



HMLM2 model for the EG data

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1. Introduction to HMLM models

One of the most frequent applications of hierarchical models involves repeated observations (level 1) nested within persons (level 2). These are described in Chapter 6 of *Hierarchical Linear Models*. In these models, the outcome Y_{ij} for occasion i within person j is conceived as a univariate outcome, observed under different conditions or at different times. An advantage of viewing the repeated observations as nested within the person is that it allows each person to have a different repeated measures design. For example, in a longitudinal study, the number of time points may vary across persons, and the spacing between time points may be different for different persons. Such unbalanced designs would pose problems for standard methods of analysis such as the analysis of variance.

Suppose, however, that the aim of the study is to observe every participant according to a fixed design with, say, T observations per person. The design might involve T observation times or T different outcome variables or even T different experimental conditions. Given the fixed design, the analysis can be reconceived as a multivariate repeated measures analysis. The multivariate model is flexible in allowing a wide variety of assumptions about the variation and covariation of the T repeated measures (Bock, 1985). In the standard application of multivariate repeated measures, there can be no missing outcomes: every participant must have a full complement of T repeated observations.

Advances in statistical computation, beginning with the EM algorithm (Dempster, Laird, & Rubin, 1977; see also Jennrich & Schluchter, 1986), allow the estimation of multivariate normal models from incomplete data. In this case, the aim of the study was to collect T observations per person, but only n_j observations were collected ($n_j \leq T$). These n_j observations are indeed collected

according to a fixed design, but $T - n_j$ data points are missing at random. Model specification for HMLM2 analysis are specified via the **Basic Settings** dialog box

2. Description of the model

HMLM allows estimation of multivariate normal models from incomplete data. Within the framework of HMLM, it is possible to estimate models having

1. An unrestricted covariance structure, that is a full $T \times T$ covariance matrix.
2. A model with homogenous level-1 variance and random intercepts and/or slopes at level-2.
3. A model with heterogeneous variances at level 1 (a different variance for each occasion) and random intercepts and/or slopes at level 2.
4. A model that includes a log-linear structure for the level-1 variance and random intercepts and/or slopes at level 2.
5. A model with first-order auto-regressive level-1 random errors and random intercepts and/or slopes at level 2.

In this illustration, we fit a model with first-order auto-regressive level-1 random errors and random intercepts and/or slopes at level 2 to the MDM file created in the first of this set of example, *i.e.* **NYS.MDM**. This is the third in a set of three HMLM models fitted to the NYS data.

6. Description of the data

Chapter 8 in *Hierarchical Linear Models* presents a series of analyses of data from the US *Sustaining Effects Study*, a longitudinal study of children's growth in academic achievement during the primary years. A level-1 model specifies the relationship between age and academic achievement for each child. At level 2, the coefficients describing each child's growth vary across children within schools as a function of demographic variables. At level 3, the parameters that describe the distribution of growth curves within each school vary across schools as a function of school-level predictors.

Level-1 file

The first few lines of the level-1 file, EG1HMLM2.SAV, are displayed below. The data for the first two children are shown below.

There are eight records listed, three for the first child and five for the second. (Typically there are four or five observations per child with a maximum of six.) The first ID is the level-3 (*i.e.*, school) ID and the second ID is the level-2 (*i.e.*, child) ID. We see that the first record comes from school 2020 and child 273026452 within that school. Notice that this child has three records, one for each of three measurement occasions. Following the two ID fields are that child's values on four variables:

- YEAR (year of the study minus 3.5)
This variable can take on values of -2.5 , -1.5 , -0.5 , 0.5 , 1.5 , and 2.5 for the six years of data collection.
- GRADE
The grade level minus 1.0 of the child at each testing occasion. Therefore, it is 0 at grade 1, 1 at grade 2, etc.
- MATH
A math test in an IRT scale score metric.
- RETAINED
An indicator that a child is retained in grade for a particular year (1 = retained, 0 = not retained).

