# Scientific Software International

Annotated example of LISREL input and output

In this example we illustrate the processes of (a) writing input for LISREL and (b) interpreting the output from the program. We use a model typical of the structural equation models often tested by researchers. It does not demonstrate all the features of LISREL: it is limited to one group; it does not model the means of the variables; it has no equality restrictions on the parameters; and it assumes that the variables are continuous and have a multivariate normal distribution.

The hypothetical model is shown below:
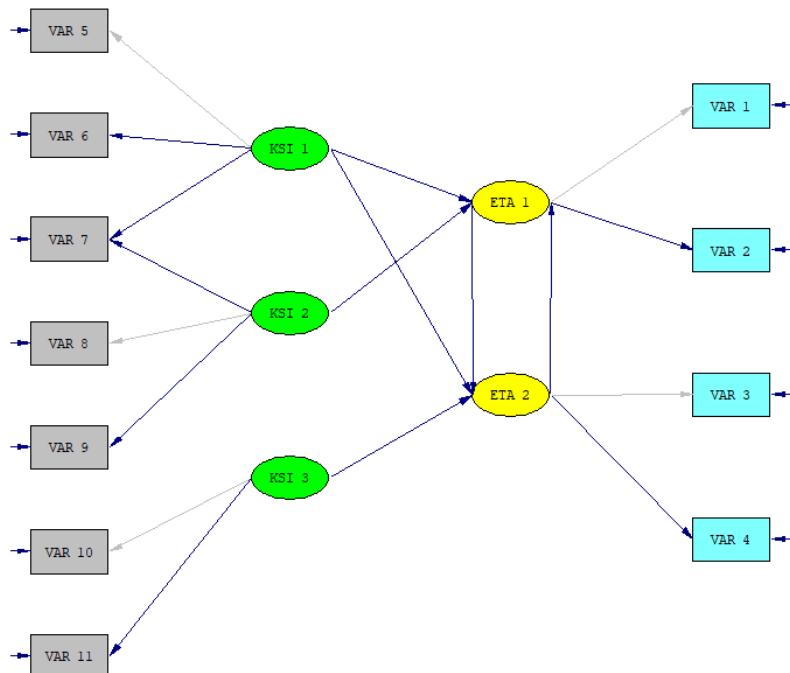
The diagram shows there are seven x-variables as indicators of three latent ξ-variables (denoted by KSI 1 to KSI 3 in the diagram). Note that x_3 is a variable measuring both ξ_1 and ξ_2. There are two latent η-variables (denoted by ETA 1 and ETA 2) each with two y-indicators. The five latent variables are connected in a two-equation interdependent system. The model involves errors in equations (the ζ's) and errors in variables (the ε's and δ's).

The *structural equations* are

$$\eta_1 = \beta_{12}\eta_2 + \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \zeta_1$$
$$\eta_2 = \beta_{21}\eta_1 + \gamma_{21}\xi_1 + \gamma_{23}\xi_3 + \zeta_2$$

or in matrix form

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} 0 & \beta_{12} \\ \beta_{21} & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ \gamma_{21} & 0 & \gamma_{23} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}.$$

Either of these forms corresponds to the structural equation model represented in the figure above,

The *measurement model* equations for y-variables are

$$y_1 = \eta_1 + \varepsilon_1$$
$$y_2 = \lambda_{21}^{(y)}\eta_1 + \varepsilon_2$$
$$y_3 = \eta_2 + \varepsilon_3$$
$$y_4 = \lambda_{42}^{(y)}\eta_2 + \varepsilon_4$$

or in matrix form

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{21}^{(y)} & 0 \\ 0 & 1 \\ 0 & \lambda_{42}^{(y)} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}.$$

The *measurement model* for *x*-variables are

$$x_1 = \xi_1 + \delta_1$$

$$x_2 = \lambda_{21}^{(x)} \xi_1 + \delta_2$$

$$x_3 = \lambda_{31}^{(x)} \xi_1 + \lambda_{32}^{(x)} \xi_2 + \delta_3$$

$$x_4 = \xi_2 + \delta_4$$

$$x_5 = \lambda_{52}^{(x)} \xi_2 + \delta_5$$

$$x_6 = \xi_3 + \delta_6$$

$$x_7 = \lambda_{73}^{(x)} \xi_3 + \delta_7$$

or in matrix form

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \lambda_{21}^{(x)} & 0 & 0 \\ \lambda_{31}^{(x)} & \lambda_{32}^{(x)} & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_{52}^{(x)} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \lambda_{73}^{(x)} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \end{bmatrix}.
$$

These equations correspond to the measurement models represented in the figure above.

One λ in each column of $\mathbf{\Lambda}_y$ and $\mathbf{\Lambda}_x$ has been set equal to 1 to fix the scales of measurement in the latent variables.

In these equations, note that the second subscript on each coefficient is always equal to the subscript of the variable that follows the coefficient. This correspondence serves to check that the terms are correct.

In the matrices $\mathbf{B}$, $\mathbf{\Gamma}$, $\mathbf{\Lambda}_y$ and $\mathbf{\Lambda}_x$, the subscripts on each coefficient, as originally defined in the path diagram, correspond to the row and column of the matrix in which they appear. Note that possible paths that are *not* included in the diagram correspond to *zeros* in these matrices.

Each of the parameter matrices contain fixed elements (the zeros and ones) and free parameters (the coefficients with subscripts).

The four remaining parameter matrices are symmetric matrices:

the covariance matrix of ξ,

$$\mathbf{\Phi} = \begin{bmatrix} \phi_{11} & & \\ \phi_{21} & \phi_{22} & \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix},$$

the covariance matrix of ς,

$$\mathbf{\Psi} = \begin{bmatrix} \psi_{11} & \\ \psi_{21} & \psi_{22} \end{bmatrix},$$

the covariance matrix of ε, a diagonal matrix,

$$\mathbf{\Theta}_\varepsilon = diag\left[\theta_{11}^{(\varepsilon)}, \theta_{22}^{(\varepsilon)}, ..., \theta_{44}^{(\varepsilon)} \right],$$

and the covariance matrix of δ, also a diagonal matrix,

$$\mathbf{\Theta}_\delta = diag\left[\theta_{11}^{(\delta)}, \theta_{22}^{(\delta)}, ..., \theta_{77}^{(\delta)} \right].$$

For most models, the commands perform the following tasks:

1. Specify characteristics of the data set, such as the number of variables, the names of the variables, the form of the data (raw data, or summary statistics such as variances and covariances).
2. Read the data.
3. Specify the general characteristics of the model: how many latent and observed exogeneous and endogenous variables are there; what will be the form of each matrix in the model (for example, if square, is it symmetric or diagonal?); will each matrix consist primarily of free or fixed parameters?
4. Specify modifications to individual elements of matrices to make them fixed in a predominantly free matrix, or vice versa; or specify sets of parameters to be equal.
5. Assign values to nonzero fixed parameters, and any other values necessary to start the iteration procedure.
6. Specify when output is desired, and nondefault settings to determine characteristics of the estimation process or output.

