Multinomial model for the Teacher data

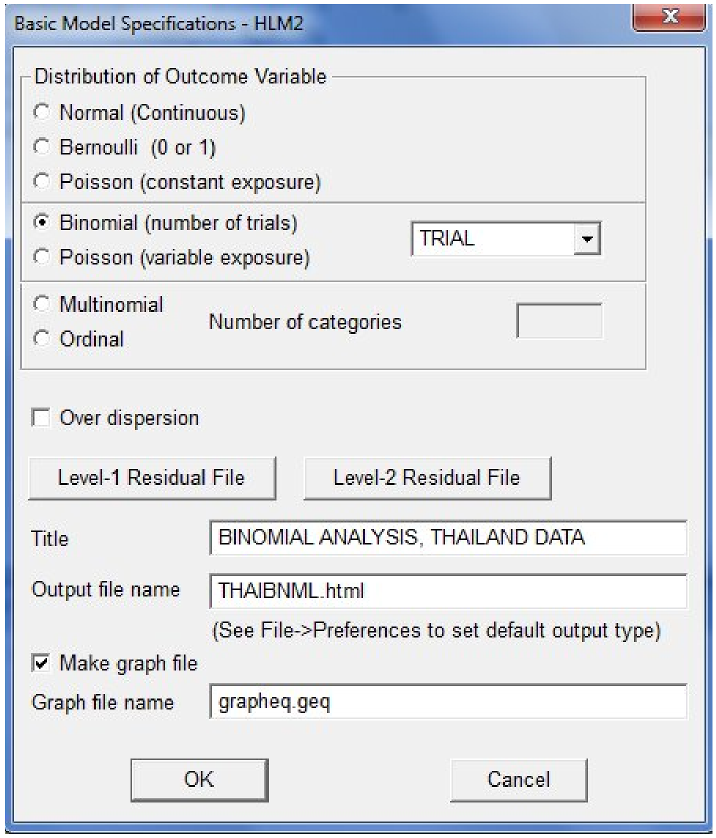
1. Introduction to HGLM models

Model specification for nonlinear analysis are specified via the **Basic Settings** dialog box as shown below. Six options are currently available. For the Binomial model and Poisson (variable exposure) the TRIAL or exposure variable is selected to the right of these two options; for the multinomial and ordinal models the number of categories should also be specified to the right of these two options.

If desired, and over-dispersion option is available for binomial and Poisson models. This option is not available with Laplace estimation. To specify over-dispersion, set the σ^2 to **computed** on the **Estimation Settings** dialog box accessed via **Other Settings** on the main menu bar.

The nonlinear analysis is doubly iterative so the maximum number of macro iterations can be specified as well as the maximum number of micro iterations Similarly, convergence criteria can be set for macro and micro iterations.

This is the fifth in a set of six examples illustrating HGLM models.



2. Description of the model

For multi-category nominal data, we use a multinomial model and a logit link function. This is an extension of the Bernoulli model with more than two possible outcomes. This feature is not available in HLM4.

Let $\text{Prob}\left(R_{ij} = m\right) = \phi_{ij}$, that is, the probability that person *i* in group *j* lands in category *m* is ϕ_{ij}, for categories *m* = 1, ..., *M*, there being *M* possible categories.

For example, R_{ij} = 1 if high school student *i* in school *j* goes on to college; R_{ij} = 2 if that student goes on to a job; R_{ij} = 3 if that student becomes unemployed. Here *M* = 3. The analysis is facilitated by constructing dummy variables $Y_1, Y_2, \ldots, Y_M$, where Y_{mij} = 1 if R_{ij} = *m*, 0 otherwise. For example, if student *ij* goes to college, R_{ij} = 1, so Y_{1ij} = 1, Y_{2ij} = 0, Y_{3ij} = 0; if student *ij* goes to work, R_{ij} = 2, so Y_{1ij} = 0, Y_{2ij} = 1, Y_{3ij} = 0; if that student becomes unemployed, R_{ij} = 3, so Y_{1ij} = 0, Y_{2ij} = 0, Y_{3ij} = 1. This leads to a definition of the probabilities as $\text{Prob}\left(Y_{mij} = 1\right) = \phi_{mij}$. For example, for *M* = 3,

