# LISREL Submodel 3

The LISREL Submodel 3A is defined by the two equations:

$$\mathbf{y} = \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon}$$
$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}$$

There are no *x*-variables in this submodel; the variables involved are:

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|---|---|
| Observed variables: | $\mathbf{y}' = \left(y_1, y_2, \ldots, y_p\right)$ |
| Latent variables: | $\mathbf{y}' = \left(\eta_1, \eta_2, \ldots, \eta_m\right)$ |
| | $\boldsymbol{\xi}' = \left(\xi_1, \xi_2, \ldots, \xi_n\right)$ |
| Error variables: | $\boldsymbol{\varepsilon}' = \left(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p\right)$ |
| | $\boldsymbol{\zeta}' = \left(\zeta_1, \zeta_2, \ldots, \zeta_m\right)$ |

Although it may seem strange that there are $\xi$-variables but no *x*-variables, this is formally possible. Solving the second equation above for $\boldsymbol{\eta}$ and substituting into the first fives the single defining equation

$$\mathbf{y} = \boldsymbol{\Lambda}_y (\mathbf{I} - \mathbf{B})^{-1} \left(\boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}\right) + \boldsymbol{\varepsilon}.$$

A special case is the so-called “ACOVS” model, in which $\mathbf{B} = \mathbf{0}$ (Default);

$$\mathbf{y} = \boldsymbol{\Lambda}_y \left(\boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}\right) + \boldsymbol{\varepsilon},$$

with covariance matrix

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}_y \left(\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Gamma}' + \boldsymbol{\Psi}\right) \boldsymbol{\Lambda}_y' + \boldsymbol{\Theta}_\varepsilon.$$

Another special case is when there are no $\xi$ -variables. This is called Submodel 3B. In this case, the previous equations reduce to:

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\zeta}$$
$$\mathbf{y} = \boldsymbol{\Lambda}_y(\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\zeta} + \boldsymbol{\varepsilon}.$$

These equations define Submodel 3B, which has only four parameter matrices – namely, $\boldsymbol{\Lambda}_y$, $\mathbf{B}$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Theta}_\varepsilon$. Under Submodel 3B, the covariance matrix of $\mathbf{y}$ is

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}_y(\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\Psi}(\mathbf{I} - \mathbf{B}^{'})^{-1}\boldsymbol{\Lambda}_y^{'} + \boldsymbol{\Theta}_\varepsilon.$$

Paradoxically, Submodel 3B is more general than the full LISREL model. There are two advantages of using this submodel rather than the full model:

1. It may be preferred because it has fewer parameter matrices, although each one is larger.
2. More important, this model provides for correlations between $\delta$ 's and $\varepsilon$ 's, which are not possible in the full LISREL model.

Since Submodel 3A has $\xi$ -variables but no *x*-variables, the LISREL method of computing TSLS and IV estimates will not work because there are no reference variables for the $\xi$ 's. Starting values must be provided by the user. The NS parameter on the OU command tells the program to use these starting values instead of TSLS and IV estimates.