A conditional 4-level model for the literacy data

1. Description of the data

To illustrate the operation of the HLM4 program, we reanalyze a subset of data from Hough, Bryk, Pinnell, Kerbow, Fountas, and Scharer (2008). Hough *et al*. used a four-level model to examine the association between school-based coaching and the development of teachers' expertise in literary instruction. The level-1 model in their study was a measurement error model associated with 1317 repeated observations on a measure of classroom instruction, which they called teaching *expertise*. (This measurement model relates the observed data to a "true" or latent score plus some error of measurement. See below.) The level-2 model represented a growth model for each teacher's "true scores" on teaching expertise, and the level-3 and level-4 models investigated the associations of the growth trajectory parameters with teacher- and school-level correlates with data from 219 teachers from 17 schools, respectively.

The example illustrates the use of a level-1 in HLM as a measurement model. In brief,

$$Y_{mtij} = \psi_{0tij} + \varepsilon_{mtij}, \quad \varepsilon_{mtij} \sim N\left(0, \sigma^2_{mtij}\right)$$

where

Y_{mtij} is the observed measure on occasion t for teacher i in school j,
ψ_{tij} is the true or latent value for teacher expertise, and
ε_{mtij} is the error of measurement associated with the observed rating m on occasion t for teacher i in school j.

(Note, in this data set there is only one observed rating per occasion. As a result the number of level-1 and level-2 units are identical.)

In most applications, ε_{mtij} is unknown and assumed normally distributed with constant variance. In contrast in this application, the Rasch measurement model for the observed outcomes, Y_{mtij}, also provides a standard error estimate for each observed measure, s_{mtij}. We explicitly represent this by multiplying both sides of the level-1 model by the inverse of the standard error, $a_{mtij} = s_{mtij}^{-1}$, yielding

$$Y_{mtij}^{*} = a_{mtij}\psi_{0tij} + e_{mtij}^{*}, \quad e_{mtij}^{*} \sim N(0,1).$$

The variance at level-1 is now assumed known and fixed at a value of 1.0.

Level-1 file. The level-1 file, MEASURE.SAV, has 1317 observations collected on 219 teachers on up to 9 different occasions. Data for the first three teachers are shown below. Each of these teachers was observed on three occasions. (Some teachers in the study were observed on as many as nine occasions over three years.)

The first column contains the level-4 (*i.e.*, school) ID, next is the level-3 (*i.e.*, teacher) ID, and this is followed by the level-2 (*i.e.*, occasion) ID. We see that the first record comes from school 1100, teacher 1100002, and occasion 11000026. Following the teacher ID fields are that teacher's values on two variables:

- expertis
  A composite Rasch measure of teachers' classroom literacy practice rated on some particular occasion (weighted by the inverse of its standard error of measurement.)
- invstder
  The inverse of the standard error of measurement associated with that individual rating (the standard errors are generated as part of the Rasch rating scale model.)

	schid	tchrid	occasid	expertis	invstderr
1	1100	1100002	11000026	-2.862	4.472
2	1100	1100002	11000027	-1.850	5.000
3	1100	1100002	11000028	-2.182	4.642
4	1100	1100011	11000116	5.750	5.000
5	1100	1100011	11000117	4.105	5.263
6	1100	1100011	11000118	7.150	5.000
7	1100	1100012	11000123	2.227	4.545
8	1100	1100012	11000124	.591	4.545
9	1100	1100012	11000125	2.913	4.348
10	1100	1100013	11000136	.400	5.000

Level-2 file. The level-2 units consisted of the 1317 occasions when measurements on classroom literary practice were made. The data are stored in the file OCCAS.SAV. The level-2 data for the first nine records are listed below. It has the same three ID's as the level-1 file. The two occasion-level variables are included in the file:

- occasion
  This variable identifies the specific data collection time point, counted up from the first study occasion in the fall of year1 (a value of 0) through the end of the study in the spring of year 3 (a value of 8).
- artifact
  A dummy variable introduced into the analysis to adjust for a measurement artifact that occurred with the first-year spring scores (at occasion = 2).