| | schoolid | childid | year | grade | math | retained | ind1 | ind2 | ind3 | ind4 | ind5 | ind6 |
|---|----------|-----------|-------|-------|-------|----------|------|------|------|------|------|------|
| 1 | 2020 | 273026452 | .50 | 2.00 | 1.15 | .00 | .00 | .00 | .00 | 1.00 | .00 | .00 |
| 2 | 2020 | 273026452 | 1.50 | 3.00 | 1.13 | .00 | .00 | .00 | .00 | .00 | 1.00 | .00 |
| 3 | 2020 | 273026452 | 2.50 | 4.00 | 2.30 | .00 | .00 | .00 | .00 | .00 | .00 | 1.00 |
| 4 | 2020 | 273030991 | -1.50 | .00 | -1.30 | .00 | .00 | 1.00 | .00 | .00 | .00 | .00 |
| 5 | 2020 | 273030991 | -.50 | 1.00 | .44 | .00 | .00 | .00 | 1.00 | .00 | .00 | .00 |
| 6 | 2020 | 273030991 | .50 | 2.00 | 2.43 | .00 | .00 | .00 | .00 | 1.00 | .00 | .00 |
| 7 | 2020 | 273030991 | 1.50 | 3.00 | 2.25 | .00 | .00 | .00 | .00 | .00 | 1.00 | .00 |
| 8 | 2020 | 273030991 | 2.50 | 4.00 | 3.87 | .00 | .00 | .00 | .00 | .00 | .00 | 1.00 |

This is followed by the 6 indicator variables IND1 to IND6:

| | |
|------|---------------------------------|
| IND1 | indicator for measure at time 1 |
| IND2 | indicator for measure at time 2 |
| IND3 | indicator for measure at time 3 |
| IND4 | indicator for measure at time 4 |
| IND5 | indicator for measure at time 5 |
| IND6 | indicator for measure at time 6 |

The indicators were created to facilitate use of HMLM2. We see that the first child, child 27306452 in school 2020, had values of 0.5, 1.5, and 2.5 on year. Clearly, that child had no data at the first three data collection waves (because we see no values of -2.5 , -1.5 , or -0.5 on year), but did have data at the last three waves. As a result, only IND4, IND5 and IND6 assume a value of 1 for measurements associated with this child. The next child had 5 measurements, and thus the value “1” appears five times for this child over IND1 to IND6.

Level-2 file

The level-2 units in the illustration are 1721 children. The data are stored in the file EG2.SAV. The level-2 data for the first eight children are listed below. The first field is the schoolid and the second is the childid. Note that each of the first ten children is in school 2020.

There are three variables:

- FEMALE (1 = female, 0 = male)
- BLACK (1 = African-American, 0 = other)
- HISPANIC (1 = Hispanic, 0 = other)

We see, for example, that child 273026452 is a Hispanic male (FEMALE = 0, BLACK = 0, HISPANIC = 1).

| | schoolid | childid | female | black | hispanic |
|----------|-----------------|----------------|---------------|--------------|-----------------|
| 1 | 2020 | 273026452 | .00 | .00 | 1.00 |
| 2 | 2020 | 273030991 | .00 | .00 | .00 |
| 3 | 2020 | 273059461 | .00 | .00 | 1.00 |
| 4 | 2020 | 278058841 | .00 | .00 | .00 |
| 5 | 2020 | 292017571 | .00 | .00 | 1.00 |
| 6 | 2020 | 292020281 | .00 | .00 | .00 |
| 7 | 2020 | 292020361 | .00 | .00 | .00 |
| 8 | 2020 | 292025081 | .00 | .00 | .00 |

Level-3 file

The level-3 units in the illustration are 60 schools. Level-3 data for the first seven schools are printed below. The full data are in the file EG3.SAV. The first field on the left is the schoolid. There are three level-3 variables:

- SIZE, number of students enrolled in the school
- LOWINC, the percent of students from low income families
- MOBILE, the percent of students moving during the course of a single academic year

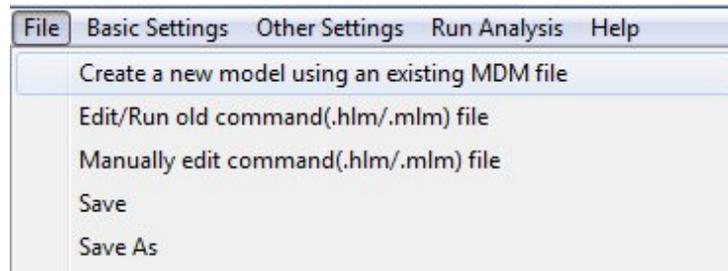
We see that the first school, school 2020, has 380 students, 40.3% of whom are low income. The school mobility rate is 12.5%.

