



HMLM model with first-order autoregressive model for level-1 variance for the NYS data

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1. Introduction to HMLM models

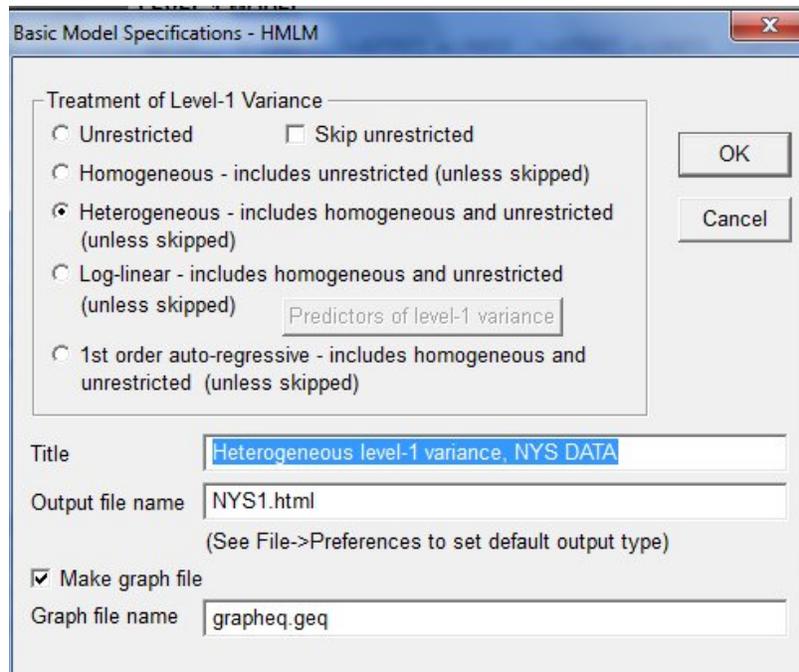
One of the most frequent applications of hierarchical models involves repeated observations (level 1) nested within persons (level 2). These are described in Chapter 6 of *Hierarchical Linear Models*. In these models, the outcome Y_{ij} for occasion i within person j is conceived as a univariate outcome, observed under different conditions or at different times. An advantage of viewing the repeated observations as nested within the person is that it allows each person to have a different repeated measures design. For example, in a longitudinal study, the number of time points may vary across persons, and the spacing between time points may be different for different persons. Such unbalanced designs would pose problems for standard methods of analysis such as the analysis of variance.

Suppose, however, that the aim of the study is to observe every participant according to a fixed design with, say, T observations per person. The design might involve T observation times or T different outcome variables or even T different experimental conditions. Given the fixed design, the analysis can be reconceived as a multivariate repeated measures analysis. The multivariate model is flexible in allowing a wide variety of assumptions about the variation and covariation of the T repeated measures (Bock, 1985). In the standard application of multivariate repeated measures, there can be no missing outcomes: every participant must have a full complement of T repeated observations.

Advances in statistical computation, beginning with the EM algorithm (Dempster, Laird, & Rubin, 1977; see also Jennrich & Schluchter, 1986), allow the estimation of multivariate normal models from incomplete data. In this case, the aim of the study was to collect T observations per person, but only n_j observations were collected ($n_j \leq T$). These n_j observations are indeed collected

according to a fixed design, but $T - n_j$ data points are missing at random.

Model specification for HMLM analysis are specified via the **Basic Settings** dialog box as shown below.



2. Description of the model

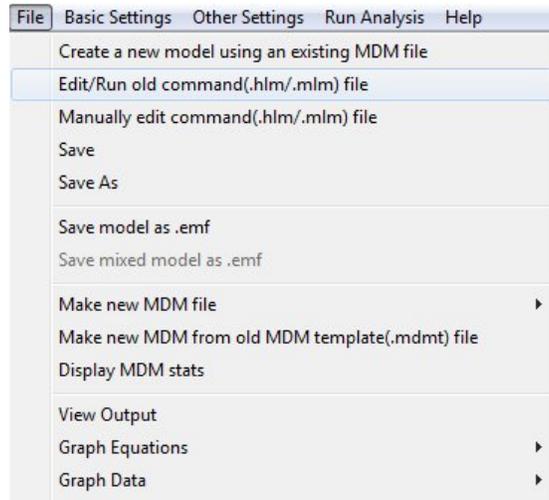
HMLM allows estimation of multivariate normal models from incomplete data. Within the framework of HMLM, it is possible to estimate models having

1. An unrestricted covariance structure, that is a full $T \times T$ covariance matrix.
2. A model with homogenous level-1 variance and random intercepts and/or slopes at level-2.
3. A model with heterogeneous variances at level 1 (a different variance for each occasion) and random intercepts and/or slopes at level 2.
4. A model that includes a log-linear structure for the level-1 variance and random intercepts and/or slopes at level 2.
5. A model with first-order auto-regressive level-1 random errors and random intercepts and/or slopes at level 2.