The command file **EX1.LIS** (see the **LISREL Examples** folder) or testing this model is listed below, followed by a detailed explanation of each line.

```
HYPOTHETICAL MODEL ESTIMATED BY ML
DA NI=11 NO=100
CM SY FI=EX1.COV
MO NY=4 NX=7 NE=2 NK=3 BE=FU PS=SY,FR
FR LY 2 1 LY 4 2 LX 2 1 LX 3 1 LX 3 2 LX 5 2 LX 7 3 BE 2 1 BE 1 2
FI GA 1 3 GA 2 2
VA 1 LY 1 1 LY 3 2 LX 1 1 LX 4 2 LX 6 3
PD
OU MI RS EF MR SS SC
```

The first line is a title. The title can extend for as many lines as needed: the program assumes that the title ends when it finds the characters "DA" as the first two characters of a line, which signals the DA command. On this command, the keyword NI indicates the number of variables, which for this problem is 11. The keyword NO indicates the number of observations.

The CM command indicates that the data are in the form of a covariance matrix. To show that the matrix is symmetric, and that only the lower triangular part (including the diagonal) is included, the option SY is specified. The data are not included in the command file; the keyword FI tells that the data are in the file **EX1.COV**. The contents of the file **EX1.COV** are:

```
(16F5.3)
 3204
 2722 2629
 3198 2875 4855
 3545 3202 5373 6315
  329  371 -357 -471 1363
  559  592 -316 -335 1271 1960
 1006 1019 -489 -591 1742 2276 3803
  468  456 -438 -539  788 1043 1953 1376
  502  539 -363 -425  838 1070 2090 1189 1741
 1050  960 1416 1714  474  694  655   71  104 1422
 1260 1154 1923 2309  686  907  917  136  162 1688 2684
```

The LA command may be used to specify that labels will be provided for the observed variables. The following line should give the labels for these variables, if requested. These labels may continue over as many lines as needed.

The MO command is used to specify the general form of the matrices in the model. There are four y-variables (NY = 4), seven x-variables (NX = 7), two η-variables (NE = 2) and three ξ-variables (NK = 3). The default form is used for most of the matrices; for example, Λ_x and Λ_y are FU (i.e., rectangular, not square) matrices with FI (fixed) elements. The nondefault matrices are specified as BE = FU and PS = SY, FR. This make BE (Beta) a full rectangular matrix instead of the default, a ZE (zero) matrix, and PS (Psi) a full symmetric matrix instead of the default, a DI (diagonal) matrix.

 The specifications for individual elements that will depart from the general form specified in the MO command are given next. The FR command allows parameters in matrices that were fixed to be free. The elements to be freed are specified by two-character names and indices to specify the row and column. For example, the first element freed is LY 2 1, which is the element in row 2, column 1, of Λ_y. The FI command serves a similar purpose: it fixes elements in matrices that were specified to be free.

The VA command fives specified values to parameters. If these parameters are fixed, then the values do not change during the estimation process. If no value is specified for a fixed parameter, its value defaults to zero. Here, the value 1 is given to various elements of Λ_x and Λ_y.

Just as labels can be given to observed variables, they can also be given to latent variables. The LE command gives labels to the η-variables, and the LK command fives labels to the ξ-variables. In the example, we have given generic names to these variables.

The OU command serves several purposes. Most commonly, it is used to specify the method of estimation (here maximum likelihood by default, since no other method is specified), and to specify what output is desired. Here we have requested the printing of the fitted covariance matrix, residuals, standardized residuals, and a Q-plot of residuals (RS), total and indirect effects (EF), miscellaneous results (MR) and a standardized solution (SS).

The printing of standard errors (SE), *t*-values (TV), and modification indices (MI) is included by default.

The (edited) output file follows. Some segments are omitted to save space.

The program first reproduces the command file, followed by the general form of the problem.

```
The following lines were read from file EX1.lis:

 HYPOTHETICAL MODEL ESTIMATED BY ML
 DA NI=11 NO=100
 CM SY FI=EX1.COV
 MO NY=4 NX=7 NE=2 NK=3 BE=FU PS=SY,FR
 FR LY 2 1 LY 4 2 LX 2 1 LX 3 1 LX 3 2 LX 5 2 LX 7 3 BE 2 1 BE 1 2
 FI GA 1 3 GA 2 2
 VA 1 LY 1 1 LY 3 2 LX 1 1 LX 4 2 LX 6 3
 PD
 OU MI RS EF MR SS SC

 HYPOTHETICAL MODEL ESTIMATED BY ML

                          Number of Input Variables 11
                          Number of Y - Variables    4
                          Number of X - Variables    7
                          Number of ETA - Variables  2
                          Number of KSI - Variables  3
                          Number of Observations    100
```

The covariance matrix is listed. In some cases, this is calculated by the program from raw data, or from correlations and standard deviations.

LISREL uses two decimals in the output by default. With the ND keyword on the OU command, you may specify a different number.

```
Covariance Matrix

             VAR  1      VAR  2      VAR  3      VAR  4      VAR  5      VAR  6

           --------    --------    --------    --------    --------    --------
   VAR  1    3.204
   VAR  2    2.722       2.629
   VAR  3    3.198       2.875       4.855
   VAR  4    3.545       3.202       5.373       6.315
   VAR  5    0.329       0.371      -0.357      -0.471       1.363
   VAR  6    0.559       0.592      -0.316      -0.335       1.271       1.960
   VAR  7    1.006       1.019      -0.489      -0.591       1.742       2.276
   VAR  8    0.468       0.456      -0.438      -0.539       0.788       1.043
   VAR  9    0.502       0.539      -0.363      -0.425       0.838       1.070
   VAR 10    1.050       0.960       1.416       1.714       0.474       0.694
   VAR 11    1.260       1.154       1.923       2.309       0.686       0.907

           Covariance Matrix

             VAR  7      VAR  8      VAR  9      VAR 10      VAR 11

           --------    --------    --------    --------    --------
   VAR  7    3.803
   VAR  8    1.953       1.376
   VAR  9    2.090       1.189       1.741
   VAR 10    0.655       0.071       0.104       1.422
   VAR 11    0.917       0.136       0.162       1.688       2.684

 Total Variance = 31.352 Generalized Variance = 0.0113

 Largest Eigenvalue = 16.796 Smallest Eigenvalue = 0.139

 Condition Number = 10.975
```

For each matrix, the free and fixed elements are listed. Each parameter is assigned a number. The number zero indicates that an element is fixed; a positive integer that an element is free. When there are equality constraints, elements restricted to have the same value are assigned the same number. The specifications are a result of the general forms for the matrices that were specified in the MO command, and the specification for individual parameters in the FI and FR commands.