| | schoolid | size | lowinc | mobility |
|---|----------|---------|--------|----------|
| 1 | 2020 | 380.00 | 40.30 | 12.50 |
| 2 | 2040 | 502.00 | 83.10 | 18.60 |
| 3 | 2180 | 777.00 | 96.60 | 44.40 |
| 4 | 2330 | 800.00 | 78.90 | 31.70 |
| 5 | 2340 | 1133.00 | 93.70 | 67.00 |
| 6 | 2380 | 439.00 | 36.90 | 39.30 |
| 7 | 2390 | 566.00 | 100.00 | 39.90 |

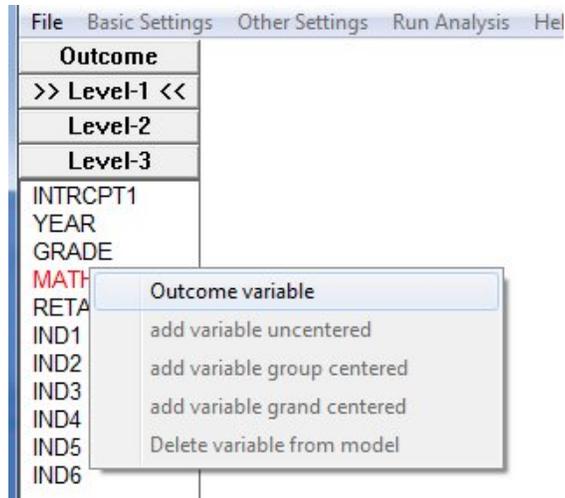
In sum, there are four variables at level 1 along with 6 indicator variables, three at level 2 and three at level 3. Note that the ID variables do not count as variables. Once the user has identified the two sets of IDs, the number of variables in each file, the variable names, and the filenames, creation of the MDM file is exactly analogous to the way an MDM is created for HMLM: care should be taken to indicate the indicator variables in the appropriate column when selecting variables. Note that the three files are linked by level-2 and level-3 IDs here.

7. Creating the command file

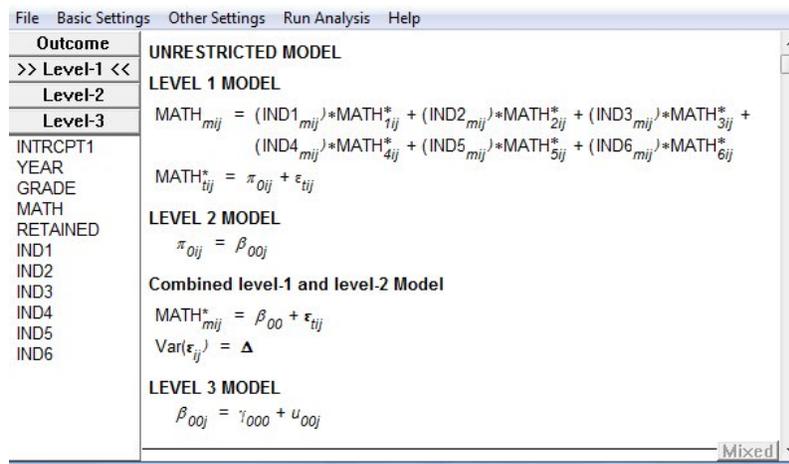
Start by selecting the **Create a new model using an existing MDM file** option from the **File** menu and open the MLM file **EGHMLM2.MDM**.



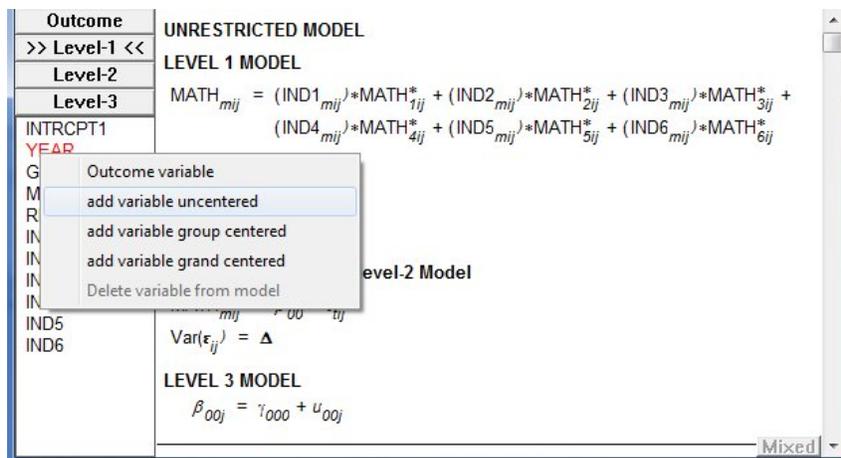
Next, select MATH as the outcome variable.



