Executing analyses based on the MDM file

Once the MDM file is constructed, it can be used as input for the analysis. As mentioned earlier, model specification has three steps:

- Specification of the level-1 model. In our example, we shall model mathematics achievement (MATHACH) as the outcome, to be predicted by student SES. Hence, the level-1 model will have two coefficients: the intercept and the SES-MATHACH slope.
- Specification of the level-2 prediction model. We shall predict each school's intercept by school SECTOR and MEANSES in our example. Similarly, SECTOR and MEANSES will predict each school's SES-MATHACH slope.
- Specification of level-1 coefficients as random or non-random. We shall model both the intercept and the slope as having randomly varying residuals. That is, we are assuming that the intercept and slope vary not only as a function of the two predictors, SECTOR and MEANSES, but also as a function of a unique school effect. The two school residuals (*e.g.*, for the intercept and slope) are assumed sampled from a bivariate normal distribution.

The procedure for executing analyses based on the MDM file is described below.

Step 1: To specify the level-1 prediction model

1. From the **HLM** window, open the **File** menu.
2. Choose **Create a new model using an existing MDM file** to open an **Open MDM File** dialog box. Open an existing MDM file (HSB.MDM in our example). The name of the MDM file will be displayed on the title bar of the main window. A list box for level-1 variables (>>**Level-1<<**) will appear (see Figure 13).
3. Click on the name of the outcome variable (MATHACH in our example). Click **Outcome variable** (see Figure 13). The specified model will appear in equation format.

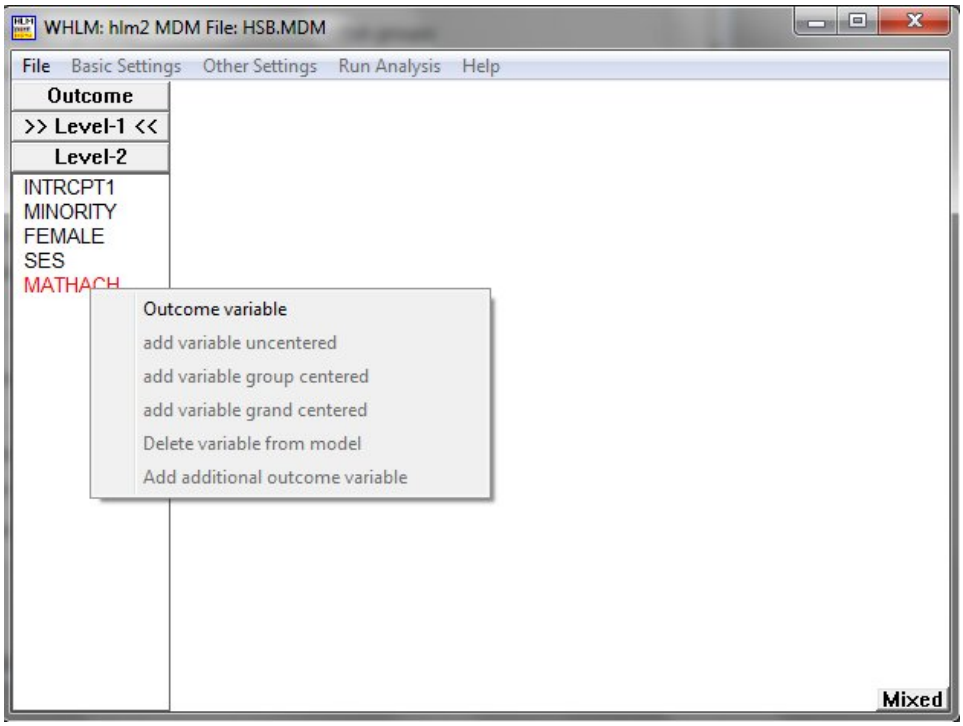

Figure 13 Model window for the HS&B example

4. Click on the name of a predictor variable and click the type of centering (SES and **add variable group centered**, see Figure 14). The predictor will appear on the equation screen and each regression coefficient associated with it will become an outcome in the Level-2 model (see Figure 15).