$$\text{Prob}\left(Y_{1ij}=1\right)=\phi_{1ij}$$
$$\text{Prob}\left(Y_{2ij}=1\right)=\phi_{2ij}$$
$$\text{Prob}\left(Y_{3ij}=1\right)=\phi_{3ij}=1-\phi_{1ij}-\phi_{2ij}$$

Note that because $Y_{3ij}=1-Y_{1ij}-Y_{2ij}$, Y_{3ij} is redundant. According to the multinomial distribution, the expected value and variance of Y_{mij} given ϕ_{mij}, are then

$$E\left(Y_{mij}\mid\phi_{mij}\right)=\phi_{mij}\qquad Var\left(Y_{mij}\mid\phi_{mij}\right)=\phi_{mij}\left(1-\phi_{mij}\right).$$

The covariance between outcomes Y_{mij} and $Y_{m'ij}$ is

$$Cov\left(Y_{mij},Y_{m'ij}\right)=-\phi_{mij}\phi_{m'ij}.$$

HGLM uses the logit link function when the level-1 sampling model is multinomial. Define η_{mij} as the log-odds of falling into category *m* relative to that of falling into category *M*. Specifically

$$\eta_{mij}=\log\left(\frac{\phi_{mij}}{\phi_{Mij}}\right)$$

where

$$\phi_{Mij}=1-\sum_{m=1}^{M-1}\phi_{mij}.$$

In words, η_{mij} is the log odds of being in *m*-th category relative to the *M*-th category, which is known as the “reference category.”

At level-1, we have

$$\eta_{mij}=\beta_{0j(m)}+\sum_{q=1}^{Q}\beta_{qj(m)}X_{qij},$$

for $m = 1, ..., (M-1)$. For example, with $M = 3$, there would be two level-1 equations, for η_{1ij} and η_{2ij}.

The level-2 model has a parallel form

$$\beta_{qj(m)}=\gamma_{q0(m)}+\sum_{s=1}^{S_q}\gamma_{qs(m)}W_{sj}+u_{qj(m)}.$$

Thus, for $M = 3$, there would be two sets of level-2 equations. The multinomial and ordinal analyses currently produce unit-specific results only. They do not provide population-average results.

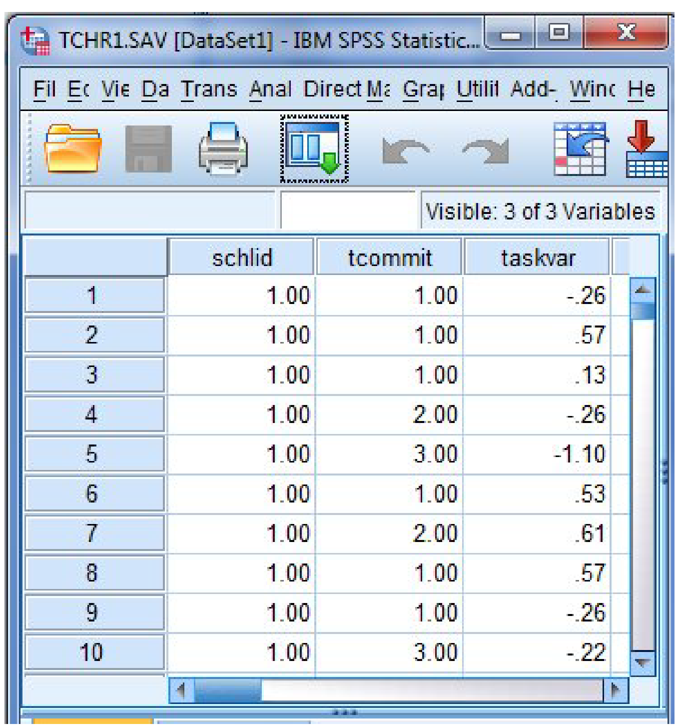
3. Description of the data

Data are from a 1990 survey of teachers in 16 high schools in California and Michigan. In the MDM file, not included with the software, there are a total of 650 teachers. The level-1 SPSS input file is TCHR1.SAV, and the level-2 file is TCHR2.SAV.