	schid	tchrid	occasid	occasion	artifact
1	1100	1100002	11000026	3.000	.000
2	1100	1100002	11000027	4.000	.000
3	1100	1100002	11000028	5.000	.000
4	1100	1100011	11000116	3.000	.000
5	1100	1100011	11000117	4.000	.000
6	1100	1100011	11000118	5.000	.000
7	1100	1100012	11000123	3.000	.000
8	1100	1100012	11000124	4.000	.000
9	1100	1100012	11000125	5.000	.000
10	1100	1100013	11000136	3.000	.000

The first teacher in this data file, Teacher 1100002 in school 1100, was observed on three occasions during the second year of the study (*i.e.* occasions 3 through 5). The same was true for the next two teachers. In general, the data collection patterns vary among teachers in this study depending upon their employment history at the school and when they first became eligible for classroom coaching.

Level-3 file. The level-3 units are the 219 teachers. The data are stored in the TCHR.SAV file. The first field is the school ID and the second is the teacher ID. Note that each of the first ten teachers is in school 1100. There are six variables in this file:

- coach
  The average number of one-on-one coaching sessions per month that each teacher received over the course of the study
- newwtch
  A dummy variable indicating that the teacher had three or fewer years of classroom teaching experience at onset of study participation
- pdpart
  A composite measure of teachers' exposure to literacy professional development prior to the onset of the study
- scmt
  A scale score on the teacher's commitment to the school measured at study onset

- y2ent
  A dummy variable indicating the teacher began work at the school during the second year of the study
- y3ent
  A dummy variable indicating the teacher began work at the school during the third year of the study

	schlid	tchrid	coach	newtchr	pdpart	scmt	y2ent	y3ent
1	1100.00	1100002	.571	.000	.842	-.292	1.000	.000
2	1100.00	1100011	.571	1.000	-.361	-.813	1.000	.000
3	1100.00	1100012	.755	.000	1.653	.267	.000	.000
4	1100.00	1100013	.571	1.000	1.115	.774	1.000	.000
5	1100.00	1100020	.496	.000	.856	1.150	.000	.000
6	1100.00	1100023	.878	.000	-.248	-1.379	.000	.000
7	1100.00	1100025	.731	.000	-.631	1.150	.000	.000
8	1100.00	1100026	.831	.000	-.248	-.813	.000	.000
9	1100.00	1100027	.736	.000	.307	.164	.000	.000
10	1100.00	1100029	.695	1.000	-.292	-.340	.000	.000

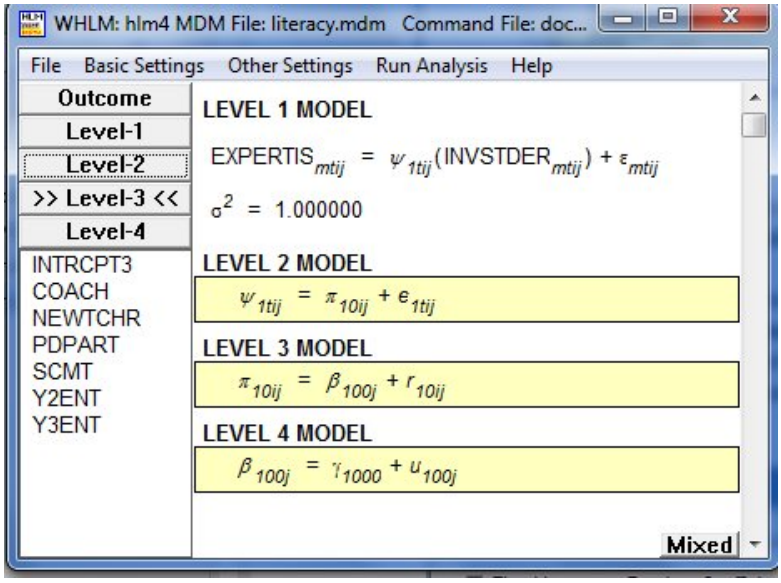
Level-4 file. The school level data from 17 schools appear in SCH.SAV. The first field is the school ID. This is followed by:

- chgcoach
  A dummy variable indicating that a coaching change occurred during the course of the study. This happened with only one school in the sample.