The unconditional unrestricted and homogeneous models are now displayed in the modeling window.



Add the level-1 predictor YEAR as uncentered predictor to the model as shown below.



Activate the random slope at level-3 by double-clicking on the term u_{10j} .

LEVEL 3 MODEL

$$\beta_{00j} = \gamma_{000} + u_{00j}$$

$$\beta_{10j} = \gamma_{100} + u_{10j}$$

The completed model is now displayed in the modeling window. The unrestricted model is shown first.

This followed by the heterogeneous model. Remember to save the command file prior to running the analysis.

8. Interpreting the output

For the HMLM2 model, three sets of output are provided. The first section contains the output for the unrestricted model. This is followed by output for the model with homogeneous level-1 variance, and finally by output for the heterogeneous level-1 variance model. At the end of the output, statistics comparing the fit of the three models fitted are provided.

Problem Title: no title

The data source for this run = EGHMLM2.MDM

The command file for this run = EG.HLM

Output file name = hmlm2.html

The maximum number of level-1 units = 7230

The maximum number of level-2 units = 1721

The maximum number of level-3 units = 60

The maximum number of iterations = 100

The outcome variable is MATH

The model specified for the fixed effects was:

The model specified for the fixed effects

| Level-1 | Level-2 | Level-3 |
|---------------------|------------------------|--------------------------|
| INTRCPT1, π_0 | INTRCPT2, β_{00} | INTRCPT3, γ_{000} |
| YEAR slope, π_1 | INTRCPT2, β_{10} | INTRCPT3, γ_{100} |

Output for the Unrestricted Model

Summary of the model specified

Level-1 Model

$$\text{MATH}_{mij} = (\text{IND1}_{mij}) * \text{MATH1}_{ij}^* + (\text{IND2}_{mij}) * \text{MATH2}_{ij}^* + (\text{IND3}_{mij}) * \text{MATH3}_{ij}^* + (\text{IND4}_{mij}) * \text{MATH4}_{ij}^* + (\text{IND5}_{mij}) * \text{MATH5}_{ij}^* + (\text{IND6}_{mij}) * \text{MATH6}_{ij}^*$$

$$\text{MATH}_{tij}^* = \pi_{0ij} + \pi_{1ij} * (\text{YEAR}_{tij}) + \epsilon_{tij}$$

Level-2 Model

$$\pi_{0ij} = \beta_{00j}$$

$$\pi_{1ij} = \beta_{10j}$$

Level-3 Model

$$\beta_{00j} = \gamma_{000} + u_{00j}$$

$$\beta_{10j} = \gamma_{100} + u_{10j}$$

$$\text{Var}(\epsilon_{ij}) = \Delta$$

$\Delta(0)$

| | | | | | | |
|------|---------|---------|---------|---------|---------|---------|
| IND1 | 0.04268 | 0.01233 | 0.01919 | 0.01968 | 0.01506 | 0.00898 |
| IND2 | 0.01233 | 0.60634 | 0.35457 | 0.42101 | 0.31132 | 0.24927 |

| | | | | | | |
|------|---------|---------|---------|---------|---------|---------|
| IND3 | 0.01919 | 0.35457 | 0.76957 | 0.62363 | 0.42394 | 0.35205 |
| IND4 | 0.01968 | 0.42101 | 0.62363 | 1.15453 | 0.67302 | 0.52773 |
| IND5 | 0.01506 | 0.31132 | 0.42394 | 0.67302 | 0.81870 | 0.55086 |
| IND6 | 0.00898 | 0.24927 | 0.35205 | 0.52773 | 0.55086 | 0.65701 |

$\tau\beta(0)$

| | |
|-------------------------|-------------------------|
| INTRCPT1 | YEAR |
| INTRCPT2 , β_{00} | INTRCPT2 , β_{10} |
| 0.20128 | 0.01542 |
| 0.01542 | 0.01608 |

