



HMLM model with log-linear model for level-1 variance for the NYS data

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1. Introduction to HMLM models

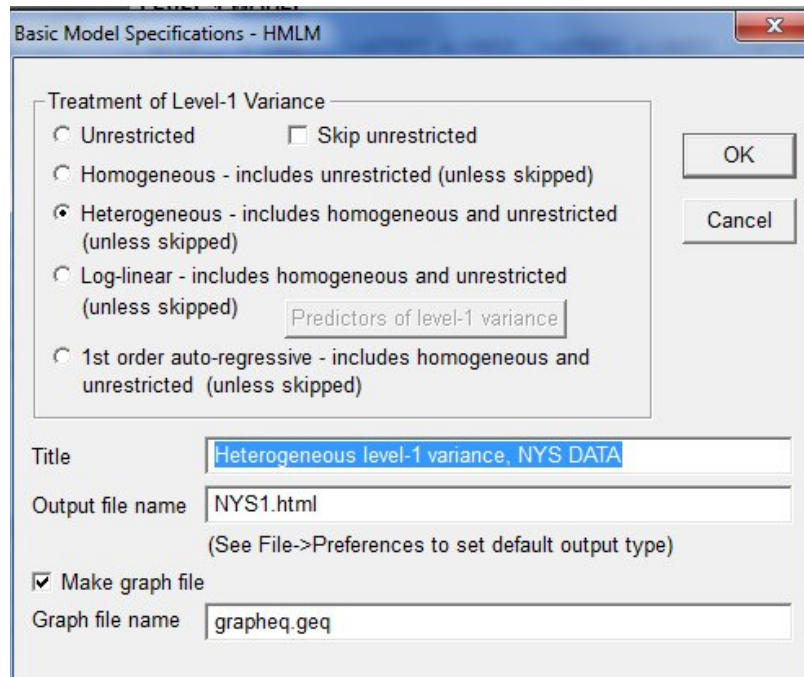
One of the most frequent applications of hierarchical models involves repeated observations (level 1) nested within persons (level 2). These are described in Chapter 6 of *Hierarchical Linear Models*. In these models, the outcome Y_{ij} for occasion i within person j is conceived as a univariate outcome, observed under different conditions or at different times. An advantage of viewing the repeated observations as nested within the person is that it allows each person to have a different repeated measures design. For example, in a longitudinal study, the number of time points may vary across persons, and the spacing between time points may be different for different persons. Such unbalanced designs would pose problems for standard methods of analysis such as the analysis of variance.

Suppose, however, that the aim of the study is to observe every participant according to a fixed design with, say, T observations per person. The design might involve T observation times or T different outcome variables or even T different experimental conditions. Given the fixed design, the analysis can be reconceived as a multivariate repeated measures analysis. The multivariate model is flexible in allowing a wide variety of assumptions about the variation and covariation of the T repeated measures (Bock, 1985). In the standard application of multivariate repeated measures, there can be no missing outcomes: every participant must have a full complement of T repeated observations.

Advances in statistical computation, beginning with the EM algorithm (Dempster, Laird, & Rubin, 1977; see also Jennrich & Schluchter, 1986), allow the estimation of multivariate normal models from incomplete data. In this case, the aim of the study was to collect T observations per person, but only n_j observations were collected ($n_j \leq T$). These n_j observations are indeed collected

according to a fixed design, but $T - n_j$ data points are missing at random.

Model specification for HMLM analysis are specified via the **Basic Settings** dialog box as shown below.



