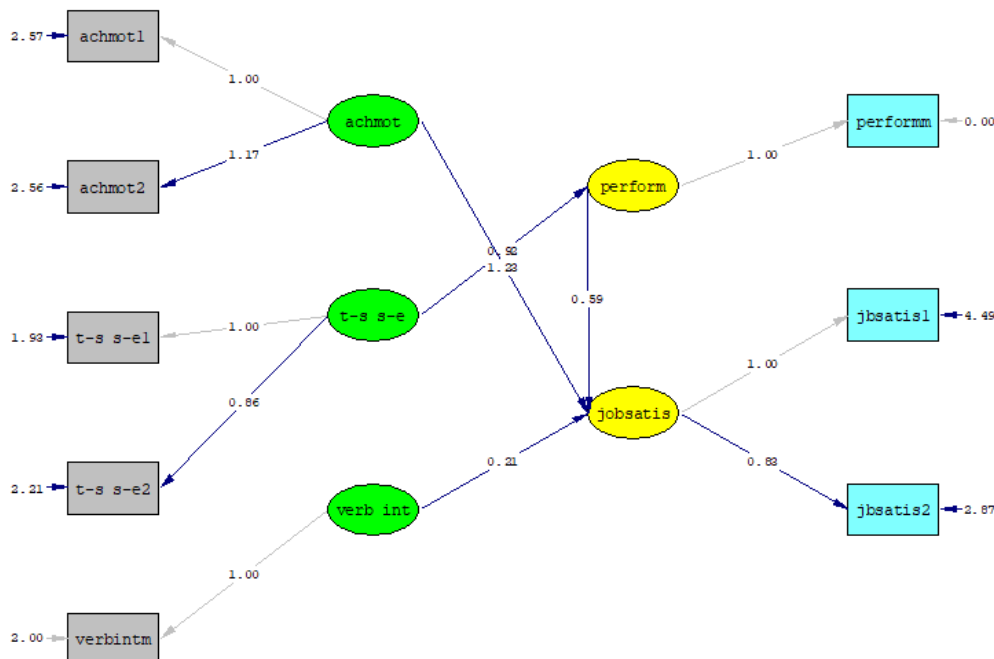


Path analysis with latent variables

For a study in an industrial sales force (Bagozzi (1980)), a structural equation model was designed specifically to answer such questions as “Is the relationship between performance and job satisfaction myth or reality? Does performance influence satisfaction, or does satisfaction influence performance?”

The variables included in the study are shown in the path diagram below.



Means, standard deviations, and product moment correlations of the observed variables based on a sample of $N = 122$ are given in the table below.

Table: Means, standard deviations and correlations for the observed variables in Bagozzi's model

Variable	y_1	y_2	y_3	x_1	x_2	x_3	x_4	x_5
PERFORMM	1.0000							
JBSATIS1	0.418	1.0000						
JBSATIS2	0.394	0.627	1.0000					
ACHMOT1	0.129	0.202	0.266	1.0000				
ACHMOT2	0.189	0.284	0.208	0.365	1.0000			
T-S S-E1	0.544	0.281	0.324	0.201	0.161	1.0000		
T-S S-E2	0.507	0.225	0.314	0.172	0.174	0.546	1.0000	
VERBINTM	-0.357	-0.156	-0.038	-0.199	-0.277	-0.294	-0.174	1.0000
Mean	720.86	15.54	18.46	14.90	14.35	19.57	24.16	21.36
Std. Dev.	2.09	3.43	2.81	1.95	2.06	2.16	2.06	3.65

The analysis of these data shown here differs slightly from that of the original source:

1. The performance measure y_1 is scaled to measure in hundreds of dollars rather than in dollars. Otherwise no scaling of the variables is used, and the analysis is based on the covariance matrix rather than the correlation matrix. When constraints are imposed on Θ_ϵ or Θ_δ , the normal theory standard errors in LISREL are valid only when the covariance matrix is analyzed.
2. Although no replicate measure is available for ξ_3 (verbal intelligence), this variable must be considered a fallible measure. A reliability of 0.85 is therefore assumed for x_3 on the grounds that a typical value of 0.85 is a better assumption than an arbitrary value of 1.00. The assumed value of the reliability will affect parameter estimates as well as standard errors. To specify a reliability 0.85 for x_3 , assign the fixed value 0.15 times the variance of x_3 to the element $\theta_{55}^{(\epsilon)}$. Then the variance is held fixed at $0.15 \times 3.65^2 = 1.998$.