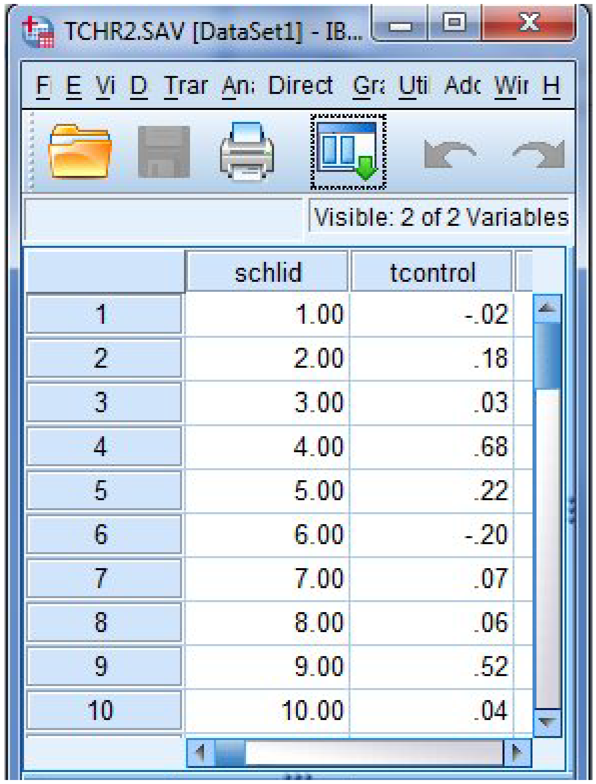
An outcome with three response categories tapping teachers' commitment to their career choice is derived from teachers' responses to the hypothetical question of whether they would become a teacher if they could go back to college and start over again. The possible responses are:

- yes, I would choose teaching again
- not sure
- no, I would not choose teaching again.

At the teacher level, it is hypothesized that teachers' perception of task variety is positively associated with greater odds of a teacher choosing the first category relative to the third category, and with greater odds of a teacher choosing the second category relative to the third category. The perception is measured by a task variety scale that assessed the extent to which teachers followed the same teaching routines each day, performed the same tasks each day, had something new happening in their job each day, and liked the variety present in their work (Rowan, Raudenbush & Cheong, 1993). The first 10 lines of the level-1 and level-2 data files are shown below.



	schlid	tcommit	taskvar
1	1.00	1.00	-.26
2	1.00	1.00	.57
3	1.00	1.00	.13
4	1.00	2.00	-.26
5	1.00	3.00	-1.10
6	1.00	1.00	.53
7	1.00	2.00	.61
8	1.00	1.00	.57
9	1.00	1.00	-.26
10	1.00	3.00	-.22



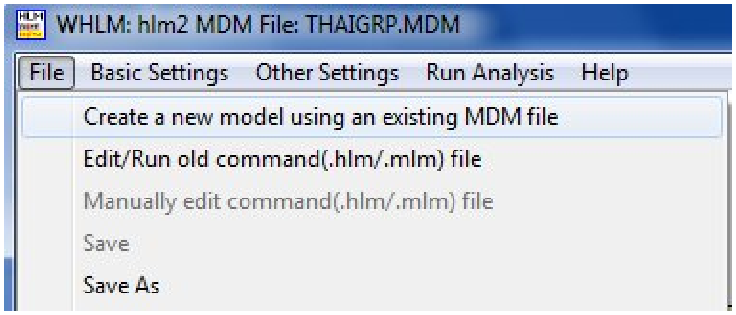
At the school level, it is postulated that the extent of teacher control has the same relationship to the two log odds as perception of task variety does. The teacher control scale is constructed by aggregating nine-item scale scores of teachers within a school. This scale indicates teacher control over school policy issues such as student behavior codes, content of in-service programs, student grouping, school curriculum, and text selection; and control over classroom issues such as teaching content and techniques, and amount of homework assigned (Rowan, Raudenbush & Kang, 1991).