Lambda-Y has only two free parameters, which were specified on the FR command in the input. Remember that the value "0" for the other six parameters merely indicates that they are fixed, NOT that they are fixed at the value 0. In fact, two of them are fixed at the value 1, as can be seen from the section with the parameter estimates.

Parameter Specifications

LAMBDA-Y

	ETA 1	ETA 2
VAR 1	0	0
VAR 2	1	0
VAR 3	0	0
VAR 4	0	2

LAMBDA-X

	KSI 1	KSI 2	KSI 3
VAR 5	0	0	0
VAR 6	3	0	0
VAR 7	4	5	0
VAR 8	0	0	0
VAR 9	0	6	0
VAR 10	0	0	0
VAR 11	0	0	7

BETA

	ETA 1	ETA 2
ETA 1	0	8
ETA 2	9	0

GAMMA

	KSI 1	KSI 2	KSI 3
ETA 1	10	11	0
ETA 2	12	0	13

For symmetric matrices such as PHI and PSI below, only the lower triangular part is specified.

PHI

	KSI 1	KSI 2	KSI 3
KSI 1	14		
KSI 2	15	16	
KSI 3	17	18	19

```
      PSI

              ETA 1      ETA 2
            --------   --------
    ETA 1       20
    ETA 2       21         22
```

For diagonal matrices such as THETA EPS and THETA DELTA, only the diagonal elements are listed. Each of these matrices is actually a square matrix.

```
      THETA-EPS

              VAR 1      VAR 2      VAR 3      VAR 4
            --------   --------   --------   --------
                23         24         25         26

      THETA-DELTA

              VAR 5      VAR 6      VAR 7      VAR 8      VAR 9     VAR 10
            --------   --------   --------   --------   --------   --------
                27         28         29         30         31         32

      THETA-DELTA

             VAR 11
            --------
                33
```

Since the free parameters are numbered consecutively, it is easy to calculate the degrees of freedom: there are (11 x 12)/2 = 66 variances and covariances, and 33 free parameters, resulting in 66 – 33 = 33 degrees of freedom.

Next the maximum likelihood estimates are reported. Starting values or initial estimates are not printed at all unless requested with PT on the OU command, i.e., starting values are only printed with the technical output.

Standard errors and *t*-values now appear together with parameter estimates within each parameter matrix and are always printed, whenever possible.

The standard errors show how accurately the values of the free parameters have been estimated. If these are small, as they mostly are here, then the parameters have been estimated accurately.

For each free parameter, the parameter estimate divided by its standard error produces a *t*-value. If a *t*-value is between -1.96 and 1.96, it is not significantly different from zero, so fixing it to zero will not make the fit of the model significantly worse.

LISREL Estimates (Maximum Likelihood)

LAMBDA-Y

	ETA 1	ETA 2
VAR 1	1.000	- -
VAR 2	0.921	- -
	(0.035)	
	26.124	
VAR 3	- -	1.000
VAR 4	- -	1.139
		(0.029)
		38.934

LAMBDA-X

	KSI 1	KSI 2	KSI 3
VAR 5	1.000	- -	- -
VAR 6	1.291	- -	- -
	(0.104)		
	12.397		
VAR 7	0.920	1.092	- -
	(0.123)	(0.117)	
	7.499	9.365	
VAR 8	- -	1.000	- -
VAR 9	- -	1.079	- -
		(0.084)	
		12.863	
VAR 10	- -	- -	1.000
VAR 11	- -	- -	1.437
			(0.092)
			15.557

BETA

	ETA 1	ETA 2
ETA 1	- -	0.538
		(0.056)
		9.573

```
ETA 2       0.937         - -
           (0.177)
            5.281


        GAMMA

            KSI 1       KSI 2       KSI 3
           --------    --------    --------
ETA 1       0.213       0.495        - -
           (0.153)     (0.147)
            1.396       3.368

ETA 2      -1.223         - -        0.996
           (0.121)                  (0.151)
          -10.102                    6.599
```

The variances and covariances among the ksi's, in the lower right-hand part, are the elements of the parameter matrix phi. The remaining elements are not part of the parameter estimates but are derived from them.

```
        Covariance Matrix of ETA and KSI

            ETA 1       ETA 2       KSI 1       KSI 2       KSI 3
           --------    --------    --------    --------    --------
ETA 1       2.957
ETA 2       3.115       4.719
KSI 1       0.482      -0.217       0.974
KSI 2       0.554      -0.312       0.788       1.117
KSI 3       0.932       1.402       0.525       0.133       1.175


        PHI

            KSI 1       KSI 2       KSI 3
           --------    --------    --------
KSI 1       0.974
           (0.187)
            5.196

KSI 2       0.788       1.117
           (0.150)     (0.194)
            5.239       5.753

KSI 3       0.525       0.133       1.175
           (0.133)     (0.125)     (0.202)
            3.947       1.064       5.813
```

```
PSI

            ETA 1      ETA 2
          --------   --------
  ETA 1     0.486
           (0.126)
            3.853

  ETA 2    -0.069      0.133
           (0.168)    (0.078)
           -0.410      1.707
```

 Now some other estimates that are derived from the parameter estimates are printed in between the
remaining estimates. These will help determine how well the observed variables measure the constructs,
both individually and as a group. Here, the multiple correlations for the observed variables are all high, so
none is a poor measure of its latent variable. The squared multiple correlations for the structural equations
indicate the proportion of variance in the endogenous variables accounted for by the variables in the
structural equations. Here they are very high.

```
Squared Multiple Correlations for Structural Equations

            ETA 1      ETA 2
          --------   --------
            0.434      0.997
```