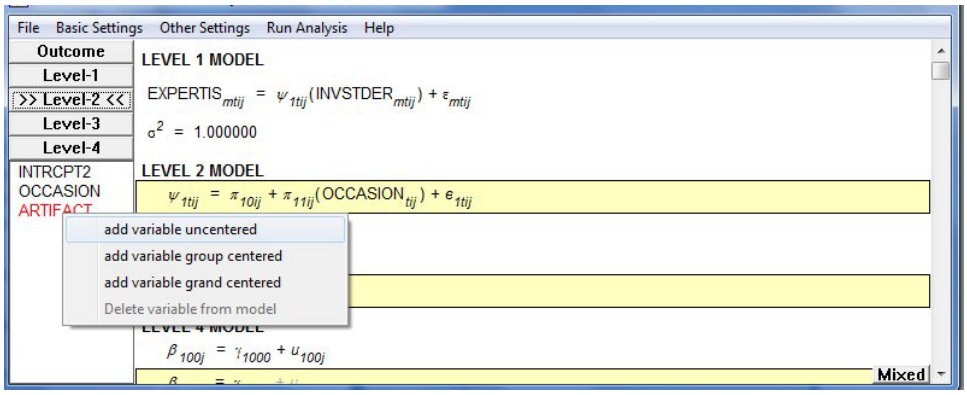
	schid	chgcoach
1	1100.00	0.000
2	1200.00	0.000
3	1300.00	0.000
4	1400.00	0.000
5	1600.00	0.000
6	1700.00	0.000
7	1800.00	0.000
8	1900.00	0.000
9	2000.00	0.000
10	2100.00	0.000

2. Creating the command file

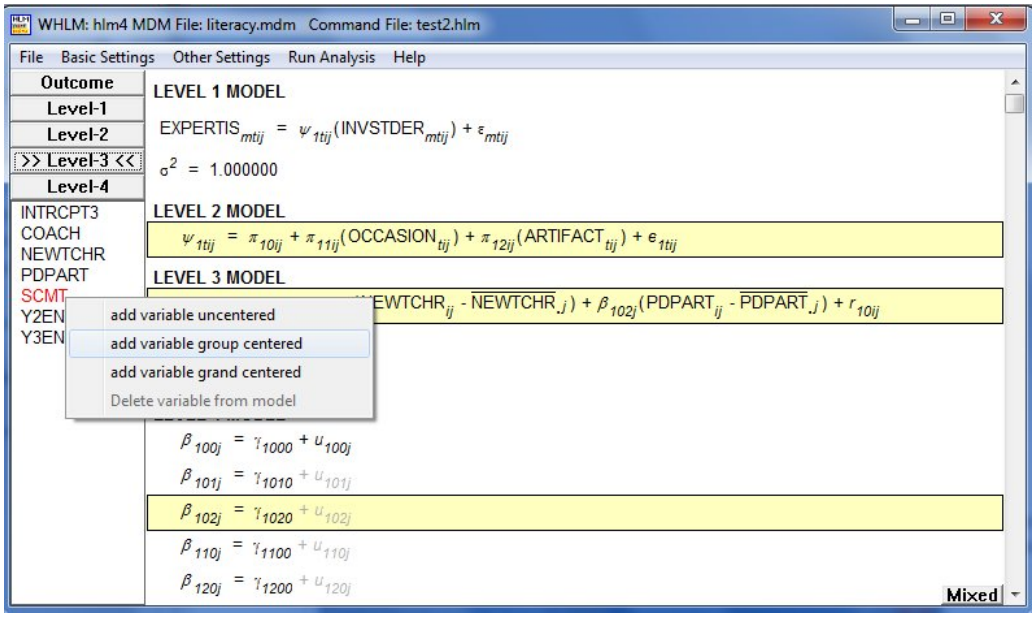
We use the command file for an unconditional model created in the first example as a starting point.