As a previous analysis showed that there is little between-teacher variability in their log-odds of choosing the second category relative to the third category, the level-1 coefficient associated with it is fixed. Furthermore, the effects associated with perception of task variety are constrained to be the same across teachers for the sake of parsimony.

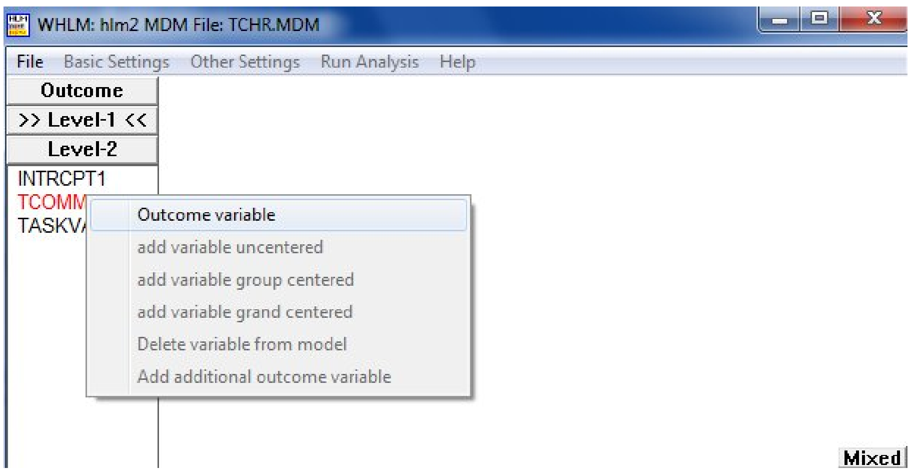
4. Creating the command file

The MDM file for a HGLM model is constructed in exactly the same way as for a linear model. The procedure is described in detail for the MDM and MDM data in other examples. Using the MDM file **TCHR.MDM**, we set up the model as shown below.

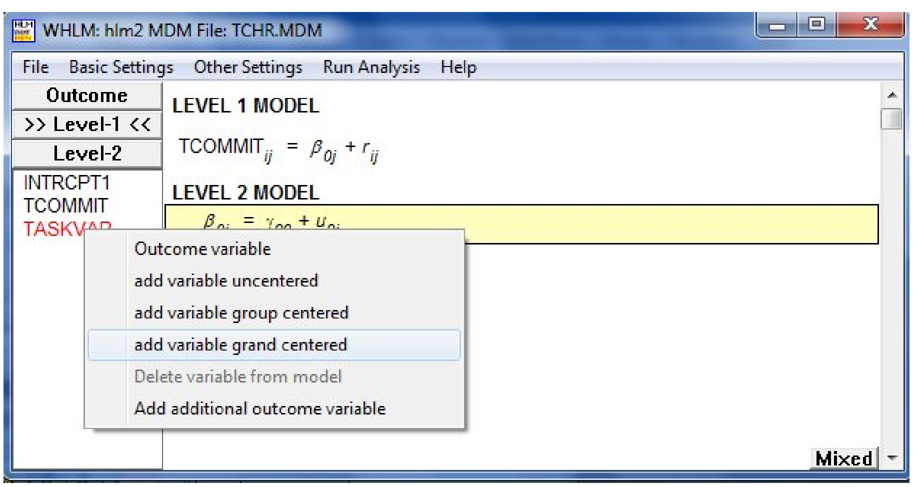
Start by selecting the **Create a new model using an existing MDM file** option from the **File** menu and open the MDM file **TCHR.MDM**.