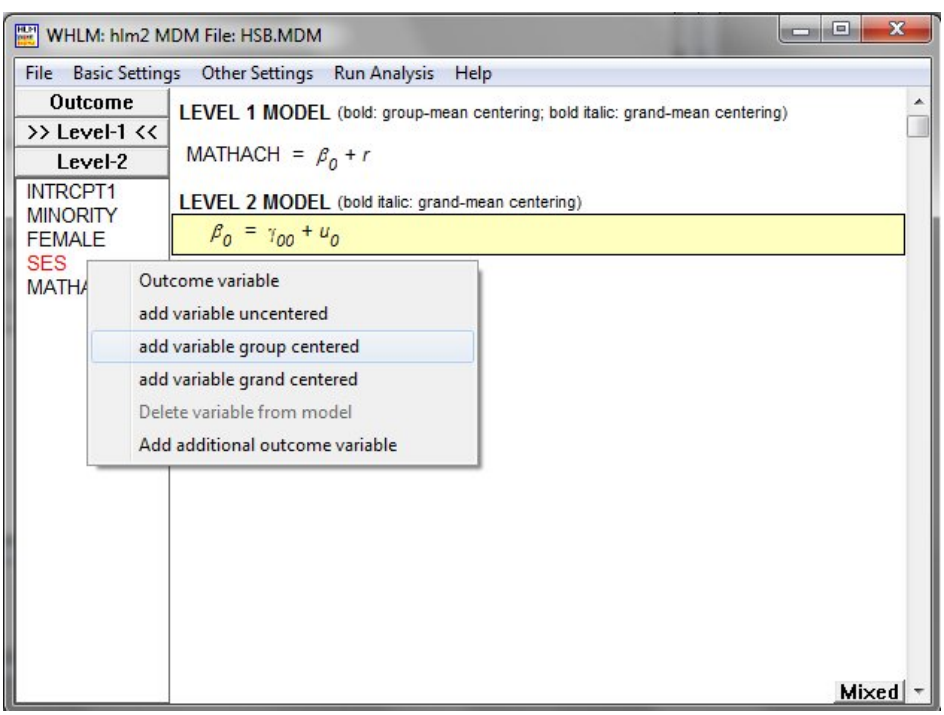

Figure 14 Specification of model predictor, SES, for the HS&B example

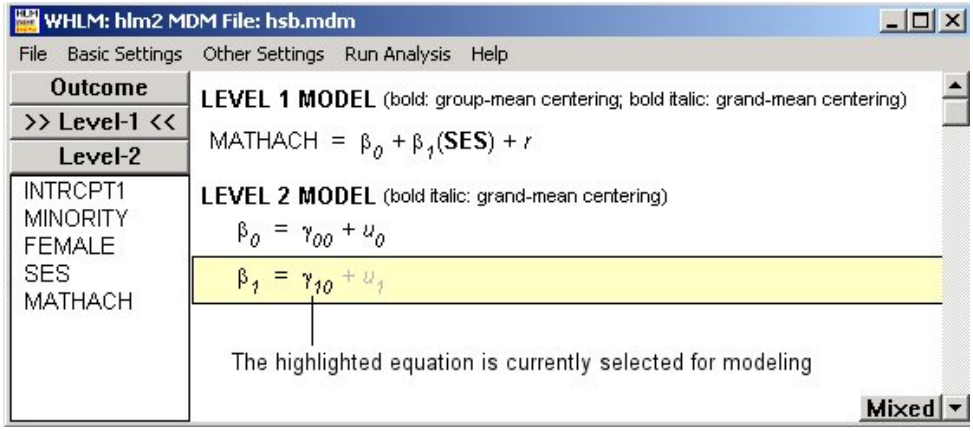

Figure 15 Model window for the HS&B example

Step 2: To specify the level-2 prediction model

1. Select the equation containing the regression coefficient(s) to be modeled by clicking on the equation (β_0 (intercept) and β_1 (SES slope) in our HS&B example). A list box for level-2 variables (>>Level-2<<) will appear (see Figure 16).
2. Click to select the variable(s) to be entered as predictor(s) and the type of centering. For our example, select SECTOR and **add variable uncentered**, and MEANSES and add variable **grand-mean centered** to model β_0 and β_1, see Figure 16.
3. HLM allows the model to be displayed in three alternative forms. Figure 17 displays the model specified in the default notation familiar to users of previous versions of HLM.

