONENT

    ASYMPTOTE =  60.01    ASYMPTOTE =  74.53    ASYMPTOTE =  20.07
    SLOPE     =  18.878   SLOPE     =   6.797   SLOPE     =   6.826
    LOCATION  =  -0.31    LOCATION  =   4.79    LOCATION  =  12.22

     DISPLAY   7.   BAYES MODAL ESTIMATES OF THE STRUCTURAL MODEL
    PARAMETERS
             1       2       3       4       5       6       7       8
             A1      B1      A2      C2      B2      A3      C3      B3

    1    60.010   1.326  74.535  -2.520   0.429  20.074 -14.599   1.153


     DISPLAY   8.    STANDARD ERRORS OF THE ESTIMATED PARAMETERS

             1       2       3       4       5       6       7       8
             A1      B1      A2      C2      B2      A3      C3      B3

    1    0.7419  0.2308  1.6332  0.0405  0.0072  1.4513  0.6108  0.0433


     STRUCTURAL MODEL:
     ROOT MEAN SQUARE ERROR (CM) = 0.5521;  MAXIMUM ERROR = -1.2 AT AGE
    1.5 YRS.

     GENERALIZED DISTANCE FROM POPULATION CENTROID =    3.564

     NUMBER OF NEWTON-RAPHSON ITERATIONS IN SOLUTION OF THE MAP
    EQUATIONS =   9

     STRUCTURAL MODEL:
      ANALYSIS OF VELOCITY CURVE OF CASE   2:  593

     SEX = F


     STRUCTURAL MODEL:
      ANALYSIS OF VELOCITY CURVE OF CASE   2:  593

     SEX = F


     EARLY CHILDHOOD MINIMUM OF GROWTH VELOCITY

          AGE     (YRS)     3.74   (MOS)     44.91
          HEIGHT  (CM)     93.68   (IN)      36.88
          VELOCITY (CM/YR)  6.939  (IN/YR)    2.732

     MIDCHILDHOOD MAXIMIM OF GROWTH VELOCITY

          AGE     (YRS)     4.48   (MOS)     53.73
          HEIGHT  (CM)     98.79   (IN)      38.89
          VELOCITY (CM/YR)  6.960  (IN/YR)    2.740

     PREPUBERTAL MINIMUM OF GROWTH VELOCITY

          AGE     (YRS)     9.49   (MOS)    113.86
          HEIGHT  (CM)    128.27   (IN)      50.50
          VELOCITY (CM/YR)  4.602  (IN/YR)    1.812

     ADOLESCENT MAXIMUM OF GROWTH VELOCITY

          AGE     (YRS)    12.00   (MOS)    144.03
          HEIGHT  (CM)    141.47   (IN)      55.70
          VELOCITY (CM/YR)  6.057  (IN/YR)    2.385

     ADOLESCENT INCREMENT       (CM)  26.35  (IN) 10.37
     PREDICTED MATURE STATURE   (CM) 154.62  (IN) 60.87
```

```
EVALUATION OF MODEL IN STEPS OF 1.00    CASE:   2  ID:  593
SEX = F
--------------------------------------------------------------------
        AGE        HEIGHT     VELOCITY      ACC.     THIRD DER.
        (YRS)       (CM)      (CM/YR)    (CM/YR/YR) (CM/YR/YR/YR)
--------------------------------------------------------------------
        1.000      69.711      14.351      -8.080       3.579
        2.000      80.815       8.706      -3.179       4.215
        3.000      88.496       7.092      -0.540       1.320
        4.000      95.468       6.946       0.040       0.056
        5.000     102.420       6.925      -0.145      -0.337
        6.000     109.212       6.600      -0.505      -0.334
        7.000     115.511       5.958      -0.741      -0.113
        8.000     121.093       5.212      -0.697       0.212
        9.000     126.007       4.680      -0.310       0.559
       10.000     130.638       4.698       0.380       0.763
       11.000     135.643       5.408       0.932       0.084
       12.000     141.452       6.057       0.006      -1.980
       13.000     147.150       4.986      -2.027      -1.292
       14.000     151.022       2.761      -2.042       0.914
       15.000     152.932       1.231      -1.036       0.865
       16.000     153.768       0.541      -0.426       0.387
       17.000     154.147       0.259      -0.176       0.148
       18.000     154.338       0.139      -0.080       0.058
       19.000     154.444       0.081      -0.041       0.025
       20.000     154.508       0.049      -0.023       0.012
       21.000     154.548       0.031      -0.014       0.007
       22.000     154.573       0.020      -0.009       0.004
       23.000     154.589       0.013      -0.006       0.002
       24.000     154.599       0.008      -0.004       0.002
       25.000     154.606       0.005      -0.002       0.001
--------------------------------------------------------------------


1STRUCTURAL GROWTH CURVE

VALUES FROM PLOT FOR CASE   2:  593

 SEX = F
--------------------------------------------------------------------
AGE        OBSERVED HEIGHT  FITTED HEIGHT  RESIDUAL  VELOCITY
(YRS) (MOS)   (CM)   (IN)     (CM)   (IN)    (CM)     (CM/YR)
OBS
VELOCITY
--------------------------------------------------------------------
1.51    18.1   74.9   29.5     76.1   29.9    -1.16      0.00
2.01    24.1   81.8   32.2     80.9   31.8     0.92      0.00
9.78
2.56    30.7   85.2   33.5     85.3   33.6    -0.11      0.00
3.02    36.2   89.5   35.2     88.6   34.9     0.87      0.00
6.78
3.50    42.1   91.6   36.1     92.0   36.2    -0.43      0.00
3.99    47.9                   95.4   37.6                6.96
8.43
4.00    48.1   95.5   37.6     95.5   37.6     0.00      0.00
4.48    53.7   99.8   39.3     98.8   38.9     1.01      0.00
6.59
4.99    59.9  102.0   40.2    102.4   40.3    -0.36      0.00
5.49    65.9  106.0   41.7    105.8   41.7     0.20      0.00
6.98
6.00    71.9  109.0   42.9    109.2   43.0    -0.18      0.00
6.47    77.7  111.4   43.9    112.3   44.2    -0.87      0.00
5.86
6.98    83.8  114.8   45.2    115.4   45.4    -0.62      0.00
7.48    89.8  117.8   46.4    118.3   46.6    -0.51      0.00
6.58
8.00    96.0  121.5   47.8    121.1   47.7     0.39      0.00
8.48   101.8  123.9   48.8    123.5   48.6     0.38      0.00
8.51   102.1                  123.7   48.7                4.60
5.20
9.02   108.3  126.8   49.9    126.1   49.6     0.69      0.00
9.48   113.7  128.5   50.6    128.2   50.5     0.28      0.00
9.52   114.2                  128.4   50.6                4.99
4.44
10.01  120.2  131.2   51.7    130.7   51.5     0.50      0.00
10.47  125.7  133.2   52.4    132.9   52.3     0.27      0.00
10.52  126.2                  133.1   52.4                5.85
4.39
11.02  132.2  135.6   53.4    135.7   53.4    -0.13      0.00
11.49  137.9  138.4   54.5    138.4   54.5    -0.01      0.00
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11.50    138.0                    138.5   54.5              5.78
5.33
11.99    143.9   140.8   55.4    141.4   55.7   -0.60   0.00
12.49    149.9   143.8   56.6    144.4   56.8   -0.57   0.00
12.51    150.1                    144.5   56.9              3.82
5.61
13.03    156.3   146.6   57.7    147.3   58.0   -0.67   0.00
13.51    162.1   149.5   58.9    149.4   58.8    0.11   0.00
5.23
14.02    168.2   151.8   59.8    151.1   59.5    0.73   0.00
14.51    174.1                    152.2   59.9              0.80
1.69
14.99    179.9   153.4   60.4    152.9   60.2    0.52   0.00
15.52    186.3                    153.5   60.4              0.36
0.14
16.05    192.6   153.6   60.5    153.8   60.5   -0.19   0.00
16.54    198.5                    154.0   60.6              0.19
0.10
17.03    204.3   153.7   60.5    154.2   60.7   -0.45   0.00
17.51    210.1                    154.3   60.7            154.25
0.57
18.00    215.9   154.3   60.7    154.3   60.8   -0.09   0.00
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   *  C A S E    3  N =  31  ID =  594
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 GENERALIZED LOGISTIC COMPONENTS OF CASE     3:   594

 SEX = F

INFANTILE COMPONENT    MIDCHILDHOOD COMPONENT   ADOLESCENT COMPONENT

ASYMPTOTE =  62.18     ASYMPTOTE =  73.82      ASYMPTOTE =  22.11
SLOPE     = 19.682     SLOPE     =  7.342      SLOPE     =  7.602
LOCATION  = -0.31      LOCATION  =   4.43      LOCATION  =  11.75
```

 DISPLAY   9.   BAYES MODAL ESTIMATES OF THE STRUCTURAL MODEL
PARAMETERS

	1	2	3	4	5	6	7	8
	A1	B1	A2	C2	B2	A3	C3	B3
1	62.179	1.334	73.820	-2.535	0.468	22.114	-14.201	1.166

 DISPLAY 10.   STANDARD ERRORS OF THE ESTIMATED PARAMETERS

	1	2	3	4	5	6	7	8
	A1	B1	A2	C2	B2	A3	C3	B3
1	0.7455	0.3036	1.4755	0.0431	0.0143	1.1068	0.1184	0.0055

 STRUCTURAL MODEL:
 ROOT MEAN SQUARE ERROR (CM) = 0.4660; MAXIMUM ERROR = -0.9 AT AGE
2.5 YRS.

 GENERALIZED DISTANCE FROM POPULATION CENTROID =   3.079

 NUMBER OF NEWTON-RAPHSON ITERATIONS IN SOLUTION OF THE MAP
EQUATIONS =   6

 STRUCTURAL MODEL:
  ANALYSIS OF VELOCITY CURVE OF CASE   3: 594

 SEX = F

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STRUCTURAL MODEL:
  ANALYSIS OF VELOCITY CURVE OF CASE   3:  594

 SEX = F

 --->NO MIDCHILDHOOD VELOCITY MAXIMUM FOUND IN CASE   3.

 PREPUBERTAL MINIMUM OF GROWTH VELOCITY

      AGE     (YRS)      9.02   (MOS)     108.28
      HEIGHT  (CM)     130.93   (IN)       51.55
      VELOCITY (CM/YR)   4.677  (IN/YR)     1.841

 ADOLESCENT MAXIMUM OF GROWTH VELOCITY

      AGE     (YRS)     11.56   (MOS)     138.77
      HEIGHT  (CM)     144.81   (IN)       57.01
      VELOCITY (CM/YR)   6.439  (IN/YR)     2.535

 ADOLESCENT INCREMENT       (CM)   27.18  (IN)  10.70
 PREDICTED MATURE STATURE   (CM)  158.11  (IN)  62.25


EVALUATION OF MODEL IN STEPS OF 1.00    CASE:   3  ID:  594
SEX = F
----------------------------------------------------------------------
        AGE        HEIGHT     VELOCITY      ACC.      THIRD DER.
        (YRS)       (CM)      (CM/YR)   (CM/YR/YR) (CM/YR/YR/YR)
----------------------------------------------------------------------
       1.000       71.696      15.126     -8.339       3.840
       2.000       83.494       9.362     -3.183       4.363
       3.000       91.843       7.775     -0.516       1.251
       4.000       99.494       7.600     -0.056      -0.094
       5.000      107.026       7.410     -0.383      -0.451
       6.000      114.171       6.809     -0.796      -0.319
       7.000      120.542       5.909     -0.943       0.047
       8.000      126.004       5.060     -0.684       0.470
       9.000      130.817       4.677     -0.020       0.835
      10.000      135.631       5.098      0.852       0.759
      11.000      141.238       6.136      0.941      -0.937
      12.000      147.583       6.202     -1.107      -2.579
      13.000      152.876       4.138     -2.554       0.043
      14.000      155.833       1.932     -1.671       1.199
      15.000      157.112       0.787     -0.711       0.668
      16.000      157.634       0.329     -0.273       0.260
      17.000      157.860       0.151     -0.109       0.095
      18.000      157.970       0.077     -0.048       0.037
      19.000      158.028       0.043     -0.024       0.016
      20.000      158.061       0.025     -0.013       0.007
      21.000      158.081       0.015     -0.007       0.004
      22.000      158.093       0.009     -0.004       0.002
      23.000      158.100       0.006     -0.003       0.001
      24.000      158.105       0.004     -0.002       0.001
      25.000      158.108       0.002     -0.001       0.001
      ----------------------------------------------------------------

1STRUCTURAL GROWTH CURVE


 VALUES FROM PLOT FOR CASE   3:  594

 SEX = F
 ----------------------------------------------------------------------
   AGE        OBSERVED HEIGHT  FITTED HEIGHT  RESIDUAL  VELOCITY
   (YRS)  (MOS)    (CM)   (IN)     (CM)   (IN)     (CM)    (CM/YR)
OBS
VELOCITY
 ----------------------------------------------------------------------
   1.50    18.0    78.4   30.9     78.3   30.8     0.08     0.00
   2.01    24.2    84.5   33.3     83.6   32.9     0.88     0.00
8.53
   2.51    30.1    87.0   34.3     87.9   34.6    -0.92     0.00
   3.02    36.2    91.2   35.9     92.0   36.2    -0.79     0.00
8.30
   3.51    42.1    95.3   37.5     95.7   37.7    -0.45     0.00
   4.01    48.1   100.2   39.4     99.6   39.2     0.62     0.00
7.75
   4.53    54.3   103.2   40.6    103.5   40.7    -0.29     0.00
   5.00    60.1   107.0   42.1    107.1   42.2    -0.06     0.00
7.82
```