2. Description of the model

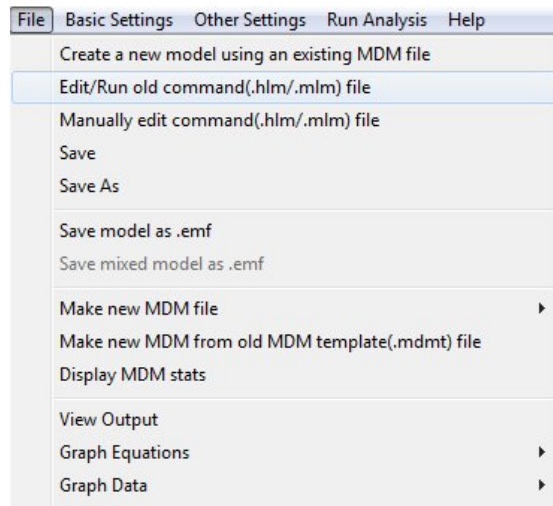
HMLM allows estimation of multivariate normal models from incomplete data. Within the framework of HMLM, it is possible to estimate models having

1. An unrestricted covariance structure, that is a full $T \times T$ covariance matrix.
2. A model with homogenous level-1 variance and random intercepts and/or slopes at level-2.
3. A model with heterogeneous variances at level 1 (a different variance for each occasion) and random intercepts and/or slopes at level 2.
4. A model that includes a log-linear structure for the level-1 variance and random intercepts and/or slopes at level 2.
5. A model with first-order auto-regressive level-1 random errors and random intercepts and/or slopes at level 2.

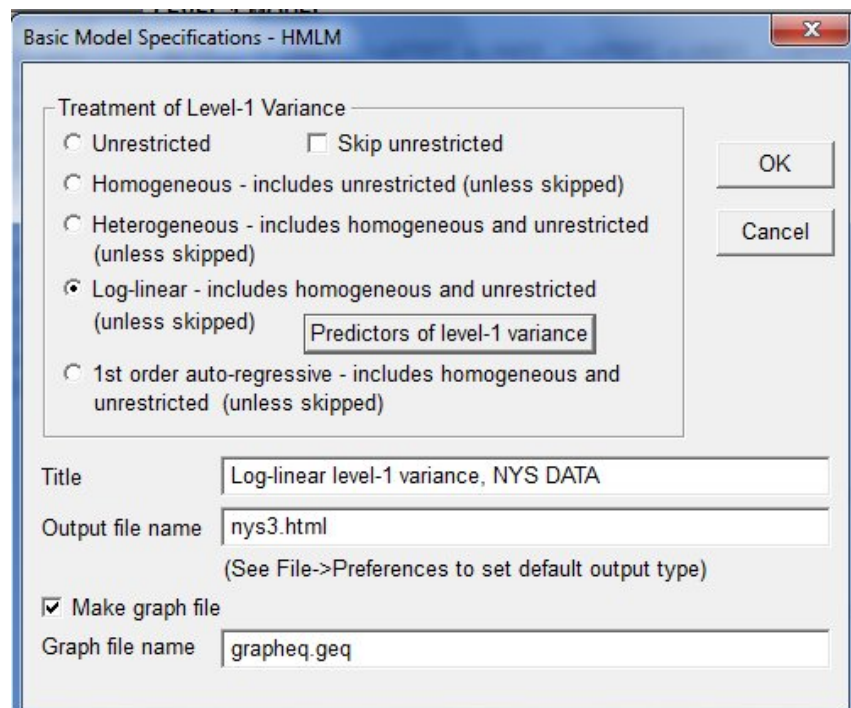
In this illustration, we fit a model with a log-linear structure for the level-1 variance to the MDM file created in the first of this set of example, *i.e.* **NYS.MDM**. This is the second in a set of three HMLM models fitted to the NYS data.

6. Creating the command file

Start by selecting the **Edit/Run old command (.hlm/.mlm) file** option from the **File** menu and open the MLM file **NYS1.MLM**.