1

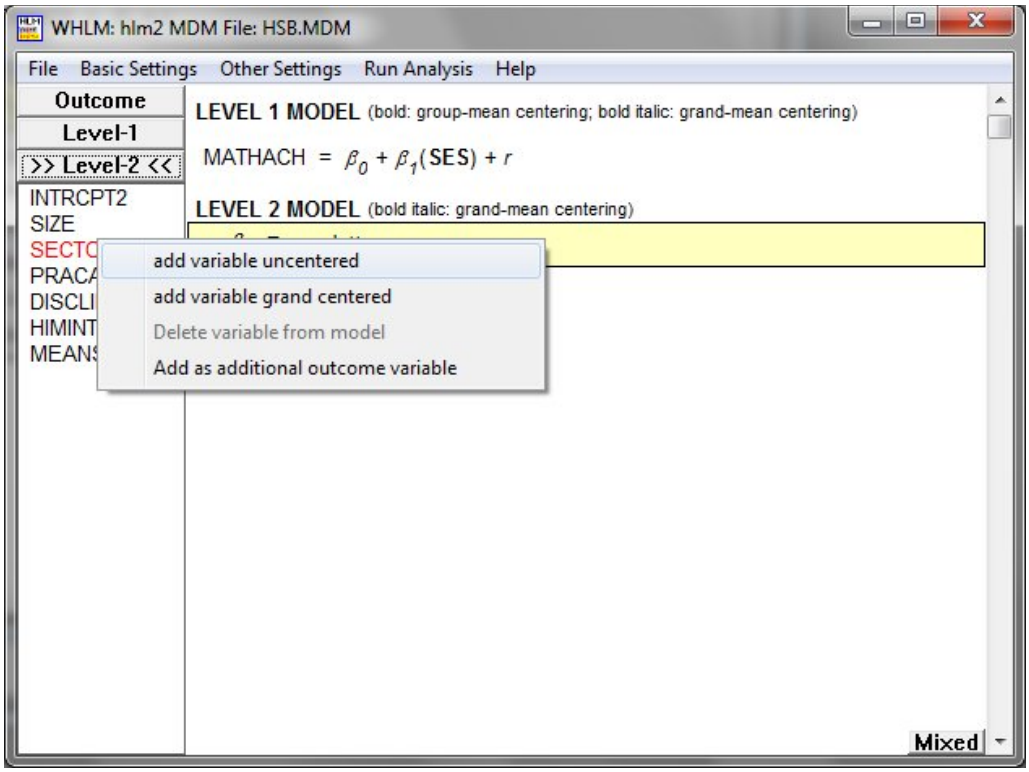

Figure 16 Specification of the level-2 model

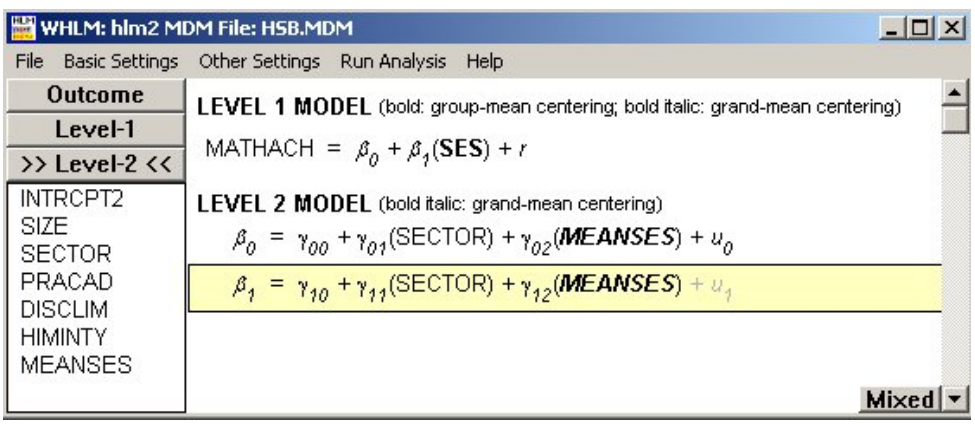

Figure 17 Model window for the HS&B example

4. In addition, the model can also be displayed in a mixed model formulation and with complete subscripts for all coefficients present in the model as illustrated in Figure 18. The mixed model is obtained by clicking the **Mixed** button at the bottom of the main window. The model is shown as a single equation, obtained by substituting the equations for β_0 and β_1 in the level-1 equation. This notation shows the model in a familiar linear regression format, and also draws attention to any cross-level interaction terms present in the combined model. By using the **Preferences** dialog box accessible via the **File** menu both the mixed model formulation and the model with subscripts for all coefficients can be displayed automatically. The model can also be saved as an EMF file for later use in reports or papers.