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    5.53     66.3    111.0    43.7    110.9    43.6     0.15     0.00
    6.02     72.3    114.9    45.2    114.3    45.0     0.58     0.00
6.74
    6.52     78.2    117.7    46.3    117.6    46.3     0.11     0.00
    6.96     83.5    121.0    47.6    120.3    47.4     0.71     0.00
6.39
    7.47     89.7    123.8    48.7    123.2    48.5     0.57     0.00
    8.00     96.0    125.1    49.3    126.0    49.6    -0.90     0.00
4.05
    8.56    102.7    128.2    50.5    128.7    50.7    -0.54     0.00
    8.95    107.4    130.5    51.4    130.6    51.4    -0.09     0.00
    9.04    108.5                     131.0    51.6              5.11
4.86
    9.53    114.3    132.9    52.3    133.3    52.5    -0.40     0.00
    9.99    119.9    135.7    53.4    135.6    53.4     0.09     0.00
   10.01    120.2                     135.7    53.4              6.14
5.55
   10.50    126.0    138.3    54.4    138.3    54.4     0.00     0.00
   10.97    131.6    141.1    55.6    141.0    55.5     0.07     0.00
   11.01    132.1                     141.3    55.6              6.19
6.71
   11.51    138.2    145.1    57.1    144.5    56.9     0.61     0.00
   12.00    144.0    148.0    58.3    147.6    58.1     0.40     0.00
5.32
   12.51    150.1    150.4    59.2    150.6    59.3    -0.15     0.00
   12.96    155.6    152.7    60.1    152.7    60.1    -0.02     0.00
4.16
   13.47    161.7    154.4    60.8    154.6    60.8    -0.15     0.00
   14.02    168.2    155.8    61.3    155.9    61.4    -0.11     0.00
   14.51    174.1                     156.6    61.7              0.51
0.86
   15.01    180.1    156.6    61.7    157.1    61.9    -0.52     0.00
   15.49    185.8                     157.4    62.0              0.22
0.84
   15.97    191.6    157.4    62.0    157.6    62.1    -0.22     0.00
   16.49    197.9                     157.8    62.1              0.11
0.81
   17.02    204.2    158.2    62.3    157.9    62.2     0.39     0.00
   18.02    216.2    158.1    62.2    158.0    62.2     0.13     0.00
   20.03    240.3    158.6    62.4    158.1    62.2     0.49     0.00
     ----------------------------------------------------------------




     ----------------------------------------------------------------
       *  C A S E    5   N =  32  ID =  596
     ----------------------------------------------------------------



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      GENERALIZED LOGISTIC COMPONENTS OF CASE    5:    596

      SEX = F

     INFANTILE COMPONENT    MIDCHILDHOOD COMPONENT    ADOLESCENT COMPONENT

     ASYMPTOTE =  61.07     ASYMPTOTE =  77.50     ASYMPTOTE =  24.18
     SLOPE     =  19.267    SLOPE     =   7.232    SLOPE     =   7.767
     LOCATION  =  -0.31     LOCATION  =   4.69     LOCATION  =  11.88

     DISPLAY  11.   BAYES MODAL ESTIMATES OF THE STRUCTURAL MODEL
     PARAMETERS


               1       2       3       4       5       6       7       8
              A1      B1      A2      C2      B2      A3      C3      B3

     1    61.075   1.330  77.502  -2.523   0.439  24.181 -13.445   1.089



      DISPLAY  12.    STANDARD ERRORS OF THE ESTIMATED PARAMETERS


               1       2       3       4       5       6       7       8
              A1      B1      A2      C2      B2      A3      C3      B3

     1    0.7824  0.3478  1.6731  0.0292  0.0088  1.3415  0.2933  0.0213
```

STRUCTURAL MODEL:

 ROOT MEAN SQUARE ERROR (CM) = 0.3386; MAXIMUM ERROR = 0.8 AT AGE
4.0 YRS.

 GENERALIZED DISTANCE FROM POPULATION CENTROID =   3.704

 NUMBER OF NEWTON-RAPHSON ITERATIONS IN SOLUTION OF THE MAP
EQUATIONS =   8

 STRUCTURAL MODEL:
  ANALYSIS OF VELOCITY CURVE OF CASE   5: 596

 SEX = F

 STRUCTURAL MODEL:
  ANALYSIS OF VELOCITY CURVE OF CASE   5: 596

 SEX = F

 EARLY CHILDHOOD MINIMUM OF GROWTH VELOCITY

      AGE   (YRS)   -0.26   (MOS)   -3.08
      HEIGHT (CM)   46.67   (IN)   18.37
      VELOCITY (CM/YR) 22.891 (IN/YR)   9.012

 MIDCHILDHOOD MAXIMIM OF GROWTH VELOCITY

      AGE   (YRS)   4.43   (MOS)   53.19
      HEIGHT (CM)   101.82   (IN)   40.09
      VELOCITY (CM/YR) 7.452 (IN/YR)   2.934

 PREPUBERTAL MINIMUM OF GROWTH VELOCITY

      AGE   (YRS)   9.07   (MOS)   108.87
      HEIGHT (CM)   131.86   (IN)   51.91
      VELOCITY (CM/YR) 5.317 (IN/YR)   2.093

 ADOLESCENT MAXIMUM OF GROWTH VELOCITY

      AGE   (YRS)   11.64   (MOS)   139.62
      HEIGHT (CM)   147.24   (IN)   57.97
      VELOCITY (CM/YR) 6.845 (IN/YR)   2.695

 ADOLESCENT INCREMENT   (CM)   30.90 (IN) 12.17
 PREDICTED MATURE STATURE   (CM) 162.76 (IN) 64.08

 EVALUATION OF MODEL IN STEPS OF 1.00   CASE: 5 ID: 596
 SEX = F

      AGE   HEIGHT   VELOCITY   ACC.   THIRD DER.
     (YRS)   (CM)   (CM/YR) (CM/YR/YR) (CM/YR/YR/YR)

      1.000   71.469   14.835   -8.199   3.702
      2.000   83.020   9.139   -3.171   4.298
      3.000   91.147   7.556   -0.501   1.311
      4.000   98.600   7.437   0.050   0.014
      5.000   106.041   7.406   -0.176   -0.371
      6.000   113.295   7.038   -0.550   -0.323
      7.000   120.014   6.368   -0.743   -0.036
      8.000   126.020   5.670   -0.589   0.350
      9.000   131.470   5.319   -0.052   0.703
     10.000   136.888   5.644   0.699   0.673
     11.000   142.959   6.529   0.851   -0.652
     12.000   149.716   6.703   -0.799   -2.313
     13.000   155.668   4.917   -2.451   -0.493
     14.000   159.374   2.586   -1.949   1.070
     15.000   161.164   1.161   -0.954   0.787
     16.000   161.958   0.519   -0.403   0.353
     17.000   162.322   0.249   -0.171   0.141
     18.000   162.505   0.131   -0.079   0.057
     19.000   162.604   0.074   -0.040   0.025
     20.000   162.662   0.044   -0.022   0.012
     21.000   162.697   0.027   -0.013   0.007
     22.000   162.719   0.017   -0.008   0.004
     23.000   162.733   0.011   -0.005   0.002
     24.000   162.742   0.007   -0.003   0.001
     25.000   162.748   0.005   -0.002   0.001

1STRUCTURAL GROWTH CURVE

 VALUES FROM PLOT FOR CASE   5: 596

  SEX = F

    AGE   OBSERVED HEIGHT FITTED HEIGHT RESIDUAL VELOCITY
   (YRS) (MOS) (CM)   (IN)   (CM)   (IN)   (CM)   (CM/YR)
OBS
VELOCITY

    1.00   12.0   70.9   27.9   71.5   28.2   -0.61   0.00
    1.49   17.8   77.2   30.4   77.8   30.6   -0.60   0.00
12.31
    1.99   23.8   83.0   32.7   82.9   32.6   0.11   0.00
    2.49   29.9   87.2   34.3   7.54
7.87
    2.52   30.2   87.8   34.6   87.4   34.4   0.37   0.00
    3.00   36.0   91.0   35.8   91.2   35.9   -0.17   0.00
7.30
    3.53   42.4   95.2   37.5   95.1   37.5   0.07   0.00
    4.00   47.9   99.4   39.1   98.6   38.8   0.84   0.00
    4.02   48.3   98.8   38.9   7.40
7.12
    4.52   54.2 102.2   40.2   102.4   40.3   -0.24   0.00
    4.97   59.6 105.4   41.5   105.8   41.7   -0.41   0.00
    5.02   60.2   106.2   41.8   7.03
7.38
    5.52   66.2 109.6   43.1   109.9   43.2   -0.25   0.00
    5.97   71.6 113.1   44.5   113.1   44.5   0.04   0.00
    6.01   72.2   113.4   44.6   6.37
7.28
    6.51   78.1 116.8   46.0   116.8   46.0   0.01   0.00
    7.00   84.1 120.0   47.2   120.0   47.3   -0.05   0.00
6.18
    7.50   89.9 122.9   48.4   123.1   48.5   -0.18   0.00
    7.98   95.8 125.8   49.5   125.9   49.6   -0.12   0.00
5.79
    8.50 102.0 128.7   50.7   128.8   50.7   -0.07   0.00
    8.99 107.9 131.7   51.9   131.4   51.7   0.29   0.00
5.66
    9.47 113.6 134.2   52.8   134.0   52.7   0.23   0.00
    9.97 119.7 137.5   54.1   136.7   53.8   0.78   0.00
5.39
   10.51 126.1 139.8   55.0   139.9   55.1   -0.06   0.00
   10.99 131.9 142.8   56.2   142.9   56.3   -0.11   0.00
   11.01 132.1   143.0   56.3   6.72
6.13
   11.50 138.0 145.9   57.4   146.3   57.6   -0.43   0.00
   11.97 143.7   149.5   58.9   6.04
7.44
   12.02 144.2 149.3   58.8   149.8   59.0   -0.54   0.00
   12.44 149.3 152.9   60.2   152.6   60.1   0.32   0.00
6.38
   12.96 155.5 155.3   61.1   155.5   61.2   -0.17   0.00
   13.50 162.0 158.1   62.2   157.8   62.1   0.29   0.00
   13.51 162.1   157.8   62.1   0.78
3.56
   14.05 168.7 159.2   62.7   159.5   62.8   -0.31   0.00
   14.58 174.9 160.6   63.2   160.6   63.2   0.02   0.00
   15.20 182.4 161.6   63.6   161.4   63.5   0.22   0.00
   16.20 194.4 162.3   63.9   162.1   63.8   0.20   0.00
   17.01 204.2 162.3   63.9   162.3   63.9   -0.03   0.00
   18.02 216.2 162.3   63.9   162.5   64.0   -0.26   0.00

 ****

 PHASE 4: SAMPLE SUMMARY
      NUMBER OF CASES =   4
      AGE INTERVAL =   1.00 (YEARS)

CROSS-SECTIONAL SUMMARIES OF HEIGHT AND VELOCITY BY AGE INTERVALS

CROSS-SECTIONAL SUMMARY OF HEIGHTS BY AGE INTERVALS

AGE (YEARS)	NUMBER OF OBS.	MEAN HEIGHT	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
1.500	7	75.93	3.64	1.38	0.08	2.25
2.500	7	84.80	1.98	0.75	-0.04	1.77
3.500	9	92.70	2.95	0.98	0.79	2.78
4.500	9	100.39	2.84	0.95	-0.01	2.08
5.500	9	107.90	2.92	0.97	-0.05	2.61
6.500	7	115.29	3.13	1.18	0.01	2.08
7.500	7	119.96	3.84	1.45	-0.02	1.69
8.500	9	125.72	3.79	1.26	0.00	1.68
9.500	8	130.96	4.09	1.45	0.01	1.46
10.500	8	135.97	4.47	1.58	0.00	1.39
11.500	8	139.28	3.80	1.34	0.25	1.88
12.500	8	149.43	3.82	1.35	-0.09	1.88
13.500	6	150.97	3.76	1.53	0.34	1.98
14.500	7	156.07	2.84	1.07	0.01	1.54
15.500	3	158.53	1.79	1.03	0.41	1.50
16.500	3	158.00	2.88	1.67	0.00	1.50
17.500	6	157.57	2.65	1.08	0.01	2.00
18.500	2	160.18	1.47	1.04	0.00	1.00
19.500	1	159.50	0.00	0.00	0.00	0.00
20.500	1	158.55	0.00	0.00	0.00	0.00

CROSS-SECTIONAL SUMMARY OF HEIGHT RESIDUALS BY AGE INTERVALS

AGE (YEARS)	NUMBER OF OBS.	MEAN RESIDUAL	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
1.500	7	-0.43	0.45	0.17	-0.01	1.44
2.500	7	0.14	0.56	0.21	-0.08	2.06
3.500	9	-0.04	0.50	0.17	0.38	2.24
4.500	9	0.15	0.49	0.16	0.26	1.67
5.500	9	0.07	0.21	0.07	0.54	3.29
6.500	7	-0.14	0.57	0.22	0.00	1.44
7.500	7	-0.26	0.42	0.16	0.09	2.54
8.500	9	-0.01	0.42	0.14	-0.43	2.39
9.500	8	0.33	0.33	0.12	-0.56	2.87
10.500	8	0.27	0.31	0.11	0.33	2.03
11.500	8	-0.29	0.60	0.21	-1.14	3.75
12.500	8	-0.14	0.32	0.11	0.08	1.85
13.500	6	-0.17	0.31	0.12	-0.03	1.74
14.500	7	0.34	0.50	0.19	0.53	2.46
15.500	3	-0.17	0.25	0.14	0.06	1.50
16.500	3	-0.03	0.14	0.08	0.31	1.50
17.500	6	-0.20	0.34	0.14	-0.01	2.31
18.500	2	-0.06	0.14	0.10	0.00	1.00
19.500	1	0.13	0.00	0.00	0.00	0.00
20.500	1	0.49	0.00	0.00	0.00	0.00

CROSS-SECTIONAL SUMMARY OF EMPIRICAL VELOCITIES BY AGE INTERVALS

AGE (YEARS)	NUMBER OF OBS.	MEAN VELOCITY	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
1.500	2	11.64	0.47	0.34	0.00	1.00
2.500	8	8.37	1.13	0.40	0.03	1.71
3.500	9	7.59	0.90	0.30	0.00	1.69
4.500	8	6.62	0.58	0.21	0.03	2.39
5.500	8	7.01	0.73	0.26	-0.48	2.23
6.500	9	6.20	0.69	0.23	-0.48	2.68
7.500	8	5.76	0.69	0.24	-2.78	4.83
8.500	8	5.39	0.69	0.24	-0.06	2.55
9.500	8	4.93	0.43	0.15	1.02	2.85
10.500	8	4.98	0.43	0.15	-0.02	1.48
11.500	9	6.09	0.83	0.28	-0.31	2.62
12.500	9	5.87	1.21	0.40	0.65	2.67
13.500	4	4.63	1.33	0.66	0.10	1.55
14.500	4	2.39	1.23	0.62	0.51	2.04
15.500	4	0.42	0.29	0.14	0.01	1.19
16.500	3	0.46	0.24	0.14	-0.01	1.50
17.500	4	0.02	0.29	0.14	0.80	2.16