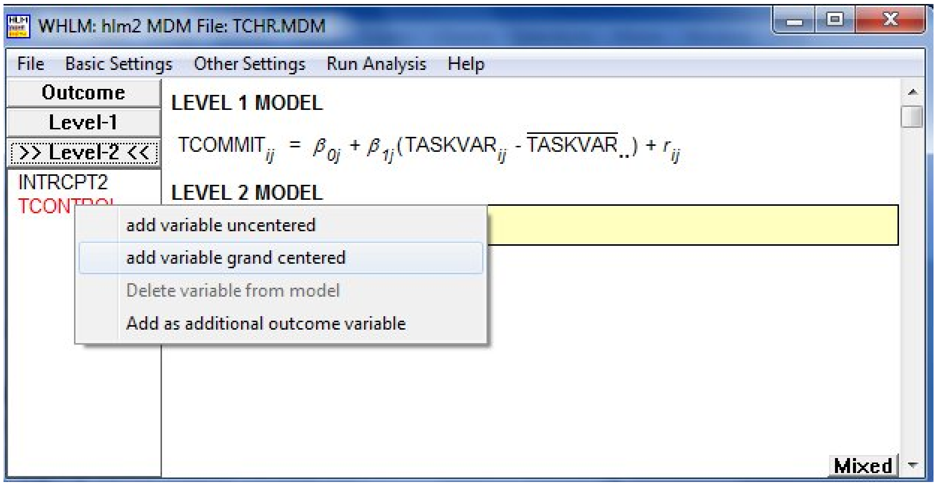
Select the outcome variable TCOMMIT by clicking on the variable name at left and selecting **Outcome variable** from the pop-up menu.



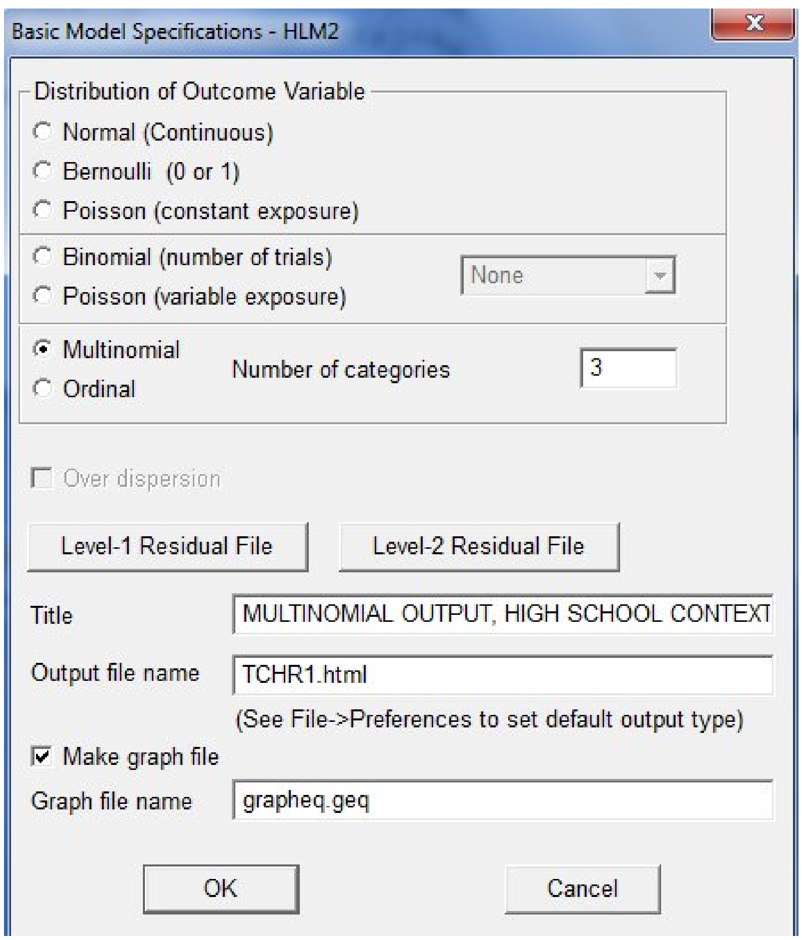
Next, add the variable TASKVAR to the model by selecting the **add variable grand centered** option from the pop-up menu.