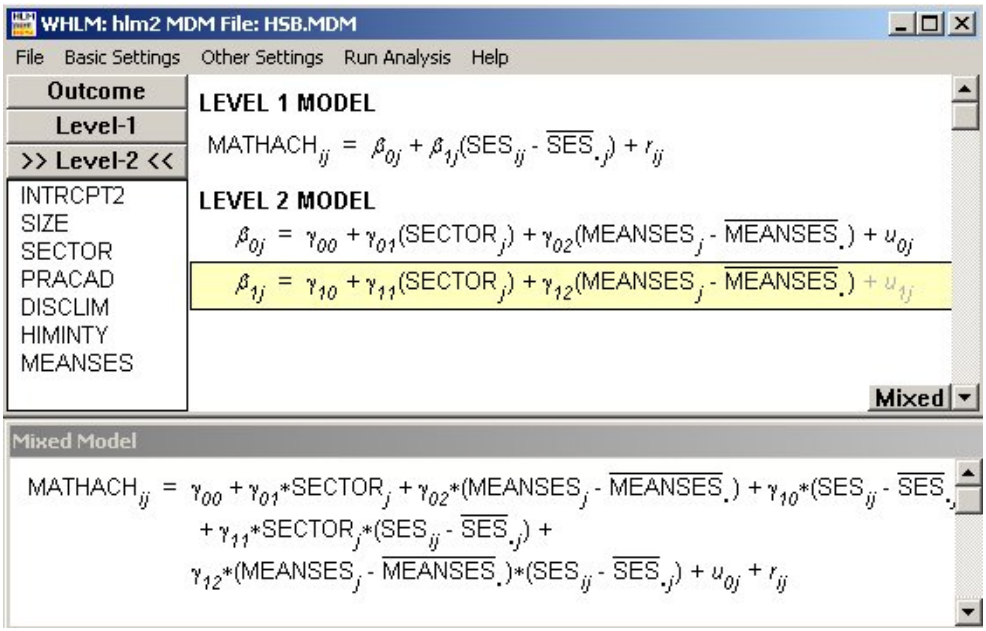

Figure 18 Alternative model window for the HS&B example

Step 3: To specify level-1 coefficients as random or non-random

The program begins by assuming that only the intercept (β_0) is specified as random. The u_1 at the end of the β_1 equation is grayed out and constrained to zero (See Figure 15), *i.e.* this level-1 coefficient is specified as "fixed". In the HS&B example, both level-1 coefficients, β_0 and β_1, are to be specified as random. To specify the SES slope as randomly varying, click on the equation for β_1 so that the error term u_1 is enabled. Note that one can toggle the error term in any of the three following ways:

- Click on the error term, u_1.
- Type *u*.
- Right-click on the yellow box, which will bring up a single-item menu **toggle error term**. Click on the button.

Steps 1 to 3 are the three major steps for executing analyses based on the MDM file. After specifying the model, a title can be given to the output and the output file can be named by the following procedure:

1. Select **Basic Settings** to open the **Basic Model Specifications – HLM2** dialog box. Enter a title in the **Title** field (for example, Intercept and slopes-as-Outcomes Model) and an output file name in **Output file name** field (see Figure 19). Click **OK**.
2. Open the **File** menu and choose **Save As** to open a **Save command file** dialog box.

3. Enter a command file name (for example, HSB1.MDM).
4. Click **Run Analysis**. A dialog box displaying the iterations will appear (see Figure 20).

Note: If you wish to terminate the computations early, press the Ctrl-C key combination once. This will stop the analysis after the current iteration and provide a full presentation of results based on that iteration. If you press Ctrl-C more than once, however, computation is terminated immediately and all output is lost.