```
     18.500     0     0.00     0.00     0.00     0.00    0.00
     19.500     0     0.00     0.00     0.00     0.00    0.00
     20.500     0     0.00     0.00     0.00     0.00    0.00
----------------------------------------------------------------
```

CROSS-SECTIONAL SUMMARY OF VELOCITY RESIDUALS BY AGE INTERVALS

```
----------------------------------------------------------------
   AGE    NUMBER   MEAN    STANDARD  STANDARD  COEFFICIENTS OF
 (YEARS) OF OBS. RESIDUAL  DEVIATION  ERROR   SKEWNESS KURTOSIS
----------------------------------------------------------------
      1.500     2     0.62     0.23     0.16     0.00    1.00
      2.500     8     0.07     0.83     0.29    -0.13    2.10
      3.500     9     0.45     0.74     0.25    -0.03    1.74
      4.500     8    -0.45     0.49     0.17     0.07    2.21
      5.500     8     0.10     0.48     0.17    -2.04    4.06
      6.500     9    -0.16     0.53     0.18    -1.15    3.31
      7.500     8     0.08     0.69     0.25    -0.78    3.30
      8.500     8     0.38     0.64     0.23    -0.16    3.13
      9.500     8     0.07     0.22     0.08     0.33    2.58
     10.500     8    -0.27     0.47     0.17     0.15    1.71
     11.500     9    -0.31     0.68     0.23    -0.28    3.04
     12.500     9     0.06     0.51     0.17     0.02    2.54
     13.500     4     0.46     0.53     0.26     0.35    1.75
     14.500     4     0.15     0.60     0.30     1.24    2.30
     15.500     4    -0.41     0.55     0.27    -0.08    1.59
     16.500     3     0.08     0.30     0.17     0.38    1.50
     17.500     4    -0.17     0.31     0.15     0.30    2.06
     18.500     0     0.00     0.00     0.00     0.00    0.00
     19.500     0     0.00     0.00     0.00     0.00    0.00
     20.500     0     0.00     0.00     0.00     0.00    0.00
----------------------------------------------------------------
```

MODEL PARAMETER UNIVARIATE STATISTICS
```
   PARAMETER    MEAN    STANDARD   STANDARD   COEFFICIENTS OF
                        DEVIATION    ERROR   SKEWNESS KURTOSIS
----------------------------------------------------------------
      1       61.24865   0.70644   0.35322   -0.20713   1.78165
      2        1.32086   0.01408   0.00000    0.00000   0.00000
      3       74.85698   1.35759   0.67880    0.97704   2.18813
      4       -2.51502   0.01720   0.00000    0.00000   0.00000
      5        0.43388   0.02169   0.00000    0.00000   0.00000
      6       22.66001   1.49354   0.74677   -0.22613   1.60854
      7      -14.64028   0.91151   0.45576   -0.38958   2.02933
      8        1.16118   0.04546   0.02273    0.01118   1.97442
```

 DISPLAY 13.   ESTIMATED POPULATION MEAN OF STRUCTURAL MODEL
PARAMETERS

```
        1       2       3       4       5       6       7       8
        A1      B1      A2      C2      B2      A3      C3      B3

1    61.249   1.321  74.857  -2.515   0.434  22.660 -14.640   1.161
```

 DISPLAY 14.   ESTIMATED COVARIANCE MATRIX OF STRUCTURAL MODEL
PARAMETERS

```
        1       2       3       4       5       6       7       8
        A1      B1      A2      C2      B2      A3      C3      B3

1   A1    1.3224
2   B1   -0.1975  0.1117
3   A2   -1.2232  0.2250  5.2338
4   C2   -0.0236  0.0016  0.0335  0.0045
5   B2    0.0096  0.0000 -0.0146 -0.0012  0.0009
6   A3    1.0786 -0.0876 -1.0210  0.0035  0.0000  4.4966
7   C3   -0.1207 -0.0025  0.9403 -0.0149  0.0213 -0.0835  1.2272
8   B3    0.0138  0.0002 -0.0594  0.0006 -0.0008 -0.0038 -0.0609
0.0033
```

DISPLAY 15.   ESTIMATED STANDARD DEVIATIONS OF STRUCTURAL MODEL
PARAMETERS

```
        1       2       3       4       5       6       7       8
       A1      B1      A2      C2      B2      A3      C3      B3

1    1.1500  0.3343  2.2877  0.0671  0.0296  2.1205  1.1078  0.0578
```

DISPLAY 16.   ESTIMATED CORRELATION MATRIX OF STRUCTURAL MODEL
PARAMETERS

```
          1       2       3       4       5       6       7       8
         A1      B1      A2      C2      B2      A3      C3      B3

1  A1   1.0000
2  B1  -0.5137  1.0000
3  A2  -0.4649  0.2942  1.0000
4  C2  -0.3056  0.0713  0.2179  1.0000
5  B2   0.2828 -0.0024 -0.2148 -0.6268  1.0000
6  A3   0.4423 -0.1236 -0.2105  0.0248  0.0005  1.0000
7  C3  -0.0948 -0.0068  0.3710 -0.2008  0.6477 -0.0355  1.0000
8  B3   0.2070  0.0088 -0.4486  0.1416 -0.4461 -0.0311 -0.9510
1.0000
```

DISPLAY 17.   AVERAGE INVERSE POSTERIOR INFORMATION MATRIX OF
PARAMETERS

```
          1       2       3       4       5       6       7       8
         A1      B1      A2      C2      B2      A3      C3      B3

1  A1   0.6570
2  B1  -0.1952  0.1115
3  A2  -0.8116  0.2141  2.7764
4  C2  -0.0248  0.0019  0.0438  0.0041
5  B2   0.0048 -0.0004 -0.0211 -0.0008  0.0003
6  A3   0.2767 -0.0746 -1.8586 -0.0127  0.0141  1.5224
7  C3   0.0439 -0.0183 -0.2718  0.0035  0.0009  0.2234  0.1194
8  B3  -0.0020  0.0009  0.0131 -0.0002  0.0000 -0.0116 -0.0080
0.0006
```

DISPLAY 18.   AVERAGE STANDARD DEVIATION OF ERRORS IN PARMETER
ESTIMATION

```
         1       2       3       4       5       6       7       8
        A1      B1      A2      C2      B2      A3      C3      B3

1    0.8106  0.3339  1.6662  0.0641  0.0158  1.2338  0.3455  0.0243
```

DISPLAY 19.   AVERAGE CORRELATIONS OF ERRORS IN PARAMETER
ESTIMATION

```
          1       2       3       4       5       6       7       8
         A1      B1      A2      C2      B2      A3      C3      B3

1  A1   1.0000
2  B1  -0.7215  1.0000
3  A2  -0.6010  0.3849  1.0000
4  C2  -0.4777  0.0896  0.4102  1.0000
5  B2   0.3725 -0.0746 -0.8009 -0.7642  1.0000
6  A3   0.2766 -0.1810 -0.9040 -0.1599  0.7226  1.0000
7  C3   0.1569 -0.1591 -0.4722  0.1599  0.1699  0.5240  1.0000
8  B3  -0.1037  0.1051  0.3248 -0.1342 -0.0934 -0.3877 -0.9588
1.0000
```

LOGISTIC COMPONENTS OF CASE   6: STRUCTURAL AVERAGE CASE

INFANTILE COMPONENT   MIDCHILDHOOD COMPONENT   ADOLESCENT COMPONENT