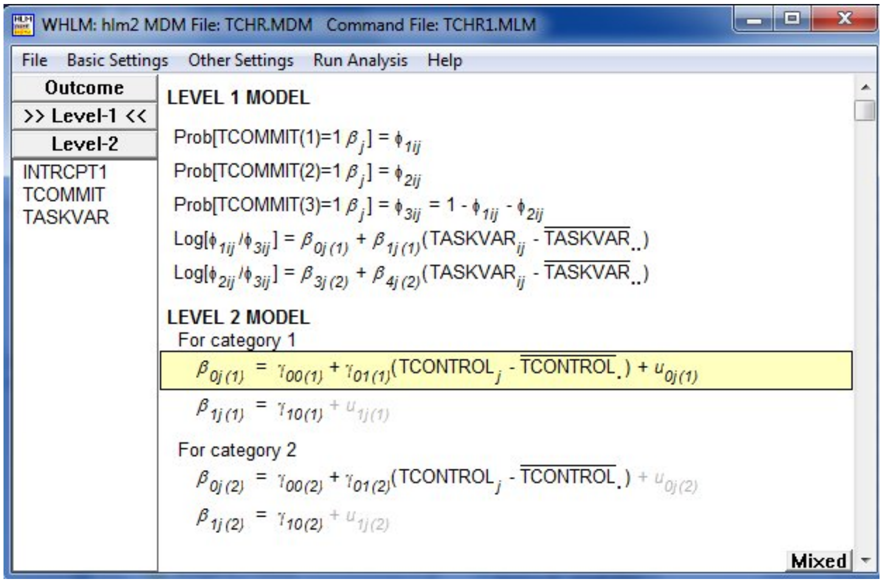
As a final step, include the level-2 predictor TCONTROL as **grand-mean centered** predictor on the level-2 intercept equation.



After specifying the basic model, click the **Outcome** button at the top of the variable list box to the left of the main HLM window to open the **Basic Model Specifications – HLM2** dialog box.



Select **Multinomial** to tell HLM that the level-1 sampling model is multinomial. Enter the value "3" in the **Number of categories** field. Click **OK** to return to the main window. The model specification is now that of a multinomial model.



Click **OK** to return to the main window and remember to save the command file prior to running the analysis.

5. Interpreting the output

The HLM output would describe the model as follows:

Summary of the model specified

Level-1 Model

Prob[TCOMMIT(1) = 1|βj] = $\phi_{1_{ij}}$

Prob[TCOMMIT(2) = 1|βj] = $\phi_{1_{ij}}$

Prob[TCOMMIT(3) = 1|βj] = $\phi_{1_{ij}}$ = 1 - $\phi_{1_{ij}}$ - $\phi_{1_{ij}}$

log[$\phi_{1_{ij}}$ / $\phi_{1_{ij}}$] = $\beta_{0j(1)}$ + $\beta_{1j(1)}$*(TASKVAR$_{ij}$)

log[$\phi_{1_{ij}}$ / $\phi_{1_{ij}}$] = $\beta_{0j(2)}$ + $\beta_{1j(2)}$*(TASKVAR$_{ij}$)

Thus, the level-1 structural models are

$$\eta_{ij(1)} = \log\left[\frac{\phi_{ij(1)}}{\phi_{ij(3)}}\right] = \beta_{0j(1)} + \beta_{1j(1)}(\text{TASKVAR})_{ij}$$

$$\eta_{ij(2)} = \log\left[\frac{\phi_{ij(2)}}{\phi_{ij(3)}}\right] = \beta_{0j(2)} + \beta_{1j(2)}(\text{TASKVAR})_{ij}$$