The value of the log-likelihood function at iteration 1 = -8.445655E+003

The value of the log-likelihood function at iteration 2 = -8.228973E+003

The value of the log-likelihood function at iteration 3 = -8.166659E+003

The value of the log-likelihood function at iteration 4 = -8.126574E+003

The value of the log-likelihood function at iteration 5 = -8.097070E+003

Iterations stopped due to small change in likelihood function

Final Results - Iteration 32

| | | | | | | |
|----------|---------|---------|---------|---------|---------|---------|
| Δ | | | | | | |
| IND1 | 0.67340 | 0.31616 | 0.38755 | 0.52412 | 0.53030 | 0.38971 |
| IND2 | 0.31616 | 0.77832 | 0.47127 | 0.56726 | 0.54171 | 0.50187 |
| IND3 | 0.38755 | 0.47127 | 0.91072 | 0.76829 | 0.66199 | 0.64640 |
| IND4 | 0.52412 | 0.56726 | 0.76829 | 1.24542 | 0.88364 | 0.81782 |
| IND5 | 0.53030 | 0.54171 | 0.66199 | 0.88364 | 1.05646 | 0.84356 |
| IND6 | 0.38971 | 0.50187 | 0.64640 | 0.81782 | 0.84356 | 0.98722 |

Standard errors of Δ

| | | | | | | |
|------|---------|---------|---------|---------|---------|---------|
| IND1 | 0.08003 | 0.05328 | 0.07256 | 0.02999 | 0.02811 | 0.04341 |
| IND2 | 0.05328 | 0.05757 | 0.06998 | 0.02542 | 0.03289 | 0.03656 |
| IND3 | 0.07256 | 0.06998 | 0.07252 | 0.02966 | 0.03284 | 0.03565 |
| IND4 | 0.02999 | 0.02542 | 0.02966 | 0.02844 | 0.03044 | 0.03913 |
| IND5 | 0.02811 | 0.03289 | 0.03284 | 0.03044 | 0.03030 | 0.03518 |
| IND6 | 0.04341 | 0.03656 | 0.03565 | 0.03913 | 0.03518 | 0.03859 |

Δ (as correlations)

| | | | | | | |
|----------------|-------|-------|-------|-------|-------|-------|
| IND1 , π_0 | 1.000 | 0.437 | 0.495 | 0.572 | 0.629 | 0.478 |
| IND2 , π_1 | 0.437 | 1.000 | 0.560 | 0.576 | 0.597 | 0.573 |
| IND3 , π_2 | 0.495 | 0.560 | 1.000 | 0.721 | 0.675 | 0.682 |
| IND4 , π_3 | 0.572 | 0.576 | 0.721 | 1.000 | 0.770 | 0.738 |
| IND5 , π_4 | 0.629 | 0.597 | 0.675 | 0.770 | 1.000 | 0.826 |
| IND6 , π_5 | 0.478 | 0.573 | 0.682 | 0.738 | 0.826 | 1.000 |

$\tau\beta$

| | |
|-------------------------|-------------------------|
| INTRCPT1 | YEAR |
| INTRCPT2 , β_{00} | INTRCPT2 , β_{10} |
| 0.14824 | 0.01268 |
| 0.01268 | 0.00935 |

Standard Errors of $\tau\beta$

| | |
|-------------------------|-------------------------|
| INTRCPT1 | YEAR |
| INTRCPT2 , β_{00} | INTRCPT2 , β_{10} |
| 0.03286 | 0.00626 |
| 0.00626 | 0.00218 |

| | | |
|---------------------------------|-------|-------|
| $\tau\beta$ (as correlations) | | |
| INTRCPT1/INTRCPT2, β_{00} | 1.000 | 0.341 |
| YEAR/INTRCPT2, β_{10} | 0.341 | 1.000 |

The value of the log-likelihood function at iteration 32 = -7.980254E+003

Final estimation of fixed effects:

| Fixed Effect | Coefficient | Standard error | t-ratio | Approx. d.f. | p-value |
|----------------------------|-------------|----------------|---------|--------------|---------|
| For INTRCPT1, π_0 | | | | | |
| For INTRCPT2, β_{00} | | | | | |
| INTRCPT3 , γ_{000} | -0.824938 | 0.054960 | -15.010 | 59 | <0.001 |
| For YEAR slope, π_1 | | | | | |
| For INTRCPT2, β_{10} | | | | | |
| INTRCPT3 , γ_{100} | 0.755026 | 0.014229 | 53.062 | 59 | <0.001 |