```
ASYMPTOTE =  61.25     ASYMPTOTE =  74.86     ASYMPTOTE =  22.66
SLOPE     = 19.1796    SLOPE     =  6.8048    SLOPE     =  7.7505
LOCATION  =  -0.35     LOCATION  =   5.33     LOCATION  =  12.10
```

SUMMARY OF DESCRIPTIVE PARAMETERS OF THE STRUCTURAL MODEL

EARLY CHILDHOOD MINIMUM OF GROWTH VELOCITY

		NUMBER OF CASES	MEAN	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
AGE	(YRS)	3	2.49	1.59	0.92	-0.49	1.50
HEIGHT	(CM)		78.52	18.40	10.62	-0.50	1.50
VELOCITY	(CM/YR)		12.05	6.26	3.61	0.50	1.50

MIDCHILDHOOD MAXIMUM OF GROWTH VELOCITY

		NUMBER OF CASES	MEAN	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
AGE	(YRS)	3	4.55	0.11	0.07	0.43	1.50
HEIGHT	(CM)		100.24	1.01	0.59	0.02	1.50
VELOCITY	(CM/YR)		6.92	0.37	0.21	-0.02	1.50

PREPUBERTAL MINIMUM OF GROWTH VELOCITY

		NUMBER OF CASES	MEAN	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
AGE	(YRS)	4	9.29	0.21	0.11	0.00	1.09
HEIGHT	(CM)		129.48	1.73	0.87	-0.01	1.32
VELOCITY	(CM/YR)		4.78	0.27	0.14	1.14	2.27

ADOLESCENT MAXIMUM OF GROWTH VELOCITY

		NUMBER OF CASES	MEAN	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
AGE	(YRS)	4	11.96	0.37	0.19	0.50	1.90
HEIGHT	(CM)		144.53	1.77	0.89	-0.06	2.01
VELOCITY	(CM/YR)		6.71	0.46	0.23	0.10	1.76

ADOLESCENT INCREMENT AND PREDICTED STATURE

	NUMBER OF CASES	MEAN	STANDARD DEVIATION	STANDARD ERROR	COEFFICIENTS OF SKEWNESS	KURTOSIS
INCREMENT	4	29.28	2.26	1.13	0.02	1.27
STATURE		158.77	2.53	1.27	-0.01	1.88

PHASE 5: STRUCTURAL AVERAGE OF STRUCTURAL GROWTH CURVES

STRUCTURAL MODEL:
 ANALYSIS OF VELOCITY CURVE OF CASE   6:   STRUCTURAL AVERAGE CURVE

SEX = F

STRUCTURAL MODEL:
 ANALYSIS OF VELOCITY CURVE OF CASE   6:   STRUCTURAL AVERAGE CURVE

SEX = F

EARLY CHILDHOOD MINIMUM OF GROWTH VELOCITY

AGE	(YRS)	3.84	(MOS)	46.02
HEIGHT	(CM)	96.05	(IN)	37.81
VELOCITY	(CM/YR)	7.075	(IN/YR)	2.785

MIDCHILDHOOD MAXIMIM OF GROWTH VELOCITY

AGE	(YRS)	4.33	(MOS)	52.00
HEIGHT	(CM)	99.57	(IN)	39.20
VELOCITY	(CM/YR)	7.081	(IN/YR)	2.788

PREPUBERTAL MINIMUM OF GROWTH VELOCITY

```
     AGE      (YRS)      9.29   (MOS)     111.51
     HEIGHT   (CM)     129.53   (IN)       51.00
     VELOCITY (CM/YR)   4.765  (IN/YR)      1.876
```

ADOLESCENT MAXIMUM OF GROWTH VELOCITY

```
     AGE      (YRS)     11.99   (MOS)     143.83
     HEIGHT   (CM)     144.64   (IN)       56.94
     VELOCITY (CM/YR)   6.686  (IN/YR)      2.632
```

```
ADOLESCENT INCREMENT        (CM)   29.24  (IN)  11.51
PREDICTED MATURE STATURE    (CM)  158.77  (IN)  62.51
```

EVALUATION OF MODEL IN STEPS OF 1.00   CASE: 6 ID:
STRUCTURAL AVERAGE CURVE   SEX = F

        AGE   HEIGHT   VELOCITY   ACC.   THIRD DER.
       (YRS)   (CM)   (CM/YR)   (CM/YR/YR) (CM/YR/YR/YR)

       1.000   70.907   14.637   -8.184   3.573
       2.000   82.248   8.902   -3.244   4.270
       3.000   90.100   7.244   -0.566   1.340
       4.000   97.213   7.077   0.019   0.049
       5.000   104.284   7.030   -0.177   -0.346
       6.000   111.164   6.671   -0.538   -0.327
       7.000   117.520   6.003   -0.756   -0.082
       8.000   123.145   5.262   -0.666   0.271
       9.000   128.135   4.795   -0.204   0.650
      10.000   132.950   4.967   0.585   0.859
      11.000   138.338   5.911   1.165   -0.005
      12.000   144.736   6.686   -0.035   -2.393
      13.000   150.980   5.402   -2.347   -1.328
      14.000   155.119   2.903   -2.233   1.100
      15.000   157.103   1.258   -1.093   0.944
      16.000   157.947   0.538   -0.438   0.408
      17.000   158.320   0.252   -0.177   0.153
      18.000   158.504   0.132   -0.079   0.059
      19.000   158.605   0.076   -0.040   0.025
      20.000   158.664   0.046   -0.022   0.012
      21.000   158.701   0.029   -0.013   0.006
      22.000   158.724   0.018   -0.008   0.004
      23.000   158.739   0.012   -0.005   0.002
      24.000   158.748   0.008   -0.003   0.001
      25.000   158.754   0.005   -0.002   0.001

```

1STRUCTURAL GROWTH CURVE

VALUES FROM PLOT FOR CASE   6:   STRUCTURAL AVERAGE CURVE

SEX = F

```
--------------------------------------------------------------------
   AGE         OBSERVED HEIGHT  FITTED HEIGHT  RESIDUAL  VELOCITY
  (YRS)  (MOS)   (CM)    (IN)     (CM)    (IN)    (CM)     (CM/YR)
OBS
VELOCITY
--------------------------------------------------------------------
   1.00   12.0                    70.9    27.9              14.64
   2.00   24.0                    82.2    32.4               8.90
   3.00   36.0                    90.1    35.5               7.24
   4.00   48.0                    97.2    38.3               7.08
   5.00   60.0                   104.3    41.1               7.03
   6.00   72.0                   111.2    43.8               6.67
   7.00   84.0                   117.5    46.3               6.00
   8.00   96.0                   123.1    48.5               5.26
   9.00  108.0                   128.1    50.4               4.80
  10.00  120.0                   133.0    52.3               4.97
  11.00  132.0                   138.3    54.5               5.91
  12.00  144.0                   144.7    57.0               6.69
  13.00  156.0                   151.0    59.4               5.40
  14.00  168.0                   155.1    61.1               2.90
  15.00  180.0                   157.1    61.9               1.26
  16.00  192.0                   157.9    62.2               0.54
  17.00  204.0                   158.3    62.3               0.25
  18.00  216.0                   158.5    62.4               0.13
  19.00  228.0                   158.6    62.4               0.08
```

```
20.00    240.0                    158.7   62.5              0.05
21.00    252.0                    158.7   62.5              0.03
-------------------------------------------------------------------


*****************************************************************


N O R M A L    E N D    O F    T H I S    P R O B L E M

         >STOP;

N O R M A L    E N D    O F    T H I S    J O B
```
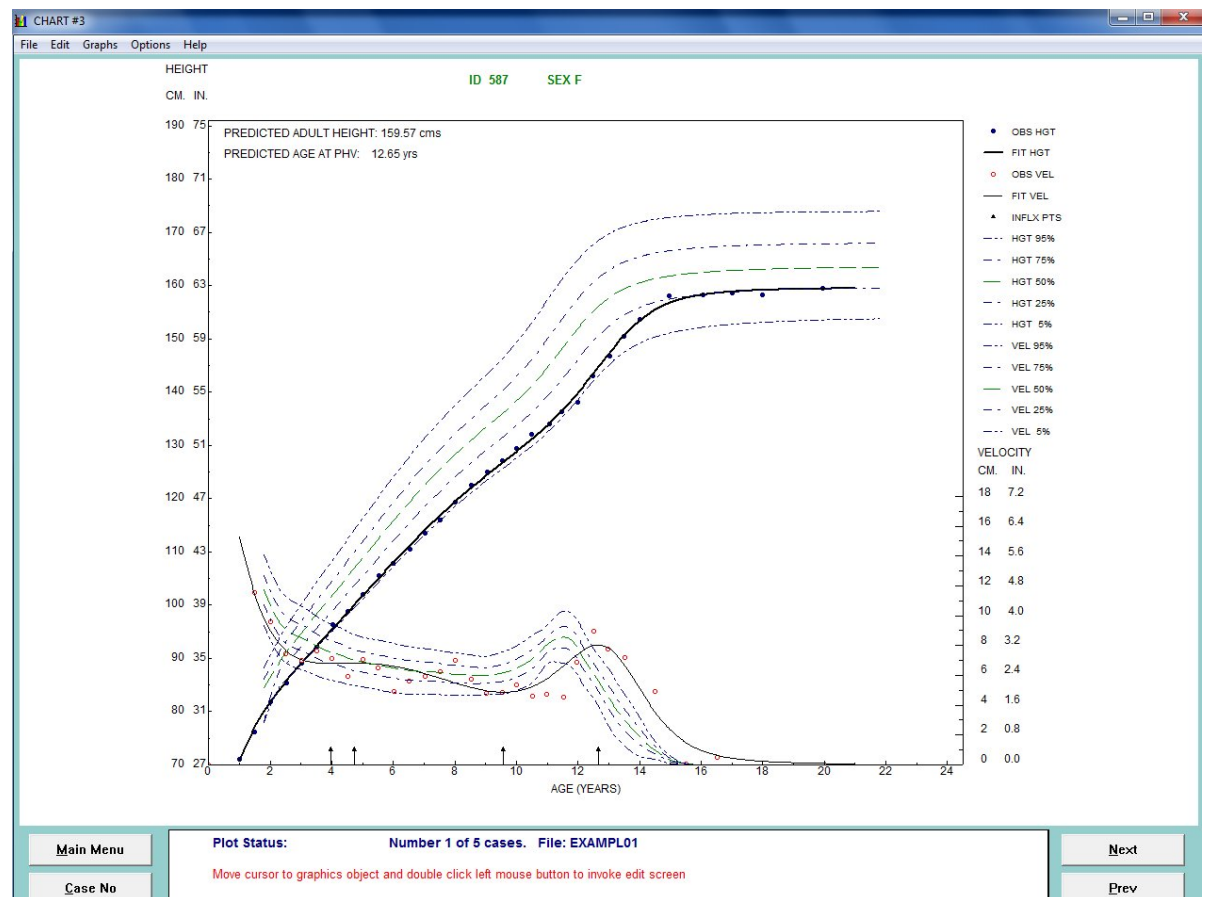
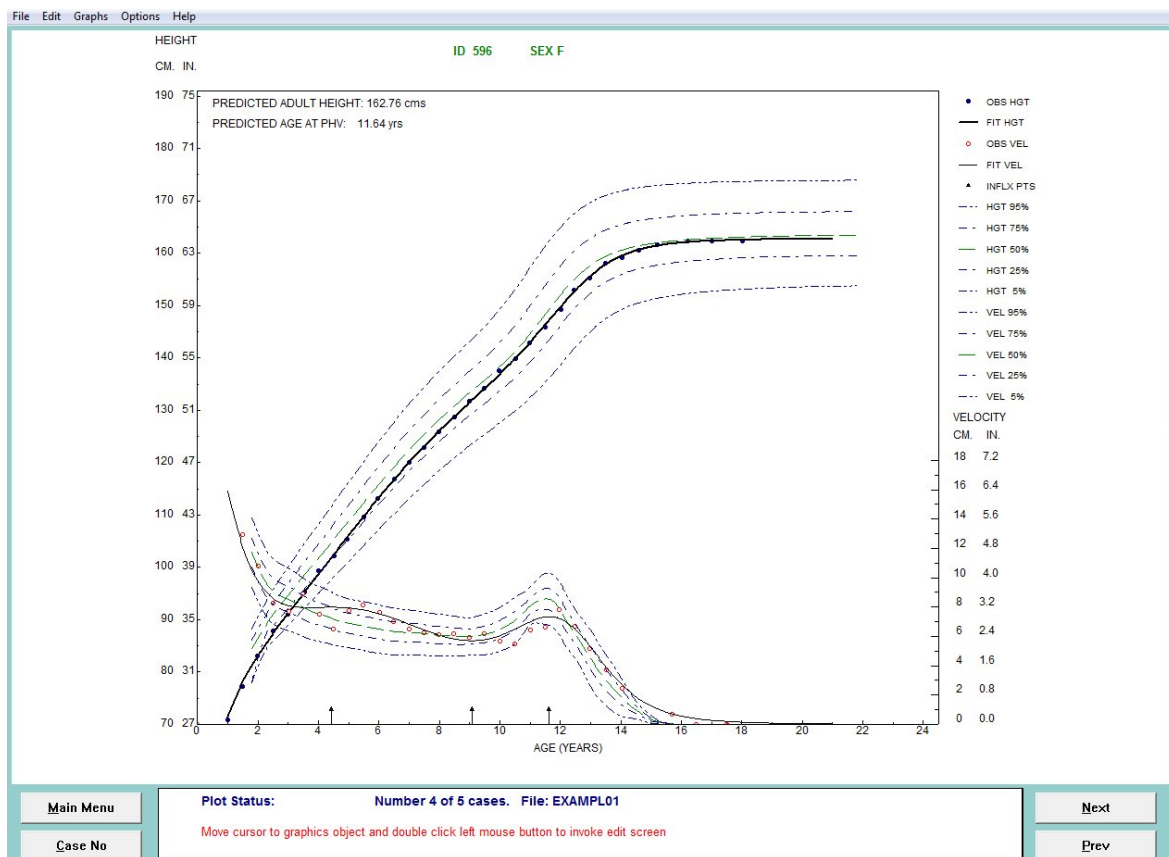## Example 1: screen plot examples