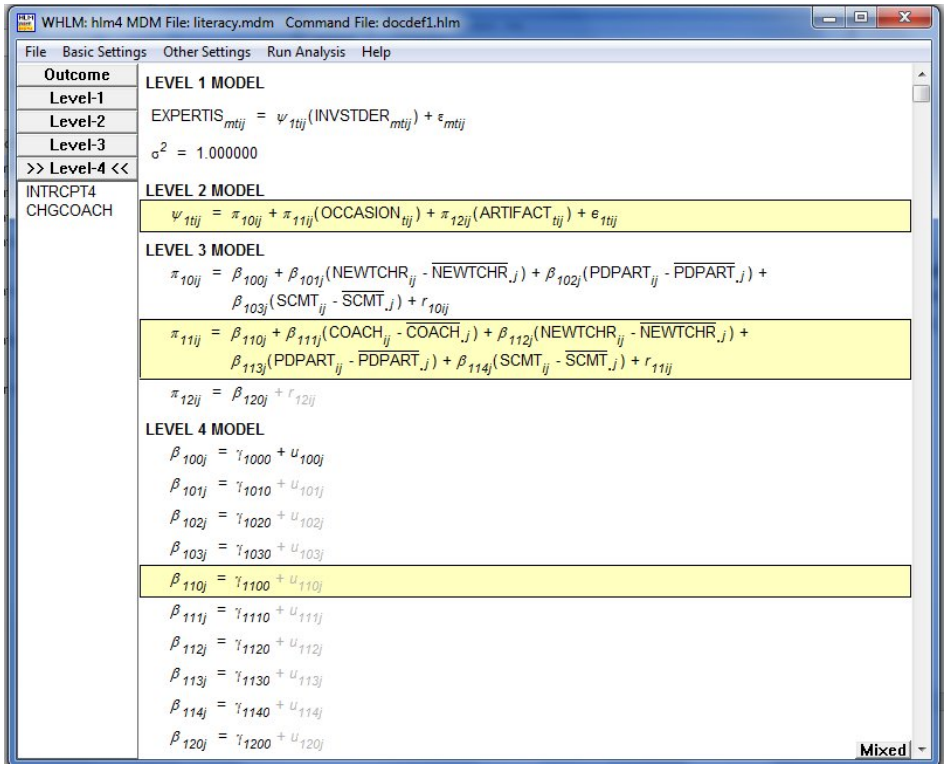
At level-2, we select the variables OCCASION and ARTIFACT as **uncentered** predictors of the intercept equation at level-2 after clicking on **>>Level-2<<** to display available predictors at this level.



