Scientific Software International

A model for tests that differ in length only

This model assumes that there is a length parameter λ_i associated with observed test score y_i in such a way that the true score variance is proportional to λ_i^4 and that the error variance is proportional to λ_i^2. It can be shown that the covariance structure for this model is of the form

$$\mathbf{\Sigma} = \mathbf{D}_\lambda \left(\boldsymbol{\lambda}\boldsymbol{\lambda}' + \psi \mathbf{I} \right) \mathbf{D}_\lambda ,$$

where $\mathbf{D}_\lambda = diag\left(\lambda_1, \lambda_2, ..., \lambda_p\right)$ and $\boldsymbol{\lambda} = diag\left(\lambda_1, \lambda_2, ..., \lambda_p\right)$. This is of the form

$$\mathbf{\Sigma} = \mathbf{\Lambda}_y \left(\mathbf{\Gamma}\mathbf{\Phi}\mathbf{\Gamma}' + \mathbf{\Psi} \right) \mathbf{\Lambda}_y' + \mathbf{\Theta}_\varepsilon .$$

with $\mathbf{\Lambda}_y = \mathbf{D}_\lambda$, $\mathbf{\Gamma} = \boldsymbol{\lambda}$, $\mathbf{\Phi} = 1$, $\mathbf{\Psi} = \psi \mathbf{I}$, and $\mathbf{\Theta}_\varepsilon = \mathbf{0}$. The model specifies equality constraints between the diagonal elements of $\mathbf{\Lambda}_y$ and the elements of the column vector $\mathbf{\Gamma}$, and also the equality of all the diagonal elements of $\mathbf{\Psi}$. The model has p + 1 parameters and is less restrictive than the parallel model but more restrictive than the congeneric model. A summary of various test theory models and their number of parameters is given in the table below. In this table, **j** denotes a column vector with all elements equal to one.

Table: Various test theory models

Model	Covariance structure	No. of parameters
Parallel	$\mathbf{\Sigma} = \lambda^2 \mathbf{j}\mathbf{j}' + \theta \mathbf{I}$	2
Tau-equivalent	$\mathbf{\Sigma} = \lambda^2 \mathbf{j}\mathbf{j}' + \mathbf{\Theta}$	p + 1
Variable-length	$\mathbf{\Sigma} = \mathbf{D}_\lambda \left(\boldsymbol{\lambda}\boldsymbol{\lambda}' + \psi \mathbf{I} \right) \mathbf{D}_\lambda$	p + 1
Congeneric	$\mathbf{\Sigma} = \boldsymbol{\lambda}\boldsymbol{\lambda}' + \mathbf{\Theta}$	2p

In this example we look at three subtests of SAT (Kristof (1971)). The covariance matrix shown below, based on candidates (n = 900) who took the January 1969 administration of the Scholastic Aptitude Test (SAT), represents the following variables:

1. Verbal Omnibus, containing 40 items administered in 30 minutes.
2. Reading Comprehension, containing 50 items administered in 45 minutes.

3. An additional section of the SAT administered experimentally.

$$\mathbf{S} = \begin{bmatrix} 54.85 & & \\ 60.21 & 99.24 & \\ 48.42 & 67.00 & 63.81 \end{bmatrix}$$

The following LISREL analysis tests whether the three tests can be considered to differ only in length (**EX61.LIS** in the **LISREL Examples** folder):

```
Kristof's Model Estimated for three Subtests of SAT
DA NI=3 NO=900
CM
54.85 60.21 99.24 48.42 67.00 63.81
MO NY=3 NE=3 NK=1 LY=DI,FR PH=ST TE=ZE
EQ LY 1 GA 1
EQ LY 2 GA 2
EQ LY 3 GA 3
EQ PS 1 - PS 3
ST 2 ALL
OU SE TV SO NS MI RS
```

There are some important differences between this example and all previous examples:

- The scales for the three η's are defined by the particular constraints imposed in the model. They are not defined in the usual way by fixing elements of $\mathbf{\Lambda}_y$. The SO option on the OU command tells the program *not* to check that scales have been defined in the usual way.
- Starting values must be specified and the NS parameter must be set on the OU command.

The following ML estimates are obtained:

```
LISREL Estimates (Maximum Likelihood)

         LAMBDA-Y

               VAR 1      VAR 2      VAR 3
            --------   --------   --------
    VAR 1      2.579       - -        - -
             (0.037)
              70.315

    VAR 2       - -       3.032       - -
                        (0.041)
                         73.437

    VAR 3       - -        - -       2.688
                                   (0.038)
                                    71.167
```

PSI

Note: This matrix is diagonal.

VAR 1	VAR 2	VAR 3
1.601	1.601	1.601
(0.068)	(0.068)	(0.068)
23.388	23.388	23.388

The χ^2 goodness-of-fit measure is

Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	2
Maximum Likelihood Ratio Chi-Square (C1)	4.917 (P = 0.0856)

and the output file shows that no standardized residuals and modification indices are significant. It appears that this model fits the data well.

Modification Indices and Expected Change

Modification Indices for LAMBDA-Y

	VAR 1	VAR 2	VAR 3
VAR 1	1.648	4.482	0.122
VAR 2	4.397	0.993	0.352
VAR 3	2.590	2.829	4.872

Expected Change for LAMBDA-Y

	VAR 1	VAR 2	VAR 3
VAR 1	0.157	-0.197	0.033
VAR 2	-0.261	0.171	0.074
VAR 3	0.163	0.168	-0.293

Modification Indices for GAMMA

	KSI 1
VAR 1	1.648
VAR 2	0.993
VAR 3	4.871

Expected Change for GAMMA

	KSI 1
VAR 1	-0.181
VAR 2	-0.191
VAR 3	0.335

No Non-Zero Modification Indices for PHI

Modification Indices for PSI

	VAR 1	VAR 2	VAR 3
VAR 1	1.648		
VAR 2	4.863	0.993	
VAR 3	1.130	1.243	4.871

Expected Change for PSI

	VAR 1	VAR 2	VAR 3
VAR 1	0.136		
VAR 2	-0.177	0.133	
VAR 3	0.080	0.091	-0.246

Modification Indices for THETA-EPS

	VAR 1	VAR 2	VAR 3
VAR 1	1.648		
VAR 2	4.863	0.993	
VAR 3	1.130	1.243	4.871

Expected Change for THETA-EPS

	VAR 1	VAR 2	VAR 3
VAR 1	1.397		
VAR 2	-1.384	1.761	
VAR 3	0.554	0.743	-2.703

Maximum Modification Index is   4.87 for Element (3, 3) of LAMBDA-Y