Figure 7.1   Example 1:ID 587; SEX F

Figure 7.2   Example 1: ID 596; SEX F
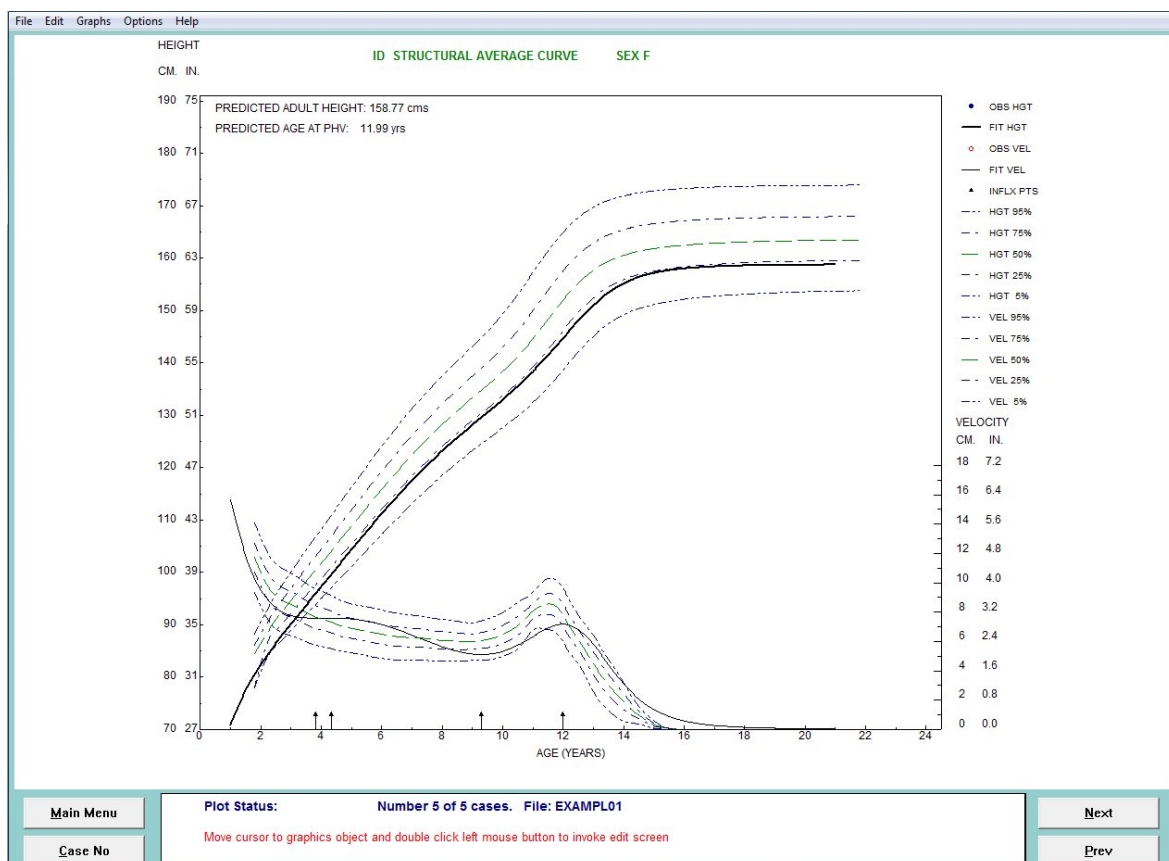
Figure 7.3   Example 1: STRUCTURAL AVERAGE CURVE; SEX F

8. Addendum: Structural and Nonstructural Analysis of Multiphasic Growth 1

by R. Darrell Bock

The orderly growth of children in height and weight is an important clinical sign of their general health and thriving. Because of its greater stability, height is more valuable than weight in this role, and charts for its comparison with population norms are available in many parts of the world. Often these norms are based on samples of children enrolled in long-term developmental studies in which height and weight are measured at regular intervals. The present paper makes use of such data for height kindly provided by Dr. Alex Roche from the records of the Fels Longitudinal Growth Study.

The statistical analysis of height measurements is complex because many sources of variation affect development over the growth span. Between the sexes, differences in physical growth are sufficiently great to require separate analysis and separate norms. Within sex, every person has his or her individual pattern of growth and attained stature. Much of this variation is heritable characteristics and can be predicted from that of close relatives. An important reason for modeling growth is to identify those features that express heritable variation. In the standard approach, one represents the pattern of growth as a parametric function of chronological age and performs conventional quantitative genetic analysis on the covariation of the parameters among relatives (Vandenberg & Falkner, 1965).

Numerous models that serve this purpose appear in the biometric literature. Recently there has been a revival of interest in the concept, originally proposed by Robertson (1908), that growth of organisms in general, and human growth in particular, occurs in a number of additive, more-or-less independent phases during the course of development. The timing and intensity of each phase is assumed genetically programmed in the individual, but they may vary in expression according to environmental conditions. Robertson suggested that human growth in stature might be described as the sum of three logistic components, but he had no suitable data with which to test the model. The first application of the concept to individual growth was that of Bock & Thissen (1976, 1980), who applied it to cases from the Berkeley and Fels growth studies. Their analyses showed that the goodness-of-fit of a triphasic logistic model over the range from one year to maturity was good: the mean-square error averaged 0.65 cm in boys and 0.58 cm in girls.

In the Bock-Thissen model, the phases represent early-childhood, middle-childhood, and adolescent growth. Karlberg (1989) argues on physiological and empirical grounds that, if growth from birth to one year is included, an additional phase would be required. Koops & Grossman (1991) have used a similar model to study additive phases of development, and the allometric relationships thereof, jointly in several dimensions of animal growth. The model of Jolicoeur, *et al*. (1992) applies asymptotic rational functions in a similar way.

A further enhancement of the Bock-Thissen model was suggested by du Toit (1993). To improve the fit of the model in the region of change-over from early to middle childhood, and to provide some asymmetry of the adolescent component, he suggested the addition of "shape" constants in the form of positive exponents to the denominators of the logistic functions. By repeated trials with the Berkeley and Fels data, we found values for these constants that tend to improve the fit of the model. We refer to this triphasic generalized logistic model as the BTT model.

Despite the good fit of the BTT model to measures of height in the Berkeley and Fels longitudinal studies, the average residuals in each yearly interval are not everywhere zero, showing that the

model has some bias, and both the size of the root mean squares, and the sign patterns in the residuals indicate the presence of some remaining systematic variation. There is some evidence of seasonal effects, with slightly increased velocities of growth in summer, and in some cases waves of slightly accelerated or decelerated growth often lasting two or three years during middle childhood (Butler, McKie, & Ratcliffe, 1990). At present the clinical or other significance of these deviations from the main trajectory of growth is obscure, but the analytical techniques described in this paper may help to elucidate them.

In addition to these short-term sources of variation, all measurements of growth contain an irreducible minimum of error because of diurnal effects on standing height or recumbent length and the technical difficulty of positioning the subject in precisely the same way on different occasions. Studies of replicate measurements by an experienced technician in a laboratory setting show that the standard error of measurement of whole body height or length cannot be reduced much below 0.3 cm.

The present paper is an attempt to account statistically for all sources of variation in measures of recumbent length between one year and 25 years of age in the records of the Fels study from 1929 through 1986. It is known that over this period a secular increase in average height amounting to about 4 cm in men and 3 cm in women per generation has occurred in this population (Bock & Sykes, 1989). For present purposes, however, we will ignore the year of birth and regard all the cases as exchangeable members of one population.

The BTT and JPA2 models may be used alone or in combination with Fourier analysis of residuals. The latter approach proceeds as follows:

1. structural analysis of population dispersion of the parameters of the triphasic logistic model describing the main trends in the growth in same-sex cases, and

2. nonstructural analysis of residuals from the model in each individual case.

In step 1, the vector mean and covariance matrix of the assumed multivariate normal distribution of model parameters are estimated by the maximum marginal likelihood (MML) method. In step 2, the model is fitted to each individual case by Bayes modal estimation, referred to here as MAP (Maximum A Posteriori) estimation, and the residuals are smoothed over the range from one to 18 years of age by filtering through a Fourier transform of rank 5.

The two-part analysis of longitudinal data by these methods has a number of advantages over purely structural or nonstructural approaches. The structural part compresses the information in the often irregular or incomplete growth records into a relatively small number of efficiently estimated parameters. These parameters represent the main features of growth that are likely to be of interest in genetic studies, and they are in a form suitable for standard multivariate statistical procedures. The MAP modal method of fitting the free parameters of the model is fast and extremely robust. Because it will fit to even a single observed measure, it provides a method of predicting height at maturity or any other age. Through conditioning of the prior distribution, these predictions can incorporate information in collateral variables such as weight, skeletal age, or heights of relatives.

In computing the finite Fourier transform to smooth the residuals after fitting the structural model, we use conventional least-squares estimation that does not require the orthogonality of the sine and cosine functions. We can therefore apply the procedure to irregular or incomplete data without attributing missing values. The addition of the smoothed residuals to the evaluated structural model then gives a fully nonstructural representation of the growth curve free of any constraints that the model alone might impose. Both the model and the inverse Fourier transform are continuous functions of time with continuous derivatives of all orders. From the sum of the corresponding derivatives of the two parts, we can readily obtain nonstructural velocity and acceleration curves for each case. Moreover, the sample means of the model parameters estimate unambiguously the structural average growth curve of the population; similarly, the inverse transform of the sample mean Fourier coefficients estimate the nonstructural average residuals and their

velocity and acceleration curves.

Finally, the autocorrelation function of the residuals required in the MAP method of fitting is estimated by inverting the spectrum of the time series obtained from the Fourier transforms (see Section *Approximating the Autocorrelation Function* on page 112).

Structural Analysis: the Triphasic Logistic Model

The Bock-Thissen-du Toit (BTT) model is the sum of three generalized logistic terms of the form

$$\frac{a}{\left(1+e^{-\frac{(bt+c)}{a}}\right)^{d}}$$

t is the time (age) variable; *a, b,* and c are parameters of individual growth having a distribution in the population of children to whom the model is applied; *d* is a fixed shape constant.

The parameter *a* is the amount of growth contributed by the term; the quantity $z = bt + c$ in the exponential function is the "logit," and *b* and *c* are its slope and intercept, respectively.

The maximum velocity of growth occurs when

$$t = \left(\log d - c\right)/b$$

at which time height is

$$y = a / D_o^d$$

and velocity is

$$\frac{dy}{dt} = abdE_0 / D_0^{d+1},$$

where $E_0 = e^{-\log d}$ and $D_0 = 1 + E_0$.

We designate the parameters to be estimated when fitting the model to observed heights as

$$a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$$

for the early, middle, and adolescent phases of growth, respectively. These are the (random) parameters that we will assume distributed $N(\mu, \Sigma)$ in the population. In both the Berkeley and Fels data, all attempts to estimate Σ under the restriction of positive definiteness leads to a near-singular matrix. To remove this over-parameterization, Bock & Thissen (1980) imposed a linear restriction on the parameters of the first and second term, but du Toit (1992) later found that setting $c_1 = 0$ serves equally well. In the present paper, we will work with the eight free parameters of this simpler form of the model.

Maximum Marginal Likelihood Estimation of μ and Σ

Bock (1989) discusses MML estimation for measurement models for a general situation in which the likelihood for a vector observation y with respect to a random effect vector θ, and some fixed technical parameters of the measurement procedure ς, is

$$f(y \mid \theta; \varsigma).$$

The probability density function of the random effects,

$$g(\theta; \eta)$$

depends on some fixed population parameters, η, in this case μ and Σ.