Level-2 Model

$\beta_{0(1)} = \gamma_{00(1)} + \gamma_{01(1)}$*(TCONTROL$_j$) + $u_{0j(1)}$
$\beta_{1(1)} = \gamma_{10(1)}$
$\beta_{0(2)} = \gamma_{00(2)} + \gamma_{01(2)}$*(TCONTROL$_j$)
$\beta_{1(2)} = \gamma_{10(2)}$

TASKVAR has been centered around the grand mean.
TCONTROL has been centered around the grand mean.

The level-2 structural models are

$$\beta_{0j(1)} = \gamma_{00(1)} + \gamma_{01(1)}(\text{TCONTROL})_{ij} + u_{0j(1)}$$
$$\beta_{1j(1)} = \gamma_{10(1)}$$
$$\beta_{0j(2)} = \gamma_{00(2)} + \gamma_{01(2)}(\text{TCONTROL})_{ij}$$
$$\beta_{1j(2)} = \gamma_{10(2)}$$

τ
INTRCPT1(1) 0.00986

Random level-1 coefficient	Reliability estimate
INTRCPT1(1), $\beta_{0(1)}$	0.083

The value of the log-likelihood function at iteration 2 = -1.246191E+003

$\gamma_{00(1)}$, the unit-specific intercept, is the expected log-odds of an affirmative response relative to a negative response for a teacher with mean perception of task variety and working in a school with average teacher control and a random effect of zero. It is adjusted for the between-school heterogeneity in the likelihood of an affirmative response relative to a negative response, which is independent of the effect of task variety and teacher control. The estimated conditional expected log-odds is 1.079269.

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For Category 1					
For INTRCPT1, $\beta_{0(1)}$					
INTRCPT2, $\gamma_{00(1)}$	1.079269	0.123439	8.743	14	<0.001
TCONTROL, $\gamma_{01(1)}$	2.090207	0.508369	4.112	14	0.001
For TASKVAR slope, $\beta_{1(1)}$					
INTRCPT2, $\gamma_{10(1)}$	0.398355	0.113650	3.505	630	<0.001
For Category 2					
For INTRCPT1, $\beta_{0(2)}$					
INTRCPT2, $\gamma_{00(2)}$	0.091930	0.141643	0.649	630	0.517
TCONTROL, $\gamma_{01(2)}$	1.057285	0.577673	1.830	630	0.068
For TASKVAR slope, $\beta_{1(2)}$					
INTRCPT2, $\gamma_{10(2)}$	0.030693	0.130029	0.236	630	0.813

The predicted probability that the same teacher responds affirmatively (Category 1) is exp{1.079269}/ (1 + exp{1.079269} + exp{0.091930}) = .584. The predicted probability of responding "not sure" (category 2) is exp{0.091930}/(1 + exp{1.079269} + exp{0.091930}) = 1 - .584 - .218 = .198.

Fixed Effect	Coefficient	Odds Ratio	Confidence Interval
For Category 1			
For INTRCPT1, $\beta_{0(1)}$			
INTRCPT2, $\gamma_{00(1)}$	1.079269	2.942528	(2.258,3.835)
TCONTROL, $\gamma_{01(1)}$	2.090207	8.086586	(2.718,24.063)
For TASKVAR slope, $\beta_{1(1)}$			
INTRCPT2, $\gamma_{10(1)}$	0.398355	1.489373	(1.191,1.862)
For Category 2			
For INTRCPT1, $\beta_{0(2)}$			
INTRCPT2, $\gamma_{00(2)}$	0.091930	1.096288	(0.830,1.448)
TCONTROL, $\gamma_{01(2)}$	1.057285	2.878545	(0.926,8.952)
For TASKVAR slope, $\beta_{1(2)}$			
INTRCPT2, $\gamma_{10(2)}$	0.030693	1.031169	(0.799,1.331)

The sets of γ_{01} and γ_{10} give the estimates of the change in the respective log-odds given one-unit change in the predictors, holding all other variables constant. For instance, all else being equal, a standard deviation increase in TCONTROL (.32) will nearly double the odds of an affirmative response to a negative response (exp{2.090207 * .32} = 1.952). Note that the partial effect associated with perception of task variety is statistically significant for the logit of affirmative versus negative responses but not for the logit of undecided versus negative responses.