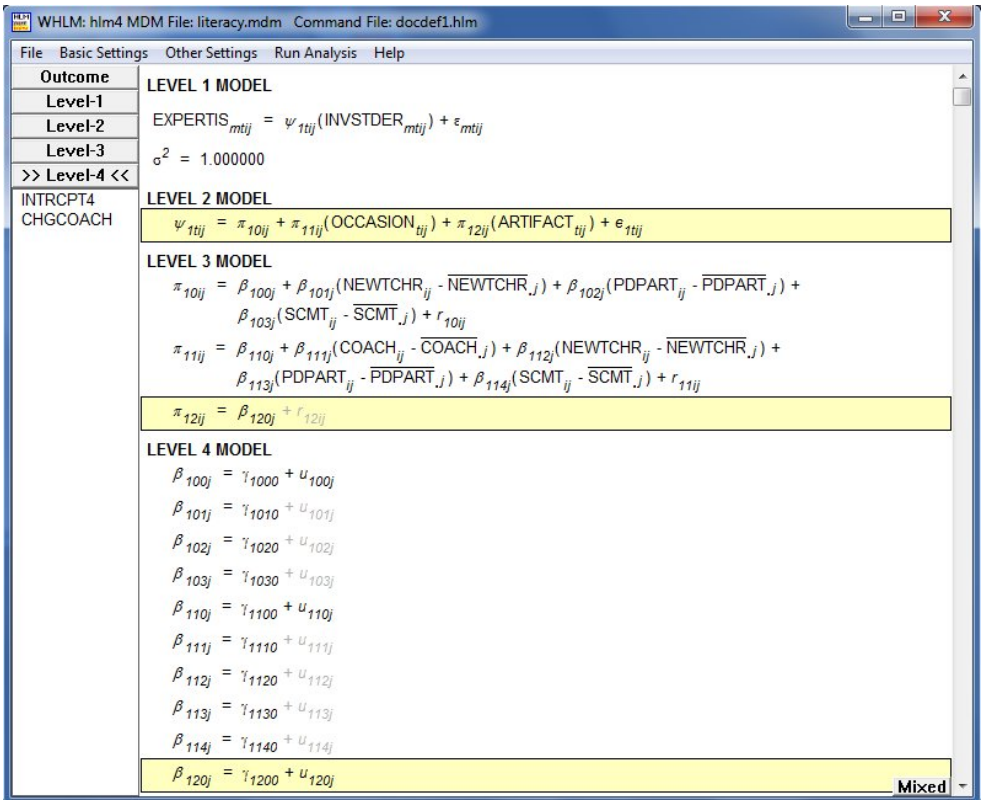
At level-3, we select the variables NEWTCH, PDPART and SCMT as **group-mean centered** variables as predictors on the equation for π_{10ij} as shown below. Next, add the same variables in the same way to the level-3 equation for π_{11ij} . Also add the variable COACH, group-mean centered, to the equation for π_{11ij} .



The completed model is shown below.



The final step is to activate the random effects associated with the equations for β_{110j} and β_{120j}. This is done by clicking on the equation and then clicking on the random terms u_{110j} and u_{120j} respectively. The final model is shown below. Remember to save the model prior to running the analysis.



3. Interpreting the output

Output after convergence is shown below.

Iterations stopped due to small change in likelihood function

σ^2_e
INVSTDER,ψ_1 0.31788

σ^2_e (as correlations)
INVSTDER,ψ_1 1.000

Random level-1 coefficient	Reliability estimate
INVSTDER	0.821

τ_π

INVSTDER INTRCPT2,π_{10}	INVSTDER OCCASION,π_{11}
0.93753	0.01861
0.01861	0.00113

τ_π (as correlations)

INVSTDER/INTRCPT2,π_{10}	1.000	0.571
INVSTDER/OCCASION,π_{11}	0.571	1.000

Random level-2 coefficient	Reliability estimate
INVSTDER/INTRCPT2	0.740
INVSTDER/OCCASION	0.077

Note: The reliability estimates reported above are based on only 214 of 219 units that had sufficient data for computation. Fixed effects and variance components are based on all the data.

Note, among teachers within schools, there is a positive correlation of 0.571 between their initial status and expertise development.

τ_β

INVSTDER INTRCPT2 INTRCPT3,β_{100}	INVSTDER OCCASION INTRCPT3,β_{110}	INVSTDER ARTIFACT INTRCPT3,β_{120}
0.28840	-0.03214	0.16341
-0.03214	0.03798	-0.05972
0.16341	-0.05972	0.22678

τ_β (as correlations)

INVSTDER/INTRCPT2/INTRCPT3,β_{100}	1.000	-0.307	0.639
INVSTDER/OCCASION/INTRCPT3,β_{110}	-0.307	1.000	-0.643
INVSTDER/ARTIFACT/INTRCPT3,β_{120}	0.639	-0.643	1.000

Random level-3 coefficient	Reliability estimate
INVSTDER/INTRCPT2/INTRCPT3	0.727
INVSTDER/OCCASION/INTRCPT3	0.965
INVSTDER/ARTIFACT/INTRCPT3	0.747

In contrast, at the school level a negative correlation, -.307, exists between school mean initial status on teachers' expertise and school-level growth rates.