In a sample of size N, the fixed quantities ς and η may be conveniently estimated by maximizing the marginal likelihood

$$L_M = \prod_{i=1}^{N} h(y_i),$$

where

$$h(y_i) = f(y_i \mid \theta; \varsigma) g(\mid \theta; \eta)$$

Differentiating and equating to zero, we obtain the marginal likelihood equations,

$$\frac{\partial \log L_M}{\partial \varsigma} = \sum_i^N \frac{1}{h(y_i)} \left[\frac{\partial h\left(y_i; \hat{\varsigma}, \hat{\eta}\right)}{\partial \varsigma} \right] = 0$$

$$\frac{\partial \log L_M}{\partial \eta} = \sum_i^N \frac{1}{h(y_i)} \left[\frac{\partial h\left(y_i; \hat{\varsigma}, \hat{\eta}\right)}{\partial \eta} \right] = 0,$$

simultaneously for the technical and population parameters.

In the present application, we assume the residuals from the growth model, $\Psi_i \lfloor \theta_i \rfloor$ fitted to n_i measurements of case i to be normally distributed with mean zero and autocovariance matrix $\sigma_\varepsilon^2 \Lambda_i$, where Λ_i is the $n_i \times n_i$ autocorrelation matrix; that is,

$$f\left(y_i \mid \theta, \sigma_\varepsilon^2, \Lambda_i\right) = \frac{|\Lambda_i|^{-\frac{1}{2}}}{\sigma_\varepsilon (2\pi)^{n_i/2}} \times$$

$$\exp\left(-\frac{1}{2\sigma_\varepsilon^2} tr\left\{\Lambda_i^{-1}\left[y_i - \Psi_i(\theta)\right]\left[y_i - \Psi_i(\theta)\right]'\right\}\right)$$

Similarly, for the m parameters of the growth model, we assume

$$g\left(\theta; \mu, \Sigma\right) = \frac{|\Sigma|^{-\frac{1}{2}}}{(2\pi)^{m/2}} \exp\left(-\frac{1}{2} tr\left\{\Sigma^{-1}(y_i - \mu)(y_i - \mu)'\right\}\right).$$

Thus, for μ we have the m likelihood equation

$$\frac{\partial \log L_M}{\partial \mu} = \sum_i^N \int_\theta \frac{f_i \cdot g}{\partial \mu} \frac{\partial \log g}{\partial \mu} d\theta$$

$$= \Sigma^{-1} \sum_i^N \left(\bar{\theta}_i - \mu\right) = 0,$$

where $\bar{\theta}_i$ is the posterior mean of θ, given y_i.

For Σ, the $m(m+1)/2$ likelihood equations are

$$\frac{\partial \log L_M}{\partial vech\Sigma} = \frac{1}{2} G' vec \sum_i^N \int_\theta \frac{f_i \cdot g}{h_i} \times$$

$$\left[-\Sigma^{-1} + \Sigma^{-1} (\theta - \mu)(\theta - \mu)' \Sigma^{-1} \right] d\theta$$

$$= \frac{1}{2} G' \sum_i^N vec \Sigma^{-1} \times$$

$$\left[-\Sigma + \Sigma_{\theta|y_i} - \left(\bar{\theta}_i - \mu \right)\left(\bar{\theta}_i - \mu \right)' \right] \Sigma^{-1} = 0,$$

where $\Sigma_{\theta|y_i}$ is the posterior covariance matrix of θ, given y_i, and $vec\Sigma = G'vech\Sigma$ (see McCulloch, 1982; Moneta, 1991).

Similarly, for σ_ε^2 provisionally assuming Λ known,

$$\frac{\partial \log L_M}{\partial \sigma_\varepsilon^2} = \frac{1}{2} \sum_i^N \int_\theta \frac{f_i \cdot g}{h_i} \times$$

$$\left[-n_i \sigma_\varepsilon^{-2} + \sigma_\varepsilon^{-4} tr \left[y_i - \Psi_i(\theta) \right]\left[y_i - \Psi_i(\theta) \right]' \right]$$

$$= \frac{1}{2} \sigma_\varepsilon^{-4} \sum_i^N \left\{ -n_i \sigma_\varepsilon^{-2} + tr \left[\bar{\Psi}_i \left(\bar{\theta}_i \right) \right] \Sigma_{\theta|y_i} \left[\bar{\Psi}_i \left(\bar{\theta}_i \right) \right]' + \left[y_i - \Psi_i \left(\bar{\theta}_i \right) \right]\left[y_i - \Psi_i \left(\bar{\theta}_i \right) \right]' \right\}$$

$$= 0,$$

where $\psi_i \left(\bar{\theta}_i \right) = \left[\partial \Psi_i \left(\bar{\theta}_i \right) / \partial \theta \right]$ is the gradient of Ψ_i at $\bar{\theta}_i$.

Provided the posterior means and covariance matrices can be evaluated, these equations may be solved efficiently by Fisher-scoring from the following elements of the information matrix (see Bock, 1989):

$$I(\mu) = \Sigma^{-1} \left(N\Sigma - \sum_i^N \Sigma_{\theta|y_i} \right) \Sigma^{-1}$$

$$I(\Sigma) = \frac{1}{2} G' \left(\Sigma^{-1} \otimes \Sigma^{-1} \right) \times$$

$$\left[\sum_i^N \left(\Sigma - \Sigma_{\theta|y_i} \right) \otimes \left(\Sigma - \Sigma_{\theta|y_i} \right) \right] \left(\Sigma^{-1} \otimes \Sigma^{-1} \right) G$$

$$I\left(\sigma_\varepsilon^2\right)=\frac{1}{2}\sigma_\varepsilon^{-8}\sum_i^N tr\left[\sigma_\varepsilon^2 I_{n_i}-\left[\psi_i\left(\bar{\theta}_i\right)\right]\Sigma_{\theta|y_i}\left[\psi_i\left(\bar{\theta}_i\right)\right]'\right]$$

$$I\left(\Sigma,\mu\right)=0$$

$$I\left(\sigma_\varepsilon^2,\mu\right)=0$$

$$I\left(\sigma_\varepsilon^2,\Sigma\right)=\frac{1}{2}\sigma_\varepsilon^{-4}G'vec\Sigma^{-1}\times$$

$$\left[\sum_i^N\Sigma_{\theta|y_i}\left[\psi_i\left(\bar{\theta}_i\right)\right]'\left[\psi_i\left(\bar{\theta}_i\right)\right]\Sigma_{\theta|y_i}\right]\Sigma^{-1}$$

For linear models, the required posterior moments are available in closed form from standard results for the conditional distribution of multivariate normal variables (see Bock, 1989, for an application to a linear model). For nonlinear models such as the logistic this is not the case. By approximating the nonlinear model with the first two terms of a Taylor expansion about a provisional value of θ, however, it is possible to apply the linear results iteratively to obtain a solution (Lindstrom & Bates, 1990), but accurate estimation of the elements of ~ is difficult by this method.

An alternative approach is a Fisher-scoring solution based on Bock (1989) in which the first two moments of the posterior distributions for each subject are computed by Gauss-Hermite quadrature. Although the high dimensionality of the problem would seem to make this impractical, du Toit (1992) has shown that for multiphasic models, which are linear combinations of nonlinear models, the moments of the combining coefficients in the multivariate normal case can be estimated from the moment generating function, leaving only those of the nonlinear parameters to be evaluated numerically. Thus, in the present application, the quadrature becomes five-dimensional and is well within the capacity of a 486 computer if the number of nodes per dimension is six, which is sufficient in this case.

More recently, Bock & du Toit (1994) have shown that, by use of quadratures adapted separately to the posterior distribution of each subject, a type of fractional quadrature requiring only a total of 243 points can be applied to any nonlinear model in five or more dimensions when the population distribution is elliptical. In the present application, both of these methods are computationally feasible for 486 and RISC computers.

Those methods of solving the marginal likelihood equations require good starting values, but they may be easily obtained by initially treating provisional estimates of the () as multivariate observations. They also benefit from imposing the constraint of positive-definiteness on ~ by means of the Gaussian decomposition,

$$\Sigma = LDL',$$

where L is unit lower triangular and

$$D = diag\left[e^{\pi j}\right].$$

Estimating Σ in this form assures its positive-definiteness without restrictions on the parameters. Only minor modifications of the likelihood equations and information matrix are required.

There are several possible approaches to approximating the auto correlation matrix, Ai, which fortunately does not have to be highly accurate in these applications. If the data points are equally and not too closely spaced over the age range of interest, there is little harm in ignoring the autocorrelation and setting $\Lambda_i = I_{n_i}$

Alternatively, we could apply conventional ARMA or Fourier analysis to the empirical residuals, after provisionally estimating θ_i, in order to estimate the autocorrelations, either specific to the individual case or assumed common to all N cases. The Fourier approach is illustrated in Section *Approximating the Autocorrelation Function*. If the observations are unequally spaced, recent results of du Toit (1992) provide for MML estimation of the α and β parameters of the ARMA(l,l) model, which describes reasonably well the autocorrelation structure of residuals from the triphasic model.

Example 1

Maximum marginal likelihood estimates of the means and covariance matrices of the model parameters, and the residual standard deviations, for the Fels male and female samples are shown in Tables A-l and A-2. The two samples consist of the oldest son and/or

Table A.1

Estimated means and covariance matrix of the BTT *parameters, and the residual standard deviation, for* 159 *male cases from the Eels Longitudinal Growth Study*

Parameter		a_1	b_1	a_2	c_2	b_2	a_3	c_3	b_3
Mean		91.4335	0.8707	66.2927	2.2650	0.3578	24.1907	19.2349	1.3730
S.E.		0.6535	0.0139	0.7318	0.0350	0.0035	0.2742	0.1566	0.0110
Covariance Matrix									
	a_1	17.0087							
	b_1	-0.2868	0.0111						
	a_2	6.6809	-0.1126	23.7315					
	c_2	-0.0293	-0.0001	-0.0170	0.0261				
	b_2	-0.0706	0.0025	-0.0108	-0.0007	0.0008			
	a_3	-4.9447	0.0834	-10.7800	0.0105	0.0134	16.9748		
	c_3	0.7283	-0.0340	0.3064	0.0010	-0.0075	-0.7701	1.3794	
	b_3	0.0021	0.0014	-0.0909	0.0074	0.0005	0.0467	0.0445	0.0094
Residual S.D.		0.7375							

daughter in each family. The estimates, obtained by the Fisher scoring solution of the MML equations evaluated by numerical integration, are based on all cases with largely complete data records between two and 22 years of age. The standard errors for the means and standard deviations show the results to be highly precise in samples of this size. These estimates assume uncorrelated residuals. Estimates of the means and covariances are essentially the same when ARMA(l,l) residual variation is assumed.

The latent roots of the correlation matrices corresponding to the two covariance matrices show appreciable variation in all eight dimensions. Elimination of an additional parameter of a three-phase generalized logistic model could not be justified in these data.

Table A.2: *Estimated means and covariance matrix of the* BTT *parameters, and the residual standard deviation, for* 132 *male cases from the Fels Longitudinal Growth Study*

Parameter		a_1	b_1	a_2	c_2	b_2	a_3	c_3	b_3
Mean		88.4897	0.8798	63.1001	2.1611	0.3613	15.8041	18.7247	1.5545
S.E.		0.0000	0.0087	0.4407	0.0193	0.0036	0.3559	0.2073	0.0178
Covariance Matrix									
	a_1	11.1069							
	b_1	-0.1731	0.0125						
	a_2	1.7811	-0.0278	17.8046					
	c_2	-0.1310	-0.0081	-0.0601	0.0063				
	b_2	-0.0273	0.0019	-0.0353	0.0018	0.0013			
	a_3	-2.4718	0.0385	-5.5053	0.0405	0.0151	9.7800		
	c_3	-0.4207	-0.0060	-0.0763	-0.0028	-0.0008	-1.3892	2.4487	
	b_3	0.0276	-0.0015	-0.1042	-0.0007	0.0001	-0.1397	0.2004	0.0266
Residual S.D.		0.6654							

Bayes Modal Estimation of θ

In routine use of the BTT model, Bayes estimation of the posterior mean θ, given y_i is not practical because of the heavy computation required. The Bayes modal, or MAP, estimator is much more satisfactory in this respect, and it gives very nearly the same values. Similarly, the inverse posterior information matrix is much easier to compute than the posterior covariance matrix. Although a biased estimator of $\Sigma_{\theta|y_i}$ it is sufficiently accurate to provide approximate standard errors for the estimated θ.