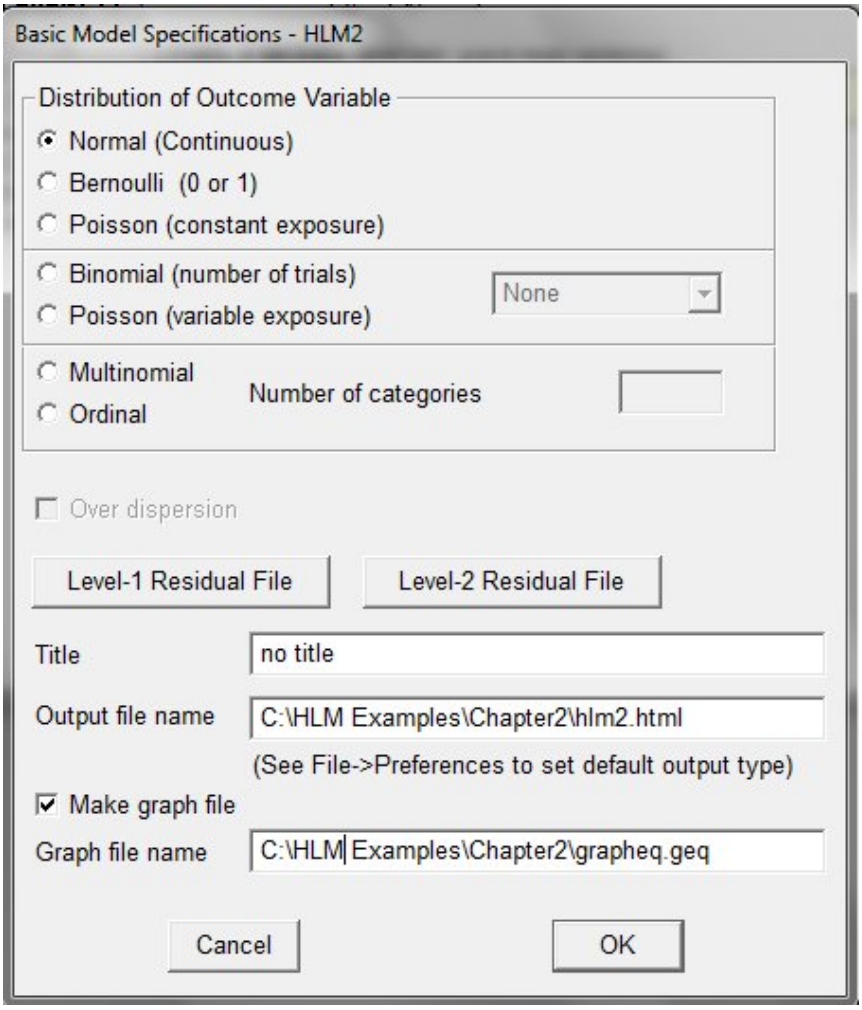

Figure 19 Basic Model Specifications – HLM2 dialog box for the HS&B example

```
C:\HLM\HLM2.EXE
The value of the likelihood function at iteration 38 = -2.325094E+004
The value of the likelihood function at iteration 39 = -2.325094E+004
The value of the likelihood function at iteration 40 = -2.325094E+004
The value of the likelihood function at iteration 41 = -2.325094E+004
The value of the likelihood function at iteration 42 = -2.325094E+004
The value of the likelihood function at iteration 43 = -2.325094E+004
The value of the likelihood function at iteration 44 = -2.325094E+004
The value of the likelihood function at iteration 45 = -2.325094E+004
The value of the likelihood function at iteration 46 = -2.325094E+004
The value of the likelihood function at iteration 47 = -2.325094E+004
The value of the likelihood function at iteration 48 = -2.325094E+004
The value of the likelihood function at iteration 49 = -2.325094E+004
The value of the likelihood function at iteration 50 = -2.325094E+004
The value of the likelihood function at iteration 51 = -2.325094E+004
The value of the likelihood function at iteration 52 = -2.325094E+004
The value of the likelihood function at iteration 53 = -2.325094E+004
The value of the likelihood function at iteration 54 = -2.325094E+004
The value of the likelihood function at iteration 55 = -2.325094E+004
The value of the likelihood function at iteration 56 = -2.325094E+004
The value of the likelihood function at iteration 57 = -2.325094E+004
The value of the likelihood function at iteration 58 = -2.325094E+004
The value of the likelihood function at iteration 59 = -2.325094E+004
The value of the likelihood function at iteration 60 = -2.325094E+004
The value of the likelihood function at iteration 61 = -2.325094E+004
```

Figure 20 Iteration screen

Annotated HLM2 output

The output file will automatically be displayed in the format specified via the **Preference** menu. It can also be opened by selecting the **View Output** option from the **File** menu. Here is the output produced by the Windows session described above (see example HSB1.MDM).