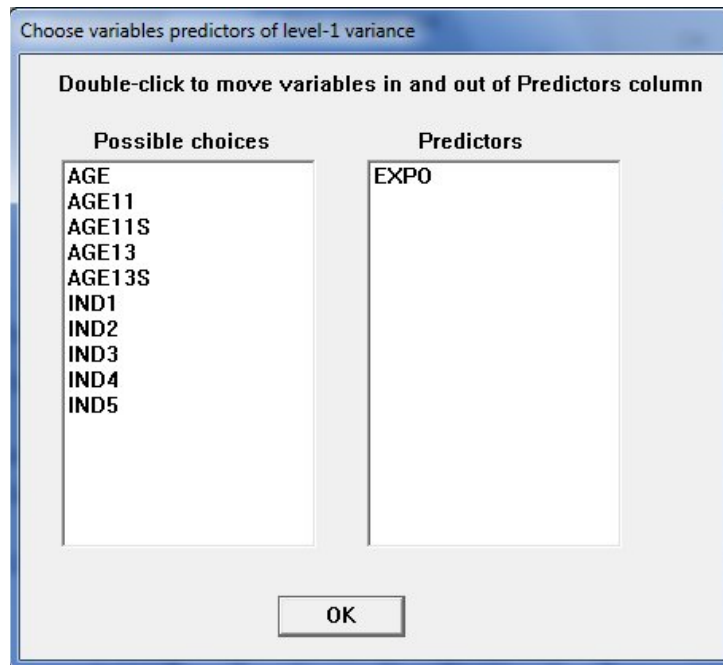


Next, click on the **Basic Settings** option on the main menu bar to open the **Basic Model Specifications - HMLM** dialog box. Check the radio button for the **Log-linear – includes homogeneous and unrestricted (unless skipped)** option.



Select the variable EXPO on the **Choose variables predictors of level-1 variance** dialog box by

double-clicking on the variable name to move it from the **Possible choices** to the **Predictors** column. Click **OK** to return to the **Basic Model Specifications - HMLM** dialog box, and again to return to the main modeling window.



The log-linear model, with EXPO as predictor of level-1 variance, is displayed in the lower part of the modeling window.

LOG-LINEAR MODEL

LEVEL 1 MODEL

$$ATTIT_{mi} = (IND1_{mi}) * ATTIT_{1i}^* + (IND2_{mi}) * ATTIT_{2i}^* + (IND3_{mi}) * ATTIT_{3i}^* + (IND4_{mi}) * ATTIT_{4i}^* + (IND5_{mi}) * ATTIT_{5i}^*$$

$$ATTIT_{ti}^* = \pi_{0i} + \pi_{1i}(AGE13_{ti}) + \pi_{2i}(AGE13S_{ti}) + \epsilon_{ti}$$

$$Var(r_{ij}) = \sigma_{ij}^2 \text{ and } \log(\sigma_{ij}^2) = \alpha_0 + \alpha_1(EXPO_{ij})$$

LEVEL 2 MODEL

$$\pi_{0i} = \beta_{00} + r_{0i}$$

$$\pi_{1i} = \beta_{10} + r_{1i}$$

$$\pi_{2i} = \beta_{20} + r_{2i}$$

Combined Model

$$ATTIT_{ti}^* = \beta_{00} + \beta_{10} * AGE13_{ti} + \beta_{20} * AGE13S_{ti} + \epsilon_{ti}$$

$$\epsilon_{ti} = r_{0i} + r_{1i} * AGE13_{ti} + r_{2i} * AGE13S_{ti} + e_{ti}$$

$$Var(\epsilon_{ij}) = Var(Ar_j + e_i) = \Delta = A \Sigma A' + \text{diag}(\sigma_1^2, \dots, \sigma_5^2)$$

$$\text{and } \log(\sigma_t^2) = \alpha_0 + \alpha_1(EXPO_t)$$

Remember to save the command file prior to running the analysis.

7. Interpreting the output

Summary of the model specified

Level-1 Model

$$ATTIT_{mi} = (IND1_{mi}) * ATTIT_{1i}^* + (IND2_{mi}) * ATTIT_{2i}^* + (IND3_{mi}) * ATTIT_{3i}^* + (IND4_{mi}) * ATTIT_{4i}^* + (IND5_{mi}) * ATTIT_{5i}^*$$

$$ATTIT_{ti}^* = \pi_{0i} + \pi_{1i} * (AGE13_{ti}) + \pi_{2i} * (AGE13S_{ti}) + \varepsilon_{ti}$$