 NOTE: R^2 for Structural Equatios are Hayduk's (2006) Blocked-Error R^2

```
        Reduced Form

            KSI 1      KSI 2      KSI 3
          --------   --------   --------
  ETA 1    -0.895      0.999      1.080
           (0.428)    (0.342)    (0.224)
           -2.091      2.916      4.815

  ETA 2    -2.062      0.936      2.008
           (0.517)    (0.403)    (0.268)
           -3.987      2.321      7.491


        Squared Multiple Correlations for Reduced Form

            ETA 1      ETA 2
          --------   --------
            0.381      0.629
```

```
THETA-EPS

              VAR 1      VAR 2      VAR 3      VAR 4
            --------   --------   --------   --------
              0.247      0.123      0.136      0.197
            (0.053)    (0.038)    (0.041)    (0.054)
              4.687      3.256      3.342      3.621


       Squared Multiple Correlations for Y - Variables

              VAR 1      VAR 2      VAR 3      VAR 4
            --------   --------   --------   --------
              0.923      0.953      0.972      0.969

THETA-DELTA

              VAR 5      VAR 6      VAR 7      VAR 8      VAR 9     VAR 10
            --------   --------   --------   --------   --------   --------
              0.389      0.336      0.063      0.259      0.440      0.247
            (0.064)    (0.067)    (0.051)    (0.050)    (0.075)    (0.053)
              6.125      5.026      1.235      5.162      5.905      4.619


THETA-DELTA

             VAR 11
            --------
              0.259
            (0.091)
              2.841


       Squared Multiple Correlations for X - Variables

              VAR 5      VAR 6      VAR 7      VAR 8      VAR 9     VAR 10
            --------   --------   --------   --------   --------   --------
              0.715      0.829      0.983      0.812      0.747      0.826

       Squared Multiple Correlations for X - Variables

             VAR 11
            --------
              0.903
```

The measures of fit for the model follow.

```
                    Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)                    33
Maximum Likelihood Ratio Chi-Square (C1)            29.394 (P = 0.6473)
```

```
Browne's (1984) ADF Chi-Square (C2_NT)              27.247 (P = 0.7488)

Estimated Non-centrality Parameter (NCP)            0.0
90 Percent Confidence Interval for NCP              (0.0 ; 12.673)

Minimum Fit Function Value                          0.294
Population Discrepancy Function Value (F0)          0.0
90 Percent Confidence Interval for F0               (0.0 ; 0.127)
Root Mean Square Error of Approximation (RMSEA)     0.0
90 Percent Confidence Interval for RMSEA            (0.0 ; 0.0620)
P-Value for Test of Close Fit (RMSEA < 0.05)        0.893

Expected Cross-Validation Index (ECVI)              0.990
90 Percent Confidence Interval for ECVI             (0.990 ; 1.117)
ECVI for Saturated Model                            1.320
ECVI for Independence Model                         14.785

Chi-Square for Independence Model (55 df)        1456.516

Normed Fit Index (NFI)                              0.980
Non-Normed Fit Index (NNFI)                         1.004
Parsimony Normed Fit Index (PNFI)                   0.588
Comparative Fit Index (CFI)                         1.000
Incremental Fit Index (IFI)                         1.003
Relative Fit Index (RFI)                            0.966

Critical N (CN)                                   185.484


Root Mean Square Residual (RMR)                     0.0652
Standardized RMR                                    0.0266
Goodness of Fit Index (GFI)                         0.953
Adjusted Goodness of Fit Index (AGFI)               0.906
Parsimony Goodness of Fit Index (PGFI)              0.476
```

The residuals compare the observed variances and covariances with those resulting from the model's parameter estimates. In a model that fits well, these will be small. The square root of the averaged squared residual (RMR) was reported above as .065; the table following the fitted covariance matrix shows where the large residuals were, and how many there were.

When examining these, keep in mind that their size will vary with the scale of the variables; changing the unit of measurement of a variable will change the variances and covariance, and thus the size of the residuals. Thus caution is needed in the interpretation of the residuals.

Fitted Covariance Matrix

	VAR 1	VAR 2	VAR 3	VAR 4	VAR 5	VAR 6
VAR 1	3.204					
VAR 2	2.722	2.629				
VAR 3	3.115	2.868	4.855			
VAR 4	3.547	3.266	5.373	6.315		
VAR 5	0.482	0.444	-0.217	-0.247	1.363	
VAR 6	0.622	0.573	-0.280	-0.318	1.258	1.960
VAR 7	1.048	0.965	-0.540	-0.614	1.757	2.269
VAR 8	0.554	0.510	-0.312	-0.355	0.788	1.018
VAR 9	0.598	0.550	-0.336	-0.383	0.851	1.098
VAR 10	0.932	0.858	1.402	1.596	0.525	0.678
VAR 11	1.339	1.233	2.014	2.293	0.754	0.974

Fitted Covariance Matrix

	VAR 7	VAR 8	VAR 9	VAR 10	VAR 11
VAR 7	3.803				
VAR 8	1.945	1.376			
VAR 9	2.099	1.206	1.741		
VAR 10	0.629	0.133	0.144	1.422	
VAR 11	0.903	0.192	0.207	1.688	2.684

Fitted Residuals

	VAR 1	VAR 2	VAR 3	VAR 4	VAR 5	VAR 6
VAR 1	0.000					
VAR 2	0.000	0.000				
VAR 3	0.083	0.007	0.000			
VAR 4	-0.002	-0.064	0.000	0.000		
VAR 5	-0.153	-0.073	-0.140	-0.224	0.000	
VAR 6	-0.063	0.019	-0.036	-0.017	0.013	0.000
VAR 7	-0.042	0.054	0.051	0.023	-0.015	0.007
VAR 8	-0.086	-0.054	-0.126	-0.184	0.000	0.025
VAR 9	-0.096	-0.011	-0.027	-0.042	-0.013	-0.028
VAR 10	0.118	0.102	0.014	0.118	-0.051	0.016
VAR 11	-0.079	-0.079	-0.091	0.016	-0.068	-0.067

Fitted Residuals

	VAR 7	VAR 8	VAR 9	VAR 10	VAR 11
VAR 7	0.000				
VAR 8	0.008	0.000			
VAR 9	-0.009	-0.017	0.000		
VAR 10	0.026	-0.062	-0.040	0.000	
VAR 11	0.014	-0.056	-0.045	0.000	0.000

```
 Summary Statistics for Fitted Residuals

 Smallest Fitted Residual =   -0.224
   Median Fitted Residual =   -0.001
  Largest Fitted Residual =    0.118
```

