



Multi-sample analyses: testing equality of factor correlation matrices

In the table below (Werts, et. al., (1976)) observed covariance matrices are given for two random samples ($N_1 = 865$, $N_2 = 900$, respectively) of candidates who took the Scholastic Aptitude Test in January 1971.

The four measures are, in order:

- x_1 = a 40-item verbal aptitude section
- x_2 = a separately timed 50-item verbal aptitude section
- x_3 = a 35-item math aptitude section
- x_4 = a separately timed 25-item aptitude section

Table: Covariance matrices for SAT Verbal and Math sections

Tests	Group 1				Group 2			
x_1	63.382				67/898			
x_2	70.984	110.237			72.301	107.330		
x_3	41.710	52.747	60.584		40.549	55.347	63.203	
x_4	30.218	37.489	36.392	32.395	28.976	38.896	39.261	35.403

We use the data to illustrate how one can test equality of factor correlations in a confirmatory factor analysis model. In problem **D** of the previous example, it was postulated that factor patterns in Λ_x and error variances in Θ_δ are invariant over groups. This means that differences between groups, if any, must manifest themselves in different factor covariance matrices Φ . By contrast, in this example, we allow Λ_x and Θ_δ to vary among groups and test the hypothesis that the correlation between the two factors is the same in both groups.

The model is a LISREL Submodel 1 with

$$\Lambda_x = \begin{bmatrix} * & 0 \\ * & 0 \\ 0 & * \\ 0 & * \end{bmatrix} \quad \Phi = \begin{bmatrix} 1 & \\ * & 1 \end{bmatrix} \quad \Theta_\delta = \begin{bmatrix} * \\ * \\ * \\ * \end{bmatrix}$$

and the test is $\phi_{21}^{(1)} = \phi_{21}^{(2)}$.

The command file **EX92.LIS** (see the **LISREL Examples** folder) is:

```

TESTING EQUALITY OF FACTOR CORRELATIONS   GROUP 1
DA NG=2 NI=4 NO=865
CM FI=EX92.COV
MO NX=4 NK=2 PH=FI
VA 1 PH 1 1 PH 2 2
FR LX 1 1 LX 2 1 LX 3 2 LX 4 2 PH 2 1
OU
TESTING EQUALITY OF FACTOR CORRELATIONS   GROUP 2
DA NI=4 NO=900
CM FI=EX92.COV
MO PH=IN
FR LX 1 1 LX 2 1 LX 3 2 LX 4 2
OU

```

The overall goodness-of-fit measure for the model with the equality constraint imposed is

Global Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	3
Maximum Likelihood Ratio Chi-Square (C1)	4.028 (P = 0.2585)

When the equality constraint is relaxed, the χ^2 drops to 2.18, so obviously the model is good and the hypothesis of equal factor correlations is tenable. The common correlation is estimated at 0.765.