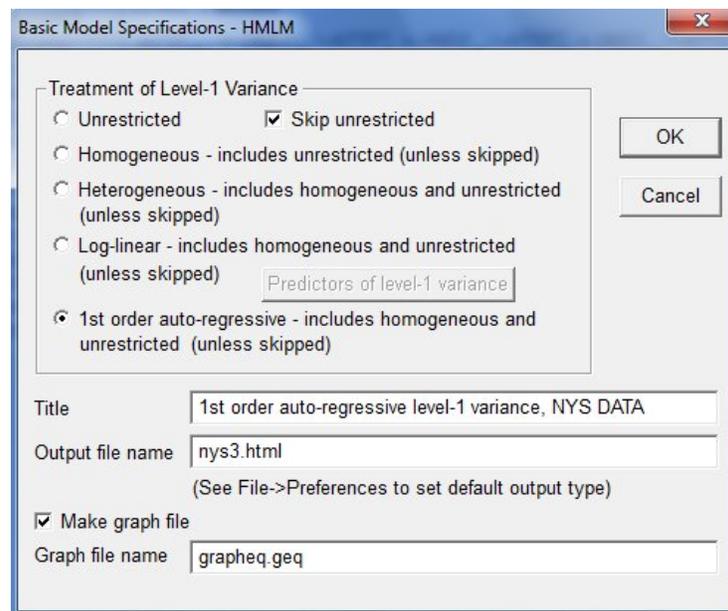
In this illustration, we fit a model with first-order auto-regressive level-1 random errors and random intercepts and/or slopes at level 2 to the MDM file created in the first of this set of example, *i.e.* **NYS.MDM**. This is the third in a set of three HMLM models fitted to the NYS data.

6. Creating the command file

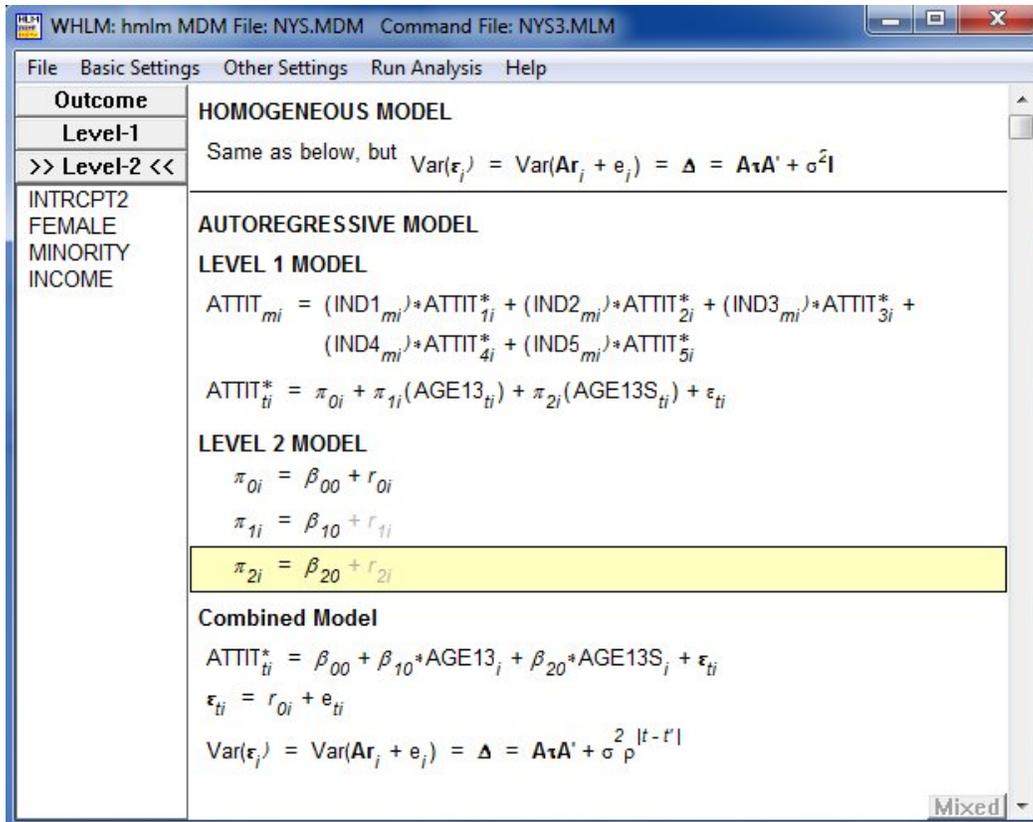
Start by selecting the **Edit/Run old command (.hlm/.mlm) file** option from the **File** menu and open the MLM file **NYS1.MLM**.