The stem-leaf plot is useful for detecting outlying residuals, and for examining the general shape of the distribution of residuals. Standardized residuals are residuals divided by their standard errors.

```
Stemleaf Plot

 - 2|2
 - 1|85
 - 1|430
 - 0|9988777666655
 - 0|444443322111110000000000000000
   0|111111222233
   0|558
   1|022
```

```
         Standardized Residuals

             VAR 1     VAR 2     VAR 3     VAR 4     VAR 5     VAR 6

             --------  --------  --------  --------  --------  --------
    VAR 1     0.000
    VAR 2     0.000     0.000
    VAR 3     0.165     0.015     0.000
    VAR 4    -0.004    -0.128     0.000     0.000
    VAR 5     - -       - -      -0.711    -0.619     - -
    VAR 6    -0.496     0.184    -0.125    -0.047     0.048     0.000
    VAR 7    -0.116     0.164     0.117     0.047    -0.051     0.021
    VAR 8    -0.396    -0.274    -0.485    -0.620    -0.001     0.155
    VAR 9    -0.630    -0.104    -0.091    -0.126    -0.027     - -
    VAR 10    0.489     0.664     0.044     0.395    -2.722     - -
    VAR 11   -0.244    -0.268    -0.220     0.034    -0.331    -0.268
```

```
         Standardized Residuals

             VAR 7     VAR 8     VAR 9     VAR 10    VAR 11

             --------  --------  --------  --------  --------
    VAR 7     0.000
    VAR 8     0.021     0.000
    VAR 9     - -       - -       - -
    VAR 10    0.110    -0.421    -0.279     0.000
    VAR 11    0.048    -0.459    -0.207     0.000     0.000
```

The summary statistics, stem-and-leaf display, and list of outlying standardized residuals below make it much easier than examining the above display to see how bad the fit is. But the display of all the standardized residuals may help locate the reasons why a model does not fit well.

```
Summary Statistics for Standardized Residuals

 Smallest Standardized Residual =   -2.722
   Median Standardized Residual =    0.000
  Largest Standardized Residual =    0.664

 Stemleaf Plot

 - 2|7
 - 2|
 - 1|
 - 1|
 - 0|7666555
 - 0|44333333222111111100000000000000000000000000000000
   0|1122224
   0|57
 Largest Negative Standardized Residuals
 Residual for   VAR 10 and    VAR 5  -2.722
```

 The plot below is another way of examining standardized residuals. If the model fits poorly, as is the case here, the plot will be shallower than the diagonal line. If the standardized residuals are very small, then the plot will be steeper than the diagonal line. An x represents a single point, an * multiple points. Non-linearities in the plotted points are indicative of specification errors in the model or of departures from linearity or normality.

When a model does not fit well, the modification indices will often be the most useful way of deciding how to change the model to improve the fit. They give an estimate of how much the chi-square will decrease if a fixed parameter is freed. Of course, a parameter should only be freed if it makes sense to do so. The estimated change shows approximately how much the parameter will change when it is freed.

Qplot of Standardized Residuals

```
        3.5.......................................................................
            .                                                                 ..
            .                                                              .   .
            .                                                           .      .
            .                                                        .         .
            .                                                     .            .
            .                                                  .               .
            .                             x                 .                  .
      N     .                                x           .                     .
      o     .                                x        .                        .
      r     .                             x        .                           .
      m     .                             *     .                              .
      a     .                             *        .                           .
      l     .                             xx          .                        .
            .                             *              .                     .
      Q     .                             x     .                              .
      u     .                             *        .                           .
      a     .                          x     .                                 .
      n     .                          x.                                      .
      t     .                           x.                                     .
      i     .                          x*                                      .
      l     .                        .  *                                      .
      e     .                     .   x                                        .
      s     .                   .    xx                                        .
            .                 .     *                                          .
            .              .      xx                                           .
            .           .        xx                                            .
            .                     x                                            .
            .                     x                                            .
            .                    x                                             .
            .           .                                                      .
            .      x          .                                                .
            .              .                                                   .
            .            .                                                     .
            .          .                                                       .
            .        .                                                         .
            .      .                                                           .
            .    .                                                             .
            .  .                                                               .
            . .                                                                .
       -3.5.......................................................................
         -3.5                                                              3.5
                          Standardized Residuals
```
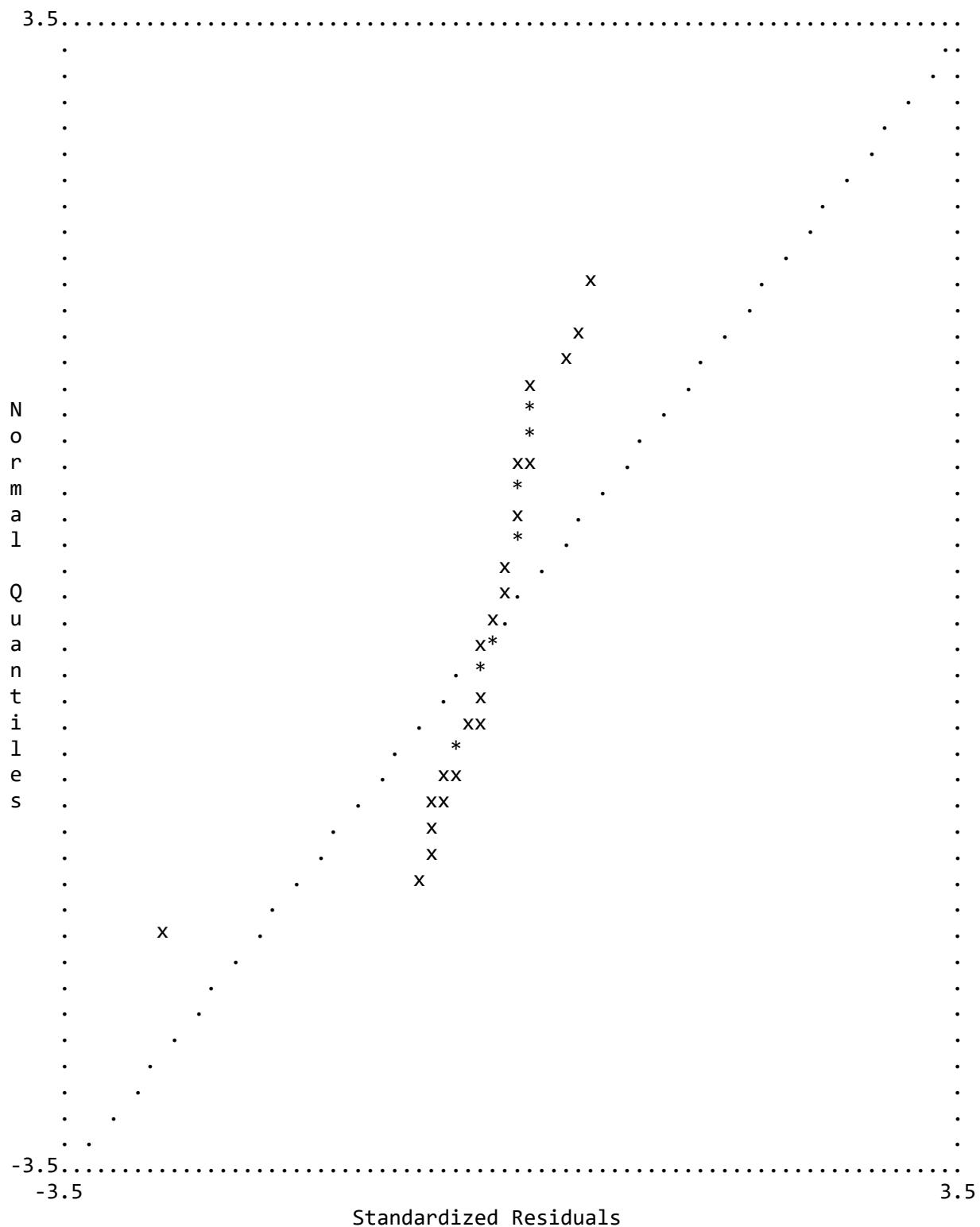
Modification Indices and Expected Change

