



Weighting in HLM 8.1

The information below replaces the current section 2.8.1 in the HLM manual.

2.8.1 Using design weights

Case weights are widely used in many branches of applied statistics. A classical application arises in sample surveys in which units have been selected with known but unequal probabilities. In these cases, it will often be desirable to weight down over-sampled units in order to produce unbiased estimates of population parameters. Another longstanding aim is to adjust for non-response. Units with low predicted probabilities of response are weighted up to increase the representativeness of the sample. More recently, it has become quite common to use weighting to remove observable confounding in the evaluation of treatment effects. Units with a high predicted probability (or “propensity”) to receive the treatment they received, based on their covariates, are weighted down. The idea here is to approximate a randomized trial with an optimal design even when such a design has not been implemented.

Example. Suppose, for instance, that in a pre-election poll, ethnic minority voters are over-sampled to insure that various ethnic groups are represented in the sample. Without weighting, the over-sampled groups would exert undue influence on estimates of the proportion of voters in the population favoring a specific candidate. Use of design weights can yield unbiased estimates of the population parameters.

2.8.1.1 Design weighting in the hierarchical context

Hierarchical data can be described as arising from a multi-stage sampling procedure. For example, schools might be sampled from a national frame of schools and then, within each school, students might then be sampled from a list of all students attending the school. Probabilities at each level might be known but unequal. For example, one might over-sample private schools and then over-sample minority students within each school. For example, in a two-level design with students nested within schools, one might compute the marginal probability that a student is selected as the product of the probability that student's school is selected multiplied by the conditional probability that the student is selected given that his or her school is selected. In another context, suppose persons are selected with known probability and then followed longitudinally over time. In this case, we have occasions at level 1 nested within persons at level 2.

Researchers must take care to clearly define the target of generalization in this kind of study. In a cross-sectional study of students nested within schools, one might wish to generalize to a population level-1 units (students) or to a population of level-2 units (schools). In a longitudinal study with repeated measures (level 1) nested within persons (level 2), one typically seeks to generalize to the level-2 units. The weights must be defined accordingly.

HLM uses a method of computation devised by Pfefferman *et al.* (1998) for hierarchical data. This method, based on weighting the information of each case in the framework of maximum likelihood, is more appropriate than the method of weighting in earlier versions of HLM, which used a more conventional approach of weighting observations.

2.1.1.2 Weighting in two-level designs

We'll illustrate the application of weights in a two-level setting where the aim is to correct for oversampling. In this context, weights might be available at level 1, at level 2 or at both levels. If weights are available at level-1, the methodology used in HLM assumes that these weights are inversely proportional to P_{ij} , the marginal probability of that student i in school j is selected into the sample. HLM will then normalize the weight to have a mean of 1.0. Thus we have

$$w_{ij} = \frac{N / P_{ij}}{\sum_{j=1}^J \sum_{i=1}^{n_j} 1 / P_{ij}} \quad (2.8.1.1)$$

in which case

$$\sum_{j=1}^J \sum_{i=1}^{n_j} w_{ij} = N \quad (2.8.1.2)$$

where N is the total sample size of level-1 units. If weights are available at level 2, the methodology assumes that these weights are inversely proportional to P_j the probability of selection of the level-2 unit. In this case, HLM will again normalize the weight to have a mean of 1.0, yielding

$$w_j = \frac{J/P_j}{\sum_{j=1}^J 1/P_j}, \quad (2.8.1.3)$$

in which case

$$\sum_{j=1}^J w_j = J. \quad (2.8.1.4)$$

where J is the total number of level-2 units.

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2.8.1.1 Weighting in three-level designs

In the three-level context, weights might be available at any one of the three levels, at any pair of them, or at all three levels. Normalization proceeds in a fashion completely analogous to that in the case of two levels. If weights are available at level 1, we assume these are inversely proportional to P_{ijk} , the marginal probability of selection of unit ijk . Similarly, if weights are available at level 2 or only at level 3, the corresponding probabilities are P_{jk} or P_k , respectively.

To apply weights

In HLM, weights are selected at the time of analysis, not when the MDM file is made:

1. Select the **Estimation Settings** option from the **Other Settings** menu.
2. Click the **Weighting** button to access the pull-down menus used to select the weighting variables at any level.

Note that the cover sheet of each HLM output reminds the user of the weighting specification chosen.