Below is a table for the results for the fixed effects with robust standard errors.

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For Category 1					
For INTRCPT1, $\beta_{0(1)}$					
INTRCPT2, $\gamma_{00(1)}$	1.079269	0.128263	8.415	14	<0.001
TCONTROL, $\gamma_{01(1)}$	2.090207	0.409607	5.103	14	<0.001
For TASKVAR slope, $\beta_{1(1)}$					
INTRCPT2, $\gamma_{10(1)}$	0.398355	0.127511	3.124	630	0.002
For Category 2					
For INTRCPT1, $\beta_{0(2)}$					
INTRCPT2, $\gamma_{00(2)}$	0.091930	0.139637	0.658	630	0.511
TCONTROL, $\gamma_{01(2)}$	1.057285	0.529606	1.996	630	0.046
For TASKVAR slope, $\beta_{1(2)}$					
INTRCPT2, $\gamma_{10(2)}$	0.030693	0.126446	0.243	630	0.808

Fixed Effect	Coefficient	Odds Ratio	Confidence Interval
For Category 1			
For INTRCPT1, $\beta_{0(1)}$			
INTRCPT2, $\gamma_{00(1)}$	1.079269	2.942528	(2.235,3.874)
TCONTROL, $\gamma_{01(1)}$	2.090207	8.086586	(3.359,19.469)
For TASKVAR slope, $\beta_{1(1)}$			
INTRCPT2, $\gamma_{10(1)}$	0.398355	1.489373	(1.159,1.913)
For Category 2			
For INTRCPT1, $\beta_{0(2)}$			
INTRCPT2, $\gamma_{00(2)}$	0.091930	1.096288	(0.833,1.442)
TCONTROL, $\gamma_{01(2)}$	1.057285	2.878545	(1.017,8.145)
For TASKVAR slope, $\beta_{1(2)}$			
INTRCPT2, $\gamma_{10(2)}$	0.030693	1.031169	(0.804,1.322)

The robust standard errors are appropriate for datasets having a moderate to large number of level 2 units. These data do not meet this criterion.

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	*p*-value
INTRCPT1(1), $u_{0(1)}$	0.09931	0.00986	14	16.16473	0.303

Note that the residual variance of $\beta_{00(1)}$ is not statistically different from zero. The model may be re-run with the coefficient set to be non-random. Below is the final results of the fixed effects for a model with the residual variance of $\beta_{00(1)}$ set to zero. Very little difference in the estimated fixed effects is observed.

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For Category 1					
For INTRCPT1, $\beta_{0(1)}$					
INTRCPT2, $\gamma_{00(1)}$	1.080218	0.127091	8.500	644	<0.001
TCONTROL, $\gamma_{01(1)}$	2.087595	0.410214	5.089	644	<0.001
For TASKVAR slope, $\beta_{1(1)}$					
INTRCPT2, $\gamma_{10(1)}$	0.398028	0.127147	3.130	644	0.002
For Category 2					
For INTRCPT1, $\beta_{0(2)}$					
INTRCPT2, $\gamma_{00(2)}$	0.092027	0.139960	0.658	644	0.511
TCONTROL, $\gamma_{01(2)}$	1.057939	0.530818	1.993	644	0.047
For TASKVAR slope, $\beta_{1(2)}$					
INTRCPT2, $\gamma_{10(2)}$	0.030788	0.126511	0.243	644	0.808