Modification Indices for LAMBDA-Y

	ETA 1	ETA 2
	--------	--------
VAR 1	- -	0.990
VAR 2	- -	0.990
VAR 3	2.115	- -
VAR 4	2.115	- -

Expected Change for LAMBDA-Y

	ETA 1	ETA 2
	--------	--------
VAR 1	- -	0.058
VAR 2	- -	-0.053
VAR 3	0.089	- -
VAR 4	-0.102	- -

Standardized Expected Change for LAMBDA-Y

	ETA 1	ETA 2
	--------	--------
VAR 1	- -	0.126
VAR 2	- -	-0.116
VAR 3	0.154	- -
VAR 4	-0.175	- -

Completely Standardized Expected Change for LAMBDA-Y

	ETA 1	ETA 2
	--------	--------
VAR 1	- -	0.070
VAR 2	- -	-0.072
VAR 3	0.070	- -
VAR 4	-0.070	- -

Modification Indices for LAMBDA-X

	KSI 1	KSI 2	KSI 3
	--------	--------	--------
VAR 5	- -	0.108	1.049
VAR 6	- -	0.108	0.342
VAR 7	- -	- -	3.376
VAR 8	0.304	- -	1.373
VAR 9	0.304	- -	0.241
VAR 10	0.208	0.001	- -
VAR 11	0.208	0.001	- -

Expected Change for LAMBDA-X

	KSI 1	KSI 2	KSI 3
VAR 5	- -	-0.044	-0.082
VAR 6	- -	0.057	-0.054
VAR 7	- -	- -	0.148
VAR 8	0.082	- -	-0.077
VAR 9	-0.089	- -	-0.038
VAR 10	0.034	-0.001	- -
VAR 11	-0.048	0.002	- -

Standardized Expected Change for LAMBDA-X

	KSI 1	KSI 2	KSI 3
VAR 5	- -	-0.047	-0.089
VAR 6	- -	0.061	-0.058
VAR 7	- -	- -	0.161
VAR 8	0.081	- -	-0.083
VAR 9	-0.087	- -	-0.041
VAR 10	0.033	-0.001	- -
VAR 11	-0.048	0.002	- -

Completely Standardized Expected Change for LAMBDA-X

	KSI 1	KSI 2	KSI 3
VAR 5	- -	-0.040	-0.076
VAR 6	- -	0.043	-0.042
VAR 7	- -	- -	0.082
VAR 8	0.069	- -	-0.071
VAR 9	-0.066	- -	-0.031
VAR 10	0.028	-0.001	- -
VAR 11	-0.029	0.001	- -

No Non-Zero Modification Indices for BETA
No Non-Zero Modification Indices for GAMMA
No Non-Zero Modification Indices for PHI
No Non-Zero Modification Indices for PSI

Modification Indices for THETA-EPS

	VAR 1	VAR 2	VAR 3	VAR 4
VAR 1	- -			
VAR 2	- -	- -		
VAR 3	0.529	0.001	- -	
VAR 4	0.550	0.002	- -	- -

Expected Change for THETA-EPS

	VAR 1	VAR 2	VAR 3	VAR 4
VAR 1	- -			
VAR 2	- -	- -		
VAR 3	0.024	-0.001	- -	
VAR 4	-0.028	0.002	- -	- -

Completely Standardized Expected Change for THETA-EPS

	VAR 1	VAR 2	VAR 3	VAR 4
VAR 1	- -			
VAR 2	- -	- -		
VAR 3	0.006	0.000	- -	
VAR 4	-0.006	0.000	- -	- -

Modification Indices for THETA-DELTA-EPS

	VAR 1	VAR 2	VAR 3	VAR 4
VAR 5	0.134	0.495	2.039	4.211
VAR 6	0.242	0.097	0.349	0.623
VAR 7	0.004	0.057	0.013	0.054
VAR 8	0.907	0.028	0.036	0.367
VAR 9	1.025	0.104	0.110	0.712
VAR 10	0.947	1.116	2.403	0.166
VAR 11	0.288	1.696	0.015	0.907

Expected Change for THETA-DELTA-EPS

	VAR 1	VAR 2	VAR 3	VAR 4
VAR 5	-0.015	0.023	0.048	-0.080
VAR 6	-0.020	0.011	-0.021	0.031
VAR 7	0.002	-0.007	0.003	0.008
VAR 8	0.032	-0.005	-0.005	-0.020
VAR 9	-0.043	0.011	-0.012	0.035
VAR 10	0.034	0.032	-0.048	0.015
VAR 11	-0.025	-0.052	0.005	0.046