Statistics for the current model

Deviance = 15960.507331
 Number of estimated parameters = 26

Output for Random Effects Model with Homogeneous Level-1 Variance

Summary of the model specified

Level-1 Model

$$\text{MATH}_{mij} = (\text{IND1}_{mij}) * \text{MATH1}_{ij}^* + (\text{IND2}_{mij}) * \text{MATH2}_{ij}^* + (\text{IND3}_{mij}) * \text{MATH3}_{ij}^* + (\text{IND4}_{mij}) * \text{MATH4}_{ij}^* + (\text{IND5}_{mij}) * \text{MATH5}_{ij}^* + (\text{IND6}_{mij}) * \text{MATH6}_{ij}^*$$

$$\text{MATH}_{tij}^* = \pi_{0ij} + \pi_{1ij} * (\text{YEAR}_{tij}) + \epsilon_{tij}$$

Level-2 Model

$$\pi_{0ij} = \beta_{00j} + r_{0ij}$$

$$\pi_{1ij} = \beta_{10j} + r_{1ij}$$

Level-3 Model

$$\beta_{00j} = \gamma_{000} + u_{00j}$$

$$\beta_{10j} = \gamma_{100} + u_{10j}$$

$$\text{Var}(\epsilon_{ij}) = \text{Var}(\text{A}_{ij} + e_{ij}) = \Delta = \text{A}\pi\text{A}' + \sigma^2\text{I}$$

A

| | | |
|------|---------|----------|
| IND1 | 1.00000 | -2.50000 |
| IND2 | 1.00000 | -1.50000 |
| IND3 | 1.00000 | -0.50000 |
| IND4 | 1.00000 | 0.50000 |
| IND5 | 1.00000 | 1.50000 |
| IND6 | 1.00000 | 2.50000 |

The value of the log-likelihood function at iteration 3 = -8.163134E+003

The value of the log-likelihood function at iteration 4 = -8.163116E+003

Iterations stopped due to small change in likelihood function

Final Results - Iteration 5

| | Parameter | Standard Error |
|------------|-----------|----------------|
| σ^2 | 0.30144 | 0.006598 |

| π | | |
|-------------|---------|---------|
| INTRCPT1,r0 | 0.64046 | 0.04679 |
| YEAR,r1 | 0.04679 | 0.01126 |

Standard errors of π

| | | |
|-------------|---------|---------|
| INTRCPT1,r0 | 0.02515 | 0.00499 |
| YEAR,r1 | 0.00499 | 0.00197 |

| π (as correlations) | | |
|-------------------------|-------|-------|
| INTRCPT1,r0 | 1.000 | 0.551 |
| YEAR,r1 | 0.551 | 1.000 |

| Δ | | | | | | |
|----------|---------|---------|---------|---------|---------|---------|
| IND1 | 0.77832 | 0.49553 | 0.51417 | 0.53282 | 0.55146 | 0.57011 |
| IND2 | 0.49553 | 0.82687 | 0.55533 | 0.58523 | 0.61513 | 0.64503 |
| IND3 | 0.51417 | 0.55533 | 0.89793 | 0.63765 | 0.67880 | 0.71996 |
| IND4 | 0.53282 | 0.58523 | 0.63765 | 0.99150 | 0.74247 | 0.79489 |
| IND5 | 0.55146 | 0.61513 | 0.67880 | 0.74247 | 1.10758 | 0.86981 |
| IND6 | 0.57011 | 0.64503 | 0.71996 | 0.79489 | 0.86981 | 1.24618 |

| Δ (as correlations) | | | | | | |
|----------------------------|-------|-------|-------|-------|-------|-------|
| IND1 , π_0 | 1.000 | 0.618 | 0.615 | 0.607 | 0.594 | 0.579 |
| IND2 , π_1 | 0.618 | 1.000 | 0.644 | 0.646 | 0.643 | 0.635 |
| IND3 , π_2 | 0.615 | 0.644 | 1.000 | 0.676 | 0.681 | 0.681 |
| IND4 , π_3 | 0.607 | 0.646 | 0.676 | 1.000 | 0.709 | 0.715 |
| IND5 , π_4 | 0.594 | 0.643 | 0.681 | 0.709 | 1.000 | 0.740 |
| IND6 , π_5 | 0.579 | 0.635 | 0.681 | 0.715 | 0.740 | 1.000 |