Final estimation of fixed effects

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INVSTDER, ψ_1					
For INTRCPT2, π_{10}					
For INTRCPT3, β_{100}					
INTRCPT4, γ_{1000}	-0.042320	0.152308	-0.278	15	0.785
For NEWTCHR, β_{101}					
INTRCPT4, γ_{1010}	-0.520219	0.226444	-2.297	144	0.022
For PDPART, β_{102}					
INTRCPT4, γ_{1020}	0.167179	0.092189	1.813	144	0.069

For SCMT, β_{103}					
INTRCPT4, γ_{1030}	0.137797	0.085591	1.610	144	0.107
For OCCASION, π_{11}					
For INTRCPT3, β_{110}					
INTRCPT4, γ_{1100}	0.208296	0.048144	4.327	15	<0.001
For COACH, β_{111}					
INTRCPT4, γ_{1110}	0.261937	0.078204	3.349	144	0.001
For NEWTCHR, β_{112}					
INTRCPT4, γ_{1120}	0.009542	0.027833	0.343	144	0.731
For PDPART, β_{113}					
INTRCPT4, γ_{1130}	0.004064	0.009894	0.411	144	0.681
For SCMT, β_{114}					
INTRCPT4, γ_{1140}	0.014517	0.010328	1.406	144	0.160
For ARTIFACT, π_{12}					
For INTRCPT3, β_{120}					
INTRCPT4, γ_{1200}	0.569328	0.133191	4.275	16	<0.001

New teachers scored considerably lower on initial status than more experienced teachers ($\gamma_{1010} = -0.520$, $t = -2.297$, p-value = 0.022.) As hypothesized by the study, both prior professional development experience PDPART and commitment to school improvement SCMT were positively related to differences among schools in initial expertise ratings (*p-values* of 0.069 and 0.107 respectively.)

In terms of teachers' growth in expertise over the course of the study, OCCASION, the study hypothesized that this would be related to differential exposure to coaching, COACH. A highly significant relationship was found, ($\gamma_{1110} = 0.262$, with associated t-value of 3.349 and a p-value = 0.001). A significant measurement artifact also occurred, see results for γ_{1200}.

Final estimation of level-1 and level-2 variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INVSTDER, e_1	0.56381	0.31788	1078	4729.76970	<0.001

Note: The chi-square statistics reported above are based on only 1312 of 1317 units that had sufficient data for computation. Fixed effects and variance components are based on all the data.

Final estimation of level-3 variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INVSTDER/INTRCPT2,r_{10}	0.96826	0.93753	193	734.15590	<0.001
INVSTDER/OCCASION,r_{11}	0.03365	0.00113	192	267.53588	<0.001

Note: The chi-square statistics reported above are based on only 214 of 219 units that had sufficient data for computation. Fixed effects and variance components are based on all the data.

The variation on among teachers within schools on expertise ratings at the study onset, $\text{var}(r_{10})$, is 0.937 and the variation within schools on teachers' rate of growth in expertise, $\text{var}(r_{11})$, is 0.001.

Both variance components are statistically significant.

Final estimation of level-4 variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INVSTDER/ INTRCPT2/INTRCPT3,u_{100}	0.53703	0.28840	16	65.90635	<0.001
INVSTDER/ OCCASION/INTRCPT3,u_{110}	0.19489	0.03798	16	599.59968	<0.001
INVSTDER/ ARTIFACT/INTRCPT3,u_{120}	0.47622	0.22678	16	71.51494	<0.001

Statistics for the current model
Deviance = 6895.349602
Number of estimated parameters = 20

We see evidence of considerable variability among schools in teachers' initial expertise ratings, u_{110}, $\left(\chi^2 = 65.906,\ p-value < 0.001\right)$. Significant variation was also found in school growth rates, u_{110}, and in the magnitude of the measurement artifact at each school, u_{120}.