Completely Standardized Expected Change for THETA-DELTA-EPS

	VAR 1	VAR 2	VAR 3	VAR 4
VAR 5	-0.007	0.012	0.019	-0.027
VAR 6	-0.008	0.005	-0.007	0.009
VAR 7	0.001	-0.002	0.001	0.002
VAR 8	0.015	-0.002	-0.002	-0.007
VAR 9	-0.018	0.005	-0.004	0.010
VAR 10	0.016	0.017	-0.018	0.005

```
VAR 11      -0.008      -0.020       0.001       0.011

    Modification Indices for THETA-DELTA

              VAR 5       VAR 6       VAR 7       VAR 8       VAR 9      VAR 10
             --------    --------    --------    --------    --------    --------
  VAR 5        - -
  VAR 6       0.341        - -
  VAR 7       0.572       0.016        - -
  VAR 8       0.000       0.091       0.183        - -
  VAR 9       0.136       0.287       2.014       1.160        - -
  VAR 10      0.459       0.806       0.079       0.586       0.161        - -
  VAR 11      0.605       1.926       0.522       0.503       0.049        - -

        Modification Indices for THETA-DELTA

              VAR 11
             --------
  VAR 11       - -

        Expected Change for THETA-DELTA

              VAR 5       VAR 6       VAR 7       VAR 8       VAR 9      VAR 10
             --------    --------    --------    --------    --------    --------
  VAR 5        - -
  VAR 6       0.034        - -
  VAR 7      -0.034       0.007        - -
  VAR 8      -0.001       0.013      -0.032        - -
  VAR 9       0.018      -0.027       0.109      -0.081        - -
  VAR 10     -0.027       0.038      -0.010      -0.026      -0.017        - -
  VAR 11      0.041      -0.079       0.032       0.031      -0.012        - -

        Expected Change for THETA-DELTA

              VAR 11
             --------
  VAR 11       - -

        Completely Standardized Expected Change for THETA-DELTA

              VAR 5       VAR 6       VAR 7       VAR 8       VAR 9      VAR 10
             --------    --------    --------    --------    --------    --------
  VAR 5        - -
  VAR 6       0.021        - -
  VAR 7      -0.015       0.003        - -
  VAR 8       0.000       0.008      -0.014        - -
  VAR 9       0.012      -0.015       0.042      -0.052        - -
  VAR 10     -0.020       0.023      -0.004      -0.018      -0.011        - -
  VAR 11      0.021      -0.034       0.010       0.016      -0.006        - -
```

```
Completely Standardized Expected Change for THETA-DELTA

              VAR 11
            --------
  VAR 11      - -

 Maximum Modification Index is    4.21 for Element ( 1, 4) of THETA DELTA-EPSILON
```

Next are the variances and covariances that are not parameters of the model but are derived from them.
These are produced by the MR option on the OU command.

```
Covariances

         Y - ETA

            VAR 1      VAR 2      VAR 3      VAR 4
           --------   --------   --------   --------
  ETA 1     2.957      2.722      3.115      3.547
  ETA 2     3.115      2.868      4.719      5.373

         Y - KSI

            VAR 1      VAR 2      VAR 3      VAR 4
           --------   --------   --------   --------
  KSI 1     0.482      0.444     -0.217     -0.247
  KSI 2     0.554      0.510     -0.312     -0.355
  KSI 3     0.932      0.858      1.402      1.596

         X - ETA

            VAR 5      VAR 6      VAR 7      VAR 8      VAR 9      VAR 10
           --------   --------   --------   --------   --------   --------
  ETA 1     0.482      0.622      1.048      0.554      0.598      0.932
  ETA 2    -0.217     -0.280     -0.540     -0.312     -0.336      1.402

         X - ETA

            VAR 11
           --------
  ETA 1     1.339
  ETA 2     2.014

         X - KSI

            VAR 5      VAR 6      VAR 7      VAR 8      VAR 9      VAR 10
           --------   --------   --------   --------   --------   --------
  KSI 1     0.974      1.258      1.757      0.788      0.851      0.525
  KSI 2     0.788      1.018      1.945      1.117      1.206      0.133
  KSI 3     0.525      0.678      0.629      0.133      0.144      1.175
```

```
  X - KSI

              VAR 11
            --------
   KSI 1      0.754
   KSI 2      0.192
   KSI 3      1.688
```

In the standardized solution, all latent variables are standardized, i.e., they have a mean of zero and a standard deviation of one.

```
Standardized Solution

        LAMBDA-Y

            ETA 1      ETA 2
          --------   --------
   VAR 1    1.719      - -
   VAR 2    1.583      - -
   VAR 3     - -      2.172
   VAR 4     - -      2.474

        LAMBDA-X

            KSI 1      KSI 2      KSI 3
          --------   --------   --------
   VAR 5    0.987      - -        - -
   VAR 6    1.274      - -        - -
   VAR 7    0.908     1.154       - -
   VAR 8     - -      1.057       - -
   VAR 9     - -      1.141       - -
  VAR 10     - -       - -       1.084
  VAR 11     - -       - -       1.557

        BETA

            ETA 1      ETA 2
          --------   --------
   ETA 1     - -      0.679
   ETA 2    0.742      - -

        GAMMA

            KSI 1      KSI 2      KSI 3
          --------   --------   --------
   ETA 1    0.123     0.304       - -
   ETA 2   -0.556      - -       0.497
```

The correlations among the latent variables presented below are often much easier to interpret than the variances and covariances shown above.

```
                  Correlation Matrix of ETA and KSI

                 ETA 1      ETA 2      KSI 1      KSI 2      KSI 3
                --------   --------   --------   --------   --------
    ETA 1        1.000
    ETA 2        0.834      1.000
    KSI 1        0.284     -0.101      1.000
    KSI 2        0.305     -0.136      0.756      1.000
    KSI 3        0.500      0.595      0.491      0.117      1.000

             PSI

                 ETA 1      ETA 2
                --------   --------
    ETA 1        0.164
    ETA 2       -0.018      0.028

              Regression Matrix ETA on KSI (Standardized)

                 KSI 1      KSI 2      KSI 3
                --------   --------   --------
    ETA 1       -0.514      0.614      0.681
    ETA 2       -0.937      0.455      1.002


Completely Standardized Solution

             LAMBDA-Y

                 ETA 1      ETA 2
                --------   --------
    VAR 1        0.961       - -
    VAR 2        0.976       - -
    VAR 3         - -       0.986
    VAR 4         - -       0.984

             LAMBDA-X

                 KSI 1      KSI 2      KSI 3
                --------   --------   --------
    VAR 5        0.845       - -        - -
    VAR 6        0.910       - -        - -
    VAR 7        0.465      0.592       - -
    VAR 8         - -       0.901       - -
    VAR 9         - -       0.864       - -
    VAR 10        - -        - -       0.909
    VAR 11        - -        - -       0.950
```

BETA

	ETA 1	ETA 2
	--------	--------
ETA 1	- -	0.679
ETA 2	0.742	- -

GAMMA

	KSI 1	KSI 2	KSI 3
	--------	--------	--------
ETA 1	0.123	0.304	- -
ETA 2	-0.556	- -	0.497

Correlation Matrix of ETA and KSI

	ETA 1	ETA 2	KSI 1	KSI 2	KSI 3
	--------	--------	--------	--------	--------
ETA 1	1.000				
ETA 2	0.834	1.000			
KSI 1	0.284	-0.101	1.000		
KSI 2	0.305	-0.136	0.756	1.000	
KSI 3	0.500	0.595	0.491	0.117	1.000

PSI

	ETA 1	ETA 2
	--------	--------
ETA 1	0.164	
ETA 2	-0.018	0.028

THETA-EPS

VAR 1	VAR 2	VAR 3	VAR 4
--------	--------	--------	--------
0.077	0.047	0.028	0.031

THETA-DELTA

VAR 5	VAR 6	VAR 7	VAR 8	VAR 9	VAR 10
--------	--------	--------	--------	--------	--------
0.285	0.171	0.017	0.188	0.253	0.174

THETA-DELTA

VAR 11

0.097

```
Regression Matrix ETA on KSI (Standardized)

              KSI 1       KSI 2       KSI 3
            --------    --------    --------
   ETA 1     -0.514       0.614       0.681
   ETA 2     -0.937       0.455       1.002
```

The effects of one variable on another can be direct, most of which are seen in a path diagram as one-way arrows; they are parameters in the model. Others are found by computing the reduced form equations. The total effects include the direct effects, as well as indirect effects that result from correlations among exogenous variables and circular or reciprocal effects.