Specifications for this HLM2 run

Problem Title: Intercepts and Slopes-as-outcomes Model

The data source for this run = HSB.MDM
The command file for this run = HSB1.MLM
Output file name = hlm2.html
The maximum number of level-1 units = 7185
The maximum number of level-2 units = 160
The maximum number of iterations = 100
Method of estimation: restricted maximum likelihood

The outcome variable is MATHACH

Summary of the model specified

Level-1 Model

$$\text{MATHACH}_{ij} = \beta_{0j} + \beta_{1j}*(\text{SES}_{ij}) + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}*(\text{SECTOR}_j) + \gamma_{02}*(\text{MEANSES}_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}*(\text{SECTOR}_j) + \gamma_{12}*(\text{MEANSES}_j) + u_{1j}$$

SES has been centered around the group mean.
MEANSES has been centered around the grand mean.

Mixed Model

$$MATHACH_{ij} = \gamma_{00} + \gamma_{01}*SECTOR_j + \gamma_{02}*MEANSES_j + \gamma_{10}*SES_{ij} + \gamma_{11}*SECTOR_j*SES_{ij} + \gamma_{12}*MEANSES_j*SES_{ij} + u_{0j} + u_{1j}*SES + r_{ij}$$

The information presented on the first page or two of the HLM2 printout summarizes key details about the MDM file (*e.g.*, number of level-1 and level-2 units, whether weighting was specified), and about both the fixed and random effects models specified for this run. In this particular case, we are estimating the model specified by Equations 4.14 and 4.15 in *Hierarchical Linear Models*.

Level-1 OLS Regressions

Level-2 Unit	INTRCPT1	SES slope
1224	9.71545	2.50858
1288	13.51080	3.25545
1296	7.63596	1.07596
1308	16.25550	0.12602
1317	13.17769	1.27391
1358	11.20623	5.06801
1374	9.72846	3.85432
1433	19.71914	1.85429
1436	18.11161	1.60056
1461	16.84264	6.26650

When first analyzing a new data set, examining the OL equations for all of the units may be helpful in identifying possible outlying cases and bad data. By default, HLM2 does not print out the ordinary least squares (OL) regression equations, based on the level-1 model. The OLS regression equations for the first 10 units, as shown here, were obtained using optional settings on the **Other Settings** menu.

The average OLS level-1 coefficient for INTRCPT1 = 12.62075
The average OLS level-1 coefficient for SES = 2.20164

This is a simple average of the OLS coefficients across all units that had sufficient data to permit a separate OLS estimation.

Least Squares Estimates

$\sigma^2 = 39.03409$

Least-squares estimates of fixed effects

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.083837	0.106889	113.050	7179	<0.001
SECTOR, γ_{01}	1.280341	0.157845	8.111	7179	<0.001
MEANSES, γ_{02}	5.163791	0.190834	27.059	7179	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.935664	0.155268	18.907	7179	<0.001
SECTOR, γ_{11}	-1.642102	0.240178	-6.837	7179	<0.001
MEANSES, γ_{12}	1.044120	0.299885	3.482	7179	<0.001

Least-squares estimates of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.083837	0.169507	71.288	7179	<0.001
SECTOR, γ_{01}	1.280341	0.299077	4.281	7179	<0.001
MEANSES, γ_{02}	5.163791	0.334078	15.457	7179	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.935664	0.147576	19.893	7179	<0.001
SECTOR, γ_{11}	-1.642102	0.237223	-6.922	7179	<0.001
MEANSES, γ_{12}	1.044120	0.332897	3.136	7179	0.002

The first of the fixed effects tables are based on OLS estimation. The second table provides robust standard errors. Note that the standard errors associated with γ_{00}, γ_{01}, and γ_{12} are smaller than their robust counterparts.