Level-2 Model

$$\pi_{0i} = \beta_{00} + r_{0i}$$

$$\pi_{1i} = \beta_{10} + r_{1i}$$

$$\pi_{2i} = \beta_{20} + r_{2i}$$

$$\text{Var}(\varepsilon_i) = \text{Var}(\mathbf{A}r_i + e_i) = \mathbf{\Delta} = \mathbf{A}\mathbf{\tau}\mathbf{A}' + \text{diag}(\sigma^2_1, \dots, \sigma^2_5)$$

The above equation, written with subscripts and Greek letters, is

$$\text{Var}(Y^*) = \mathbf{A}\mathbf{T}\mathbf{A}' + \Sigma$$

where $\Sigma = \text{diag}(\sigma_i^2)$, and

$$\log(\sigma_i^2) = \alpha_0 + \alpha_1 (EXPO)_i.$$

A

IND1	1.00000	-2.00000	4.00000
IND2	1.00000	-1.00000	1.00000
IND3	1.00000	0.00000	0.00000
IND4	1.00000	1.00000	1.00000
IND5	1.00000	2.00000	4.00000

Iterations stopped due to small change in likelihood function

Final results – Iteration 7

	Parameter	Standard Error
α_0	-3.72883	0.069238
α_1	-1.43639	1.053241

	σ^2
IND1	0.02690
IND2	0.02677
IND3	0.02419
IND4	0.02188
IND5	0.02136

From the results above, we see that the estimated relationship between σ_t^2 and the variable EXPO can be described as

$$\log(\hat{\sigma}_t^2) = -3.72883 - 1.43639 (EXPO)_t.$$

Inspection of the data shows that the values of EXPO ranges between -0.37 and 1.04. This implies that $\hat{\sigma}_t^2$ ranges between 0.0401 and 0.0054.

τ

INTRCPT1, r_0	0.04255	0.00831	-0.00257
AGE13, r_1	0.00831	0.00277	-0.00005
AGE13S, r_2	-0.00257	-0.00005	0.00051

Standard errors of τ

INTRCPT1, r_0	0.00517	0.00128	0.00089
AGE13, r_1	0.00128	0.00054	0.00025
AGE13S, r_2	0.00089	0.00025	0.00025

τ (as correlations)

INTRCPT1, r_0	1.000	0.766	-0.549
AGE13, r_1	0.766	1.000	-0.042
AGE13S, r_2	-0.549	-0.042	1.000

Δ

IND1	0.03576	0.01267	0.01566	0.01782	0.01917
IND2	0.01267	0.05095	0.03168	0.03516	0.03464
IND3	0.01566	0.03168	0.06674	0.04829	0.04889
IND4	0.01782	0.03516	0.04829	0.07909	0.06192
IND5	0.01917	0.03464	0.04889	0.06192	0.09510

The 5×5 matrix above contains the variance and covariance estimates implied by the “log-linear” model for the level-1 variance.

Δ (as correlations)

IND1	1.000	0.297	0.320	0.335	0.329
IND2	0.297	1.000	0.543	0.554	0.498
IND3	0.320	0.543	1.000	0.665	0.614
IND4	0.335	0.554	0.665	1.000	0.714
IND5	0.329	0.498	0.614	0.714	1.000

The value of the log-likelihood function at iteration 7 = 1.749582E+002

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, π_0					
INTRCPT2, β_{00}	0.328946	0.015379	21.390	238	<0.001
For AGE13 slope, π_1					
INTRCPT2, β_{10}	0.064661	0.004923	13.135	238	<0.001
For AGE13S slope, π_2					
INTRCPT2, β_{20}	-0.000535	0.003222	-0.166	238	0.869

Statistics for the current model

Deviance = -349.916489

Number of estimated parameters = 11

There are 3 fixed effects ($f = 3$), the dimension of τ is 3 ($r = 3$), and there is 1 intercept and 1 explanatory ($H = 1$) variable. Thus, there are a total of $f + r(r+1)/2 + 1 + H = 3 + 3(3+1)/2 + 1 + 1 = 11$ parameters.