Eight or ten Newton-Raphson iterations are generally enough to approximate the MAP estimate of θ with a maximum error less than 0.0001 for any parameter, starting from $\hat{\mu}$ as the first provisional value. The gradient vector is

$$G(\theta) = \sigma_\varepsilon^{-2} \left[\psi_i(\theta_i) \right] \Lambda^{-1} \left[y_i - \psi_i(\theta_i) \right] - \Sigma^{-1} \left(\theta - \hat{\mu} \right)$$

and the expected Hessian matrix is

$$H(\theta) = -\sigma_\varepsilon^{-2} \left[\psi_i(\theta_i) \right]' \Lambda^{-1} \left[\psi_i(\theta_i) \right] - \Sigma^{-1}$$

where the elements in the row of matrix derivatives, $\psi_i(\theta_i)$, corresponding to time t_j are:

$$D_1^{-1}, \quad a_1 d_1 t_j E_1 D_1^{-(d_1+1)}$$

$$D_2^{-1}, \quad a_2 d_2 E_2 D_2^{-(d_2+1)}, \quad a_2 d_2 t_j E_2 D_2^{-(d_2+1)}$$

$$D_3^{-1}, \quad a_3 d_3 E_3 D_3^{-(d_3+1)}, \quad a_3 d_3 t_j E_3 D_3^{-(d_3+1)}$$

With E and D defined as in Table A.3.

Table A.3

Derivatives of the Fourier Nonstructural Growth Curves

Order	BTT Model	Fourier Series
1	$\Sigma_{l=1}^3 a_l b_l d_l E_l D_l^{-(d_l+1)}$	$+2C\sum_{k=1}^{r} k\left(A_k \sin CKt^* + B_k \cos CKt^*\right)$
2	$\Sigma_{l=1}^3 a_l b_l^2 d_l E_l D_l^{-(d_l+2)}\left[(d_l+1)E_l - D_l\right]$	$+2C^2\sum_{k=1}^{r} k^2\left(-A_k \cos CKt^* + B_k \sin CKt^*\right)$
3	$\Sigma_{l=1}^3 a_l b_l^3 d_l E_l D_l^{-(d_l+3)}\left[(d_l+1)(d_l+2)E_l^2 - 3(d_l+1)E_l D_l + D_l^2\right]$ $+2C^3\sum_{k=1}^{r} k^3\left(A_k \sin CKt^* - B_k \cos CKt^*\right)$	
	$E_l = e^{-(b_l t + c_l)}, \quad D_l = 1 + E_l, \quad c_1 = 0$ $C = 2\pi/(t_U - t_L + 1.0); \quad t^* = t - (t_U - t_L)/2$	

The k-th Newton step is then

$$\hat{\theta}_i^{(k+1)} = \hat{\theta}_i^{(k)} - \left[H\left(\hat{\theta}_i^{(k)}\right)\right]^{-1} G\left(\hat{\theta}_i^{(k)}\right).$$

Example 2: Structural analysis of height data from one year to maturity

Figure A-1 is a plot of the BTT model fitted by the MAP method to 44 observed heights. The subject was the first boy in the Fels Longitudinal Study for whom a complete record to maturity was obtained.

He was born in 1930. The prior mean and covariance matrix for the MAP fitting were those for male cases shown in Table A-1. The assumed measurement error variance was 1 cm.
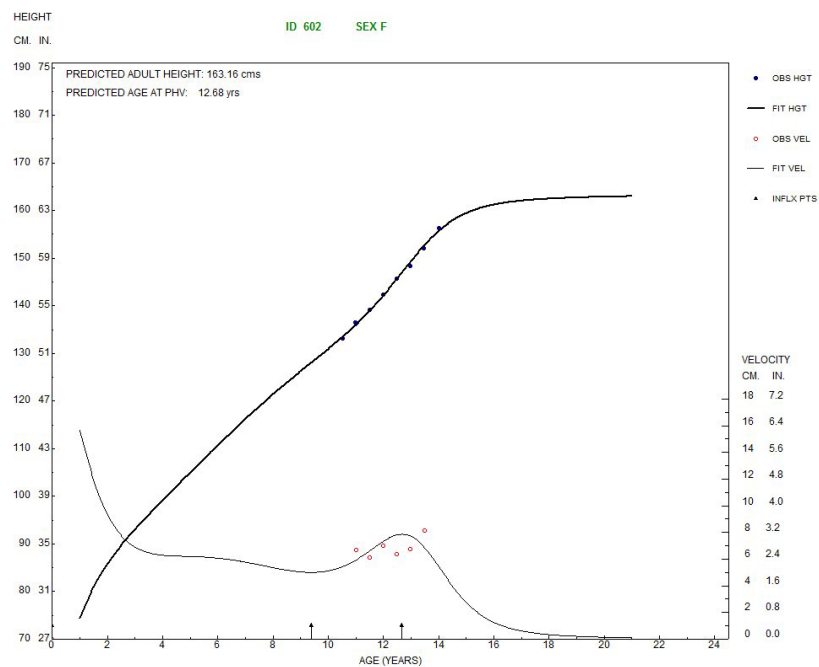
Figure A.1 *Structural curves of a boy in the Fels Longitudinal Study*

The root-mean-square error of fit is 0.716 cm. Although the model gives a good account of the observed heights, the residual variance is significantly in excess of the measurement error of approximately 0.4 cm, which is typical of the Fels anthropometric procedure *(p <* .01). The derivative of the fitted curve shows only a trivial indication of a midchildhood growth velocity maximum (the so-called "midspurt") but the adolescent peak height velocity (PHV) and prepubertal velocity minimum (PPM) are clearly evident at 13.89 and 11.20 years, respectively. The velocities at these ages are 8.32 and 4.34 cm/year, respectively.

Nonstructural Analysis

In fitting the structural model, we regard the residuals as non-informative and seek only to minimize their mean square error. Inasmuch as the error with the **BTI** model is generally in excess of the measurement error, however, the residuals may have some significance for physiological or genetic studies of the growth process and should not be ignored. A way to extract the potentially interesting information from the background of measurement error is to smooth the residuals nonstructurally. A number of standard methods, including spline and kernel smoothing (Largo, *et al.,* 1978; Gasser, *et al.,* 1984b), can be applied to this problem. We shall use Fourier analysis for this purpose and compare the results with optimal kernel estimates of the distance, velocity, and acceleration curves.

Depending on the rank (number of harmonics) chosen for the finite Fourier transform, the inverse transform provides a continuous approximation to the residuals that retains more or less of their details. We have found in the Fels and Berkeley data that a rank 5 transform smoothes the residuals in a plausible way, while reducing the mean square to the known magnitude of the measurement error and rendering the sign pattern of the final residuals essentially random.

Our objective is to smooth the residuals with the lowest rank transform that will reduce the unaccounted-for variance to about $0.1 \ \text{cm}^2$, The range of the smoothing will extend from the earliest observation, y_L, at an age greater than or equal to one year, to the latest observation, y_U, at an age less than or equal to 18 years. We therefore define the transform on the interval $C = 2\pi / (t_U + t_L) / 2$ and measure time $t^* = t - t_0$ continuously from the midpoint $t_0 = (t_U + t_L) / 2$, where t_U and t_L are the ages in decimal years corresponding to y_L and y_U. The additional half year beyond the observation at each end of the interval permits the terminal smoothed residuals to be nonzero. The smoothed residual at time *t* is

$$\tilde{u}(t) + A_0 + 2\sum_{k=1}^{r}\left(A_k \cos Ckt^* + B_k \sin Ckt^*\right),$$

Where $r < n_i / 2 - 1$ is the rank of the transform A_0 and A_k and B_k are its coefficients.

To avoid the requirement of equal spacing of the n observations, we estimate the $2n_i + 1$ coefficients by ordinary least squares on the assumption that the time points are all distinct and the ultimate residuals, after the smoothing, are independent with mean zero and variance σ^2. For this purpose, we construct for case i the $n_i \times (2r+1)$ full-rank basis

$$X_i = \left[1, 2\cos Ckt_{ij}^*, 2\sin Ckt_{ij}^*\right], \quad k = 1, 2, \ldots, r; \; j = 1, 2, \ldots, n_i$$

and estimate the $2r + 1$ vector of coefficients, β_i as

$$\hat{\beta}_i = \left(X_i'X_i\right)^{-1} X_i'u_i, \quad \text{for } \left|X_i'X_i\right| \neq 0,$$

where $u_i = y_i - \Psi_i\left(\hat{\theta}_i\right)$ is the n_i vector of residuals from the MAP fitted structural model. To better condition this calculation, we first perform the Cholesky decomposition $S_iS_i' = X_i'X_i$ and compute the coefficients

$$\hat{\xi}_i = S_i^{-1}X_i'u_i,$$

with respect to an orthonormal basis. Then

$$\hat{\beta}_i = \left(S_i^{-1}\right)'\hat{\xi}_i.$$

In the Fels data, $r = 5$ gives about the right amount of detail in the curves. The minimum number of observations necessary to estimate r_i is therefore $2r + 1 = 11$. But to allow some degrees of freedom for estimating σ, we generally require a minimum of $3r = 15$ observations at distinct time points, preferably with no gaps in excess of two years.

Because this smoothing process is effectively nonstructural, the sum of the triphasic logistic model and the smoothed residuals is a nearly nonstructural representation of the original data. The representation has derivatives of all orders - the first three of which are given in Table A-3 in terms of the phases of the BTT model and of the Fourier series on the interval $t_U + t_L + 1.0$.

Example 3: Fourier smoothing of residuals from the BTT model

Figure A-2 shows height and growth velocity curves for Example 1 computed as the sum of the BTT curve and a finite Fourier series to the fifth harmonic fitted to the residuals. The interval of the combined fitting is from one to 18 years. The dotted lines beyond 18 years derive from the BTT model alone. The root-mean-square error is reduced to 0.460 cm from the previous value, a significant reduction in variance $(p < .01)$, assuming that the Fourier residuals are uncorrelated normal.

The fit of the resulting nonstructural curve is visibly better than the BTT curve in the region of the adolescent growth spurt. The corresponding region of the velocity curve is more peaked than in Figure A-1, and its maximum is advanced to 14.26 years - to the preceding minimum 12.34 years. The corresponding PHV and PPM of 8.55 and 4.11 cm/year, are more extreme than those in Example 1. The most noticeable difference between the representations of growth is in the velocity curves. The nonstructural curve shows two well-defined preadolescent maxima at 7.0 and 10.75 years, with preceding minima at 5.75 and 9.25 years, respectively. The mid-childhood growth cycles of this boy resemble closely the patterns found by Butler, McKie, & Ratcliffe (1990) in their recent study of preadolescent growth in Scottish children.