The least-squares likelihood value = -2.336211E+004
Deviance = 46724.22267
Number of estimated parameters = 1

Starting Values

$\sigma^2_{(0)}$ = 36.72025

$\tau_{(0)}$

INTRCPT1,β_0	2.56964	0.28026
SES,β_1	0.28026	-0.01614

New $\tau_{(0)}$

INTRCPT1,β_0	2.56964	0.28026
SES,β_1	0.28026	-0.01614

The initial starting values failed to produce an appropriate variance-covariance matrix ($\tau_{(0)}$). An automatic fix-up was introduced to correct this problem (New $\tau_{(0)}$).

Estimation of fixed effects
(Based on starting values of covariance components)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.094864	0.204326	59.194	157	<0.001
SECTOR, γ_{01}	1.226266	0.315204	3.890	157	<0.001
MEANSES, γ_{02}	5.335184	0.379879	14.044	157	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.935219	0.168674	17.402	157	<0.001
SECTOR, γ_{11}	-1.634083	0.260672	-6.269	157	<0.001
MEANSES, γ_{12}	1.015061	0.323523	3.138	157	0.002

Above are the initial estimates of the fixed effects. These are not to be used in drawing substantial conclusions.

```
The value of the log-likelihood function at iteration 1 = -2.325199E+004
The value of the log-likelihood function at iteration 2 = -2.325182E+004
The value of the log-likelihood function at iteration 3 = -2.325174E+004
The value of the log-likelihood function at iteration 4 = -2.325169E+004
The value of the log-likelihood function at iteration 5 = -2.325154E+004
. . .
The value of the log-likelihood function at iteration 57 = -2.325094E+004
The value of the log-likelihood function at iteration 58 = -2.325094E+004
The value of the log-likelihood function at iteration 59 = -2.325094E+004
The value of the log-likelihood function at iteration 60 = -2.325094E+004
```

Below are the estimates of the variance and covariance components from the final iteration and selected other statistics based on them.

```
******* ITERATION 61 *******
```

σ^2 = 36.70313 *Level-1 variance components*

τ

INTRCPT1, β_0	2.37996	0.19058	*Level-2 variance-covariance components*
SES, β_1	0.19058	0.14892	

τ (as correlations) *Level-2 variance-covariance components expressed as correlations*

INTRCPT1, β_0	1.000	0.320
SES, β_1	0.320	1.000

These are average reliability estimates for the random level-1 coefficients

Random level-1 coefficient	Reliability estimate
INTRCPT1, β_0	0.733
SES, β_1	0.073

The value of the log-likelihood function at iteration 61 = -2.325094E+004

The next three tables present the final estimates for: the fixed effects with GLS and robust standard errors, variance components at level-1 and level-2, and related test statistics.

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.095006	0.198717	60.865	157	<0.001
SECTOR, γ_{01}	1.226384	0.306272	4.004	157	<0.001
MEANSES, γ_{02}	5.333056	0.369161	14.446	157	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.937787	0.157119	18.698	157	<0.001
SECTOR, γ_{11}	-1.640954	0.242905	-6.756	157	<0.001
MEANSES, γ_{12}	1.034427	0.302566	3.419	157	<0.001

Final estimation of fixed effects (with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.095006	0.173688	69.637	157	<0.001
SECTOR, γ_{01}	1.226384	0.308484	3.976	157	<0.001
MEANSES, γ_{02}	5.333056	0.334600	15.939	157	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.937787	0.147615	19.902	157	<0.001
SECTOR, γ_{11}	-1.640954	0.237401	-6.912	157	<0.001
MEANSES, γ_{12}	1.034427	0.332785	3.108	157	0.002

The first table provides model-based estimates of the standard errors while the second table provides robust estimates of the standard errors. Note that the two sets of standard errors are similar. If the robust and model-based standard errors are substantively different, it is recommended that the tenability of key assumptions should be investigated.

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	*p*-value
INTRCPT1, u_0	1.54271	2.37996	157	605.29503	<0.001
SES slope, u_1	0.38590	0.14892	157	162.30867	0.369
level-1, r	6.05831	36.70313			

Statistics for current covariance components model

Deviance = 46501.875643
Number of estimated parameters = 4