$\tau\beta$

| INTRCPT1 | YEAR |
|-------------------------|-------------------------|
| INTRCPT2 , β_{00} | INTRCPT2 , β_{10} |
| 0.16532 | 0.01705 |
| 0.01705 | 0.01102 |

Standard Errors of $\tau\beta$

| | |
|-------------------------|-------------------------|
| INTRCPT1 | YEAR |
| INTRCPT2 , β_{00} | INTRCPT2 , β_{10} |
| 0.03641 | 0.00720 |
| 0.00720 | 0.00252 |

| | | |
|---------------------------------|-------|-------|
| $\tau\beta$ (as correlations) | | |
| INTRCPT1/INTRCPT2, β_{00} | 1.000 | 0.399 |
| YEAR/INTRCPT2, β_{10} | 0.399 | 1.000 |

The value of the log-likelihood function at iteration 5 = -8.163116E+003

Final estimation of fixed effects:

| Fixed Effect | Coefficient | Standard error | t-ratio | Approx. d.f. | p-value |
|----------------------------|-------------|----------------|---------|--------------|---------|
| For INTRCPT1, π_0 | | | | | |
| For INTRCPT2, β_{00} | | | | | |
| INTRCPT3 , γ_{000} | -0.779305 | 0.057829 | -13.476 | 59 | <0.001 |
| For YEAR slope, π_1 | | | | | |
| For INTRCPT2, β_{10} | | | | | |
| INTRCPT3 , γ_{100} | 0.763028 | 0.015262 | 49.996 | 59 | <0.001 |

Statistics for the current model

Deviance = 16326.231108
 Number of estimated parameters = 9

Output for Random Effects Model with Heterogeneous Level-1 Variance

Summary of the model specified

Level-1 Model

$$\text{MATH}_{mij} = (\text{IND1}_{mij}) * \text{MATH1}_{ij}^* + (\text{IND2}_{mij}) * \text{MATH2}_{ij}^* + (\text{IND3}_{mij}) * \text{MATH3}_{ij}^* + (\text{IND4}_{mij}) * \text{MATH4}_{ij}^* + (\text{IND5}_{mij}) * \text{MATH5}_{ij}^* + (\text{IND6}_{mij}) * \text{MATH6}_{ij}^*$$

$$\text{MATH}_{tij}^* = \pi_{0ij} + \pi_{1ij} * (\text{YEAR}_{tij}) + \epsilon_{tij}$$

Level-2 Model

$$\pi_{0ij} = \beta_{00j} + r_{0ij}$$

$$\pi_{1ij} = \beta_{10j} + r_{1ij}$$

Level-3 Model

$$\beta_{00j} = \gamma_{000} + u_{00j}$$

$$\beta_{10j} = \gamma_{100} + u_{10j}$$

$$\text{Var}(\epsilon_{ij}) = \text{Var}(\text{Arij} + e_{ij}) = \Delta = \text{A} * \tau\tau * \text{A}' + \text{diag}(\sigma_{21}, \dots, \sigma_{26})$$

A

| | | |
|------|---------|----------|
| IND1 | 1.00000 | -2.50000 |
| IND2 | 1.00000 | -1.50000 |
| IND3 | 1.00000 | -0.50000 |
| IND4 | 1.00000 | 0.50000 |
| IND5 | 1.00000 | 1.50000 |
| IND6 | 1.00000 | 2.50000 |

The value of the log-likelihood function at iteration 4 = -8.070086E+003

The value of the log-likelihood function at iteration 5 = -8.070080E+003

Iterations stopped due to small change in likelihood function

Final Results - Iteration 7

| | σ^2 | Standard Error |
|------|------------|----------------|
| IND1 | 0.34891 | 0.059597 |
| IND2 | 0.38314 | 0.020556 |
| IND3 | 0.31846 | 0.014915 |
| IND4 | 0.37849 | 0.015840 |
| IND5 | 0.20344 | 0.011466 |
| IND6 | 0.15546 | 0.014216 |

| $\pi\pi$ | | |
|-------------|---------|---------|
| INTRCPT1,r0 | 0.62722 | 0.04769 |
| YEAR,r1 | 0.04769 | 0.01386 |

| Standard errors of $\pi\pi$ | | |
|-----------------------------|---------|---------|
| INTRCPT1,r0 | 0.02499 | 0.00495 |
| YEAR,r1 | 0.00495 | 0.00205 |

| $\pi\pi$ (as correlations) | | |
|----------------------------|-------|-------|
| INTRCPT1,r0 | 1.000 | 0.511 |
| YEAR,r1 | 0.511 | 1.000 |