Kernel estimation of growth curves serves much the same purpose as Fourier smooth but is entirely nonparametric in the sense that points on the curves are simply weighted averages of the observed heights. The velocity and acceleration curves are not exact derivatives of the distance curve, but close correspondence between the Fourier and kernel velocity curves in Examples 3 and 4 suggest that differences are slight.
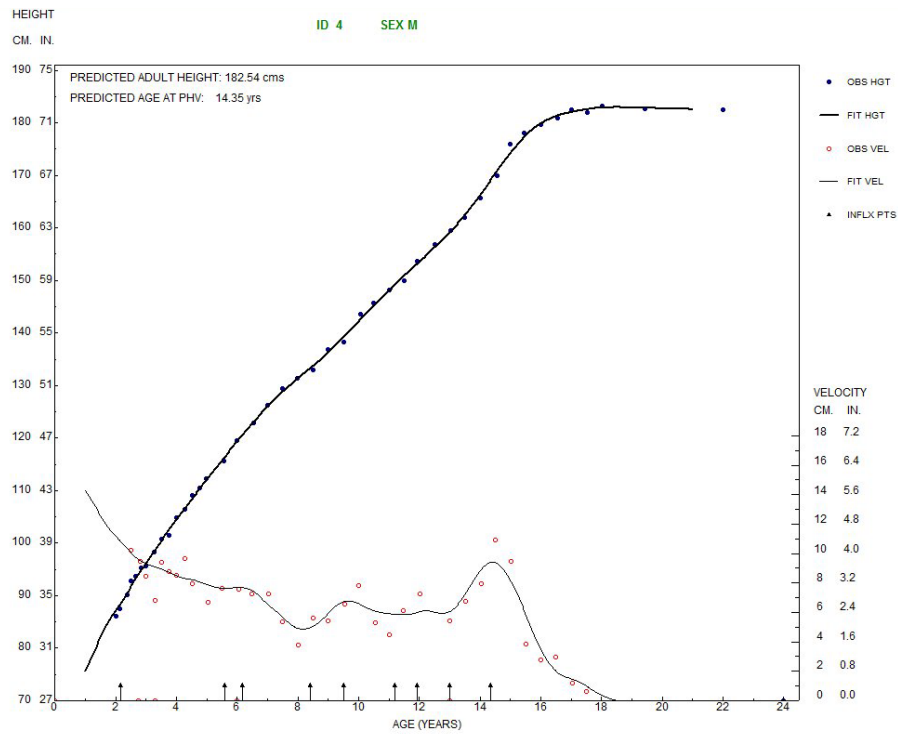
Figure A.2 *Nonstructural Fourier curves of a boy in the Fels Longitudinal Study.*
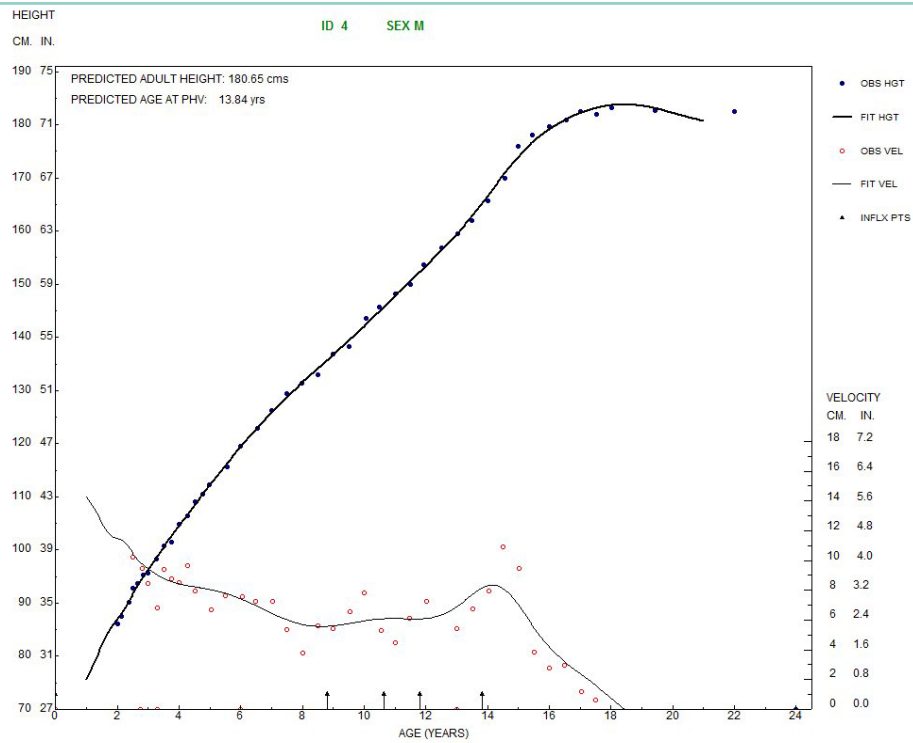

Figure A.3 *Nonstructural kernel curves of a boy in the Fels Longitudinal Study.*

Example 4: Estimation of growth distance, velocity and acceleration by optimal kernel smoothing

For the data of Example 1, Figure A-3 shows smoothed curves computed with optimal kernels for distance and velocity derived by Gasser, *et al.* (1984b).[2] The kernel for acceleration was also applied to the data in order to locate the turning points of the velocity curve. The band width for each of these smooths was 2.5 years. As is apparent from comparison of Figures *A-2* and *A-3,* this choice of band width produces almost the same result as Rank 5 Fourier smoothing when the intervals between data points are small relative to the band width. The distance curves of the two methods are not visibly different nor are the velocity curves different up to 10 years. After that age, the kernel velocity maxima and minima are somewhat less extreme than their Fourier counterparts. Their locations also differ somewhat, but as descriptions of the course of growth, they are essentially equivalent.

Approximating the Autocorrelation Function

The likelihood term shown above in MML estimation of the population parameters and in MAP estimation of the individual growth parameters requires the matrix Λ, which contains the pairwise correlations between residuals. As in most time series analyses, we assume the distribution of the residuals to be at least weakly stationary; that is, that the expected values and variances, and the correlations of the same lag, are constant. Thus, to estimate the elements of Λ, we need only estimate the autocorrelation function (ACF), $\rho\left(t_{ik}\right)$ giving the correlation for lag $t_{ik} = t_i - t_k$.

In the present application, we must also make the rather strong assumption that the ACF of the residuals is the same for all same-sex subjects in the population. Estimating an individual ACF would be possible if the observations for that case were closely and uniformly spaced over the full growth cycle, but we do not wish to limit the MAP fitting of individual growth curves in that way. We will therefore assume that the common ACF is estimated in a sample of complete, equally spaced records from a large-scale growth study. It can then be applied in clinical situations where data are much less extensive.

A way of estimating the common autocorrelation function exists in the Wiener-Khinchin theorem, which establishes the reciprocal relationship between the spectrum of a stationary time series and t~e corresponding auto covariance function (ACVF) (see Bloomfield, 1976; Fuller, 1976; Jenkins & Watts, 1968). In the case *ofn* equally spaced observations, the spectrum is the additive partition of the residual sum of squares into $r + 1$ terms attributable to successive harmonics of the Fourier series. Because the basis matrix is orthogonal in this case, the ordinates of the point spectrum at the successive harmonics are the squares of the corresponding coefficients:

Harmonics$_{(h)}$	Spectrum S_h
0	A_0^2
1	$A_1^2 + B_1^2$
2	$A_2^2 + B_2^2$
$\vdots$	$\vdots$
r	$A_r^2 + B_r^2$

If $r = n/2 - 1$, $\sum_{h=0}^{r} S_h = \sum_j^n u_i^2 = n\sigma_\varepsilon^2$, the sum of ordinates equals the residual sum of squares, and the Fourier series reproduces the observed residuals at the *n* discrete time points exactly; otherwise, it approximates them.

Let $\gamma(t)$ and $\rho(t)$ be the respective autocovariance and autocorrelation functions for a continuously measured lag *t,* and let γ_k and ρ_k be their values at lag *k* of *n* discrete, equally spaced observations. Then

$$\gamma_k = \frac{1}{n}\sum_j^{n-k} u_j u_{j+k}$$

and

$$\rho_k = \gamma_k / \sigma_\varepsilon^2 = \gamma_k / \gamma_0.$$

The essential result relating the ACVF and the spectrum in discrete time is

$$s_h = \sum_{k=a}^{b} \gamma_k \cos\frac{2\pi kh}{n},$$

where $a = 1 + n/2$, $b = n/2$, and $h = 0, 1, \ldots, n/2$, for n even, and $a = b = (n-1)/2$ and $h = 0, 1, \ldots, (n-1)/2$, for n odd. In this formula, $s_h = S_h / n$ are the ordinates of the mean-square spectrum.

Since $\gamma(t)$ is an even function, these results simplify to

$$s_h = \gamma_0 + 2 \sum_{k=1}^{b} \gamma_k \cos \frac{2\pi k h}{n},$$

where $b = n/2$ for n even, and $b = (n-1)/2$ for n odd.

The Fourier transform of the spectrum then gives back the ACVF,

$$\gamma_k = s_0 + \sum_{k=1}^{b} \gamma_k \cos \frac{2\pi k h}{n}.$$

Here we are assuming the transform is of full rank, in which case the sum of the mean square spectral ordinates equals σ_ε^2 and $\rho_k = \gamma_k / \gamma_0$. Applied to growth data measured with independent error of variance σ^2, however, only that part of the residual variance equal to $\sigma_\varepsilon^2 - \sigma^2$ contributes to the autocorrelation. For this reason, we use only the leading $r + 1$ ordinates of the spectrum in computing the autocovariances, where r is considerably smaller than $n/2$ or $(n-1)/2$. In that case, $\gamma_0 < \sigma_\varepsilon^2$ and $\rho_k = \gamma_k / \sigma_\varepsilon^2$.

When applying these results to the construction of Λ, we robustify the calculation of the spectrum by weighting the cross-products with the Tukey window,

$$W_{ijk} = \frac{1}{2}\left(1 + \cos \frac{c k t_{ij}}{2}\right),$$

to decrease the influence of extreme lags. Then the j, k element of the autocovariance function for the continuous lag $t_{ik} = t_i - t_k$ is approximated by

$$\hat{\gamma}(t_{jk}) = s_0 + \sum_{h=1}^{r} s_h \cos C h t_{jk},$$

where $C = 2\pi / \left(t_U - t_L + 1.0\right)$. The corresponding autocorrelation

is $\tilde{\rho}\left(t_{jk}\right) = \tilde{\gamma}\left(t_{jk}\right) / \sigma_\varepsilon^2$. Generally, we set the correlations for lags greater than five years equal to zero. The resulting Λ matrix is strictly positive-definite for any lags in this range, including $t_{jk} = 0$, provided the transform is less than full rank, so that $\tilde{\gamma}\left(t_{jj}\right) < \sigma_\varepsilon^2$.

To aggregate the results for a sample of same sex cases, we average the spectral ordinates, s_h and divide by $\sum_i^N n_i$ obtain the average mean-square spectrum. Its inverse then estimates the assumed common autocovariance function. We then divide the autocovariances by the total variance of the residuals to obtain the autocorrelations with respect to variation not attributable to measurement error.

Example 5: Estimation of a common autocorrelation function

The calculations for this example are based on 66 male and 64 female cases in the Fels archive for which observed heights were available for each year and half year ($\pm$ 0.1 year) between the ages of two and 18 years. To meet the requirement of equal spacing, all age points were set exactly to their yearly and half yearly values. From these data spectral ordinates were computed for the first five harmonics and averaged over the within each sex group. The resulting values are shown in Table A-4. Inversion of the spectra yielded the autocorrelations in monthly lags up to four years. Smooth curves connecting these values are shown in Figure A-4. The value for lag zero represents the average coefficient of determination of the residuals by the fitted finite series; *i.e.,* the covariance at that point is the sum of the six average ordinates in Table A-4. The variation not accounted for is assumed to be independent measurement error.

Conclusion

The procedures for structural and nonstructural analysis described here provide a unified methodology for the study of complex pat-

Table A.4

Average spectral ordinates

| | N | Harmonics | | | | | |
		0	1	2	3	4	5
Male	67	.0117	.3162	1.1017	1.9525	.9768	.8540
Female	64	.0097	.3551	.6996	1.0980	1.1868	.6948

terns of growth. Based on proven and well-understood statistical techniques, they serve both to describe growth and to predict its future course. Although the analytical methods benefit from complete and regular data, their Bayesian origin enables them to make the best use of the observations available, however fragmentary, when fitting individual growth curves. When estimating the population distributions of growth parameters or other features of the growth curves, the use of maximum marginal likelihood estimation insures consistent and efficient estimation.

The procedures for fitting growth curves are implemented in the AUXAL program of Bock, du Toit, & Thissen (1994).
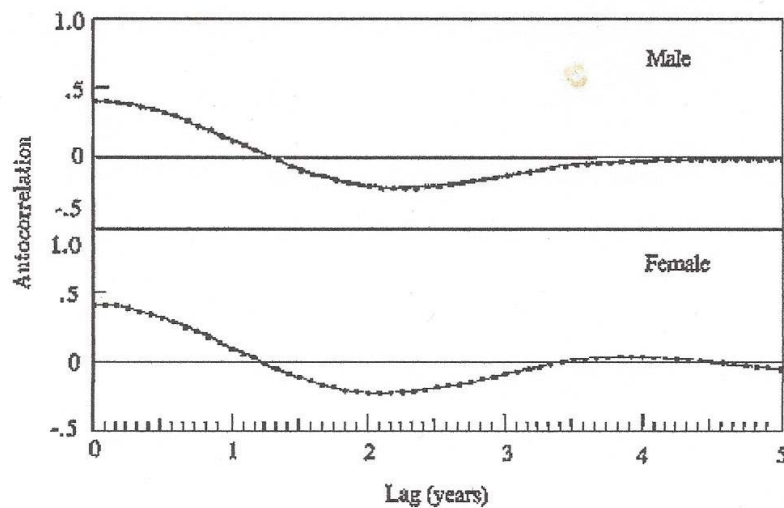

Figure A.4: *Estimated autocorrelation functions for 67 male and 64 female cases from the Fels Longitudinal Study*

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