Next, click on the **Basic Settings** option on the main menu bar to open the **Basic Model Specifications - HMLM** dialog box. Check the radio button for the **1st order auto-regressive – includes homogeneous and unrestricted (unless skipped)** option.



The final model is now displayed in the modeling window.



Remember to save the command file prior to running the analysis.

7. Interpreting the output

Output for Random Effects Model First-order Autoregressive Model for Level-1 Variance

Summary of the model specified

Level-1 Model

$$ATTIT_{mi} = (IND1_{mi}) * ATTIT_{1i}^* + (IND2_{mi}) * ATTIT_{2i}^* + (IND3_{mi}) * ATTIT_{3i}^* + (IND4_{mi}) * ATTIT_{4i}^* + (IND5_{mi}) * ATTIT_{5i}^*$$

$$ATTIT_{ti}^* = \pi_{0i} + \pi_{1i} * (AGE13_{ti}) + \pi_{2i} * (AGE13S_{ti}) + \epsilon_{ti}^*$$

Level-2 Model

$$\pi_{0i} = \beta_{00} + r_{0i}$$

$$\pi_{1i} = \beta_{10}$$

$$\pi_{2i} = \beta_{20}$$

Note that β_1 and β_2 are specified as non-random due to the fact that the time-series is relatively short and therefore the data do not allow the estimation of both random slopes and an autocorrelation parameter.

$$\text{Var}(\varepsilon_i) = \text{Var}(\mathbf{A}r_i + e_i) = \mathbf{\Delta} = \mathbf{A}\tau\mathbf{A}' + \sigma^2\rho^{|t-t'|}$$

The above equation, written with subscripts and Greek letters, is

$$\text{Var}(Y^*) = \mathbf{A}\tau\mathbf{A}' + \Sigma$$

where

$$\Sigma = \sigma^2\rho^{|t-t'|}$$

A

IND1	1.00000
IND2	1.00000
IND3	1.00000
IND4	1.00000
IND5	1.00000

Iterations stopped due to small change in likelihood function

Final Results - Iteration 6

	Parameter	Standard Error
ρ	0.39675	0.053849
σ^2	0.04158	0.003582

Note that the maximum-likelihood estimate of $\hat{\rho} = 0.397$ is much larger than its standard error (0.054), suggesting a significantly positive autocorrelation.

τ

INTRCPT1, r_0	0.02427
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Standard error of τ

INTRCPT1, r_0	0.00450
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Δ

IND1	0.06585	0.04077	0.03081	0.02686	0.02530
IND2	0.04077	0.06585	0.04077	0.03081	0.02686
IND3	0.03081	0.04077	0.06585	0.04077	0.03081
IND4	0.02686	0.03081	0.04077	0.06585	0.04077
IND5	0.02530	0.02686	0.03081	0.04077	0.06585

The 5×5 matrix above contains the variance and covariance estimates implied by the “auto-correlation” model for the level-1 variance.

Δ (as correlations)

IND1	1.000	0.619	0.468	0.408	0.384
IND2	0.619	1.000	0.619	0.468	0.408
IND3	0.468	0.619	1.000	0.619	0.468
IND4	0.408	0.468	0.619	1.000	0.619
IND5	0.384	0.408	0.468	0.619	1.000

The value of the log-likelihood function at iteration 6 = 1.471600E+002

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, π_0					
INTRCPT2, β_{00}	0.327579	0.015265	21.459	238	<0.001
For AGE13 slope, π_1					
INTRCPT2, β_{10}	0.061428	0.004836	12.703	1076	<0.001
For AGE13S slope, π_2					
INTRCPT2, β_{20}	0.000211	0.003373	0.062	1076	0.951

Statistics for the current model

Deviance = -294.319916
 Number of estimated parameters = 6

Summary of Model Fit

Model	Number of Parameters	Deviance
1. Unrestricted	18	-378.26694
2. Homogeneous σ^2	5	-229.01630
3. First order Autoregressive	6	-294.31992

At the end of the output file, a comparison is given of the three models. The unrestricted will, by its very nature, always fit the best. This comes at the cost of a high number of parameters to be estimated. The first order autoregressive model fits the data better than the one with homogeneous level-1 variance.