Δ

| | | | | | | |
|------|---------|---------|---------|---------|---------|---------|
| IND1 | 0.82432 | 0.48844 | 0.50148 | 0.51451 | 0.52755 | 0.54058 |
| IND2 | 0.48844 | 0.89848 | 0.54224 | 0.56913 | 0.59603 | 0.62293 |
| IND3 | 0.50148 | 0.54224 | 0.90146 | 0.62376 | 0.66452 | 0.70528 |
| IND4 | 0.51451 | 0.56913 | 0.62376 | 1.05687 | 0.73300 | 0.78762 |
| IND5 | 0.52755 | 0.59603 | 0.66452 | 0.73300 | 1.00493 | 0.86997 |
| IND6 | 0.54058 | 0.62293 | 0.70528 | 0.78762 | 0.86997 | 1.10778 |

| Δ (as correlations) | | | | | | |
|----------------------------|-------|-------|-------|-------|-------|-------|
| IND1 , π_0 | 1.000 | 0.568 | 0.582 | 0.551 | 0.580 | 0.566 |
| IND2 , π_1 | 0.568 | 1.000 | 0.603 | 0.584 | 0.627 | 0.624 |
| IND3 , π_2 | 0.582 | 0.603 | 1.000 | 0.639 | 0.698 | 0.706 |
| IND4 , π_3 | 0.551 | 0.584 | 0.639 | 1.000 | 0.711 | 0.728 |
| IND5 , π_4 | 0.580 | 0.627 | 0.698 | 0.711 | 1.000 | 0.825 |
| IND6 , π_5 | 0.566 | 0.624 | 0.706 | 0.728 | 0.825 | 1.000 |

| | |
|-------------------------|-------------------------|
| $\tau\beta$ | |
| INTRCPT1 | YEAR |
| INTRCPT2 , β_{00} | INTRCPT2 , β_{10} |
| 0.16531 | 0.01552 |
| 0.01552 | 0.00971 |

Standard Errors of $\tau\beta$

| | |
|-------------------------|-------------------------|
| INTRCPT1 | YEAR |
| INTRCPT2 , β_{00} | INTRCPT2 , β_{10} |
| 0.03637 | 0.00677 |
| 0.00677 | 0.00225 |

| | | | |
|---------------------------------|-------|-------|--|
| $\tau\beta$ (as correlations) | | | |
| INTRCPT1/INTRCPT2, β_{00} | 1.000 | 0.387 | |
| YEAR/INTRCPT2, β_{10} | 0.387 | 1.000 | |

The value of the log-likelihood function at iteration 7 = -8.070079E+003

Final estimation of fixed effects:

| Fixed Effect | Coefficient | Standard error | t-ratio | Approx. d.f. | p-value |
|----------------------------|-------------|----------------|---------|--------------|---------|
| For INTRCPT1, π_0 | | | | | |
| For INTRCPT2, β_{00} | | | | | |
| INTRCPT3 , γ_{000} | -0.781960 | 0.057792 | -13.531 | 59 | <0.001 |
| For YEAR slope, π_1 | | | | | |
| For INTRCPT2, β_{10} | | | | | |
| INTRCPT3 , γ_{100} | 0.751231 | 0.014452 | 51.983 | 59 | <0.001 |

Statistics for the current model

Deviance = 16140.158919
Number of estimated parameters = 14

Summary of Model Fit

| Model | Number of Parameters | Deviance |
|-----------------------------|----------------------|-------------|
| 1. Unrestricted | 26 | 15960.50733 |
| 2. Homogeneous σ^2 | 9 | 16326.23111 |
| 3. Heterogeneous σ^2 | 14 | 16140.15892 |

| Model Comparison | χ^2 | d.f. | p-value |
|--------------------|-----------|------|---------|
| Model 1 vs Model 2 | 365.72378 | 17 | <0.001 |
| Model 1 vs Model 3 | 179.65159 | 12 | <0.001 |
| Model 2 vs Model 3 | 186.07219 | 5 | <0.001 |