The standard errors (in parentheses) and *t*-values are listed below each effect.

```
Total and Indirect Effects

          Total Effects of KSI on ETA

              KSI 1       KSI 2       KSI 3
            --------    --------    --------
   ETA 1     -0.895       0.999       1.080
            (0.426)     (0.341)     (0.223)
             -2.102       2.931       4.839


   ETA 2     -2.062       0.936       2.008
            (0.515)     (0.401)     (0.267)
             -4.007       2.333       7.529



          Indirect Effects of KSI on ETA

              KSI 1       KSI 2       KSI 3
            --------    --------    --------
   ETA 1     -1.109       0.503       1.080
            (0.338)     (0.238)     (0.223)
             -3.281       2.112       4.839


   ETA 2     -0.839       0.936       1.012
            (0.464)     (0.401)     (0.308)
             -1.808       2.333       3.289



          Total Effects of ETA on ETA

              ETA 1       ETA 2
            --------    --------
   ETA 1      1.016       1.084
            (0.408)     (0.280)
             2.489       3.866
```

```
ETA 2       1.890      1.016
          (0.714)    (0.408)
            2.649      2.489
```

The stability index is used in models with reciprocal or circular paths; as long as it is less than 1, there is no problem: the system is stable, and the total effects are finite.

```
Largest Eigenvalue of B*B' (Stability Index) is   0.879

          Indirect Effects of ETA on ETA

              ETA 1      ETA 2

             --------   --------
  ETA 1       1.016      0.546
            (0.408)    (0.246)
              2.489      2.224

  ETA 2       0.953      1.016
            (0.545)    (0.408)
              1.747      2.489


          Total Effects of ETA on Y

              ETA 1      ETA 2

             --------   --------
  VAR 1       2.016      1.084
            (0.408)    (0.280)
              4.938      3.866

  VAR 2       1.856      0.998
            (0.383)    (0.257)
              4.852      3.877

  VAR 3       1.890      2.016
            (0.714)    (0.408)
              2.649      4.938

  VAR 4       2.152      2.296
            (0.813)    (0.469)
              2.648      4.899


          Indirect Effects of ETA on Y

              ETA 1      ETA 2

             --------   --------
  VAR 1       1.016      1.084
            (0.408)    (0.280)
              2.489      3.866
```

```
VAR 2        0.936        0.998
            (0.378)      (0.257)
             2.478        3.877

VAR 3        1.890        1.016
            (0.714)      (0.408)
             2.649        2.489

VAR 4        2.152        1.157
            (0.813)      (0.466)
             2.648        2.484


        Total Effects of KSI on Y

             KSI 1        KSI 2        KSI 3
            --------     --------     --------
VAR 1       -0.895        0.999        1.080
            (0.426)      (0.341)      (0.223)
            -2.102        2.931        4.839

VAR 2       -0.824        0.919        0.994
            (0.392)      (0.313)      (0.205)
            -2.103        2.936        4.861

VAR 3       -2.062        0.936        2.008
            (0.515)      (0.401)      (0.267)
            -4.007        2.333        7.529

VAR 4       -2.348        1.066        2.287
            (0.586)      (0.457)      (0.304)
            -4.006        2.333        7.521


HYPOTHETICAL MODEL ESTIMATED BY ML

Standardized Total and Indirect Effects

        Standardized Total Effects of KSI on ETA

             KSI 1        KSI 2        KSI 3
            --------     --------     --------
ETA 1       -0.514        0.614        0.681
ETA 2       -0.937        0.455        1.002

        Standardized Indirect Effects of KSI on ETA

             KSI 1        KSI 2        KSI 3
            --------     --------     --------
ETA 1       -0.636        0.309        0.681
ETA 2       -0.381        0.455        0.505
```

```
             Standardized Total Effects of ETA on ETA

                    ETA 1      ETA 2
                   --------   --------
         ETA 1      1.016      1.370
         ETA 2      1.496      1.016

             Standardized Indirect Effects of ETA on ETA

                    ETA 1      ETA 2
                   --------   --------
         ETA 1      1.016      0.690
         ETA 2      0.754      1.016

             Standardized Total Effects of ETA on Y

                    ETA 1      ETA 2
                   --------   --------
         VAR 1      3.467      2.355
         VAR 2      3.192      2.168
         VAR 3      3.250      4.380
         VAR 4      3.700      4.987

          Completely Standardized Total Effects of ETA on Y

                    ETA 1      ETA 2
                   --------   --------
         VAR 1      1.937      1.316
         VAR 2      1.969      1.337
         VAR 3      1.475      1.988
         VAR 4      1.473      1.985

             Standardized Indirect Effects of ETA on Y

                    ETA 1      ETA 2
                   --------   --------
         VAR 1      1.747      2.355
         VAR 2      1.609      2.168
         VAR 3      3.250      2.207
         VAR 4      3.700      2.514

          Completely Standardized Indirect Effects of ETA on Y

                    ETA 1      ETA 2
                   --------   --------
         VAR 1      0.976      1.316
         VAR 2      0.992      1.337
         VAR 3      1.475      1.002
         VAR 4      1.473      1.000
```

```
Standardized Total Effects of KSI on Y

             KSI 1      KSI 2      KSI 3
            --------   --------   --------
   VAR 1     -0.884     1.055      1.170
   VAR 2     -0.814     0.972      1.078
   VAR 3     -2.035     0.989      2.177
   VAR 4     -2.317     1.127      2.479


      Completely Standardized Total Effects of KSI on Y

             KSI 1      KSI 2      KSI 3
            --------   --------   --------
   VAR 1     -0.494     0.590      0.654
   VAR 2     -0.502     0.599      0.665
   VAR 3     -0.924     0.449      0.988
   VAR 4     -0.922     0.448      0.986
```

This analysis shows that the model fits the data well. Some of the following examples illustrate what can be done when this is not the case. For instance, detecting misspecification of a model, evaluating alternative models, and how to deal with a nonidentified model.