The correlations between y_1 and y_2 and between y_1 and y_3 are 0.418 and 0.394, respectively,,,,, and are both very significant. Once a correlation between η_1 and η_2 has been established, four rival hypotheses may be considered to account for it, namely:

The latent variables η_1 and η_2 are correlated because

- H_1 : they have ξ_1 , ξ_2 , and ξ_3 as common causes (spurious correlation)
- H_{2a} : η_2 influences η_1
- H_{2b} : η_1 influences η_2
- H_3 : η_1 and η_2 influence each other reciprocally

H_1 is tested with $\mathbf{B} = \mathbf{0}$, $\Gamma(2 \times 3) = \text{FU, FR}$ and $\Psi(2 \times 2) = \text{SY, FR}$. This hypothesis is rejected because

$\hat{\psi}_{21}$ is significant. To test H_{2a} , set $\psi_{21} = 0$ and free β_{12} . To fix H_{2b} , fix ψ_{21} and free β_{21} . In both

models the freed **B** element is significant, so neither hypothesis can be rejected, nor can one be preferred over the other on statistical grounds.

The H_3 model can only be identified if one of the three ξ -variables in each equation is excluded. A good choice is the effect of ξ_1 on η_1 , least significant in H_{2a} , and the effect of ξ_2 on η_2 , least significant in H_{2b} . Then the following structural equations are estimated in the H_3 model.

$$\begin{aligned}\eta_1 &= -0.136(0.151)\eta_2 + 0.931(0.220)\xi_2 - 0.090(0.062)\xi_3 + \zeta_1 & R^2 &= 0.444 \\ \eta_2 &= 0.748(0.220)\eta_1 + 1.113(0.452)\xi_1 + 0.243(0.109)\xi_3 + \zeta_2 & R^2 &= 0.447\end{aligned}$$

This model has an overall χ^2 of 10.31, the same value as for the previous models, but it has one parameter less. The probability level of this χ^2 is 0.668.

In the first equation the effect of η_2 on η_1 is not significant, indicating the causal relationship between η_1 and η_2 is indeed one-way rather than reciprocal. Also, in the first equation, the effect of ξ_3 is not significant. In the second equation all included variables are significant. If we clean up the model by eliminating the direct paths which are insignificant, we obtain a model estimated as follows.

$$\begin{aligned}\eta_1 &= 0.923(0.140)\xi_2 + \zeta_1 & R^2 &= 0.533 \\ \eta_2 &= 0.594(0.140)\eta_1 + 1.228(0.477)\xi_1 + 0.213(0.107)\xi_3 + \zeta_2 & R^2 &= 0.478\end{aligned}$$

This model has a χ^2 of 14.19 with 15 degrees of freedom, which still represent on s a good fit. The sum of the two R^2 's, 1.011, is now higher than for all the previous models. The effect of ξ_3 on η_2 is barely significant but this effect is likely to be larger in a large sample.

A path diagram for the final model is the one shown at the beginning of this example. The specification for the model is:

$$\begin{aligned}\mathbf{LY} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & * \end{bmatrix} & \mathbf{TE} &= \begin{bmatrix} 0 \\ * \\ * \end{bmatrix} (diag) \\ \mathbf{LX} &= \begin{bmatrix} 1 & 0 & 0 \\ * & 0 & 0 \\ 0 & 1 & 0 \\ 0 & * & 0 \\ 0 & 0 & 1 \end{bmatrix} & \mathbf{TD} &= \begin{bmatrix} * \\ * \\ * \\ * \\ 1.998 \end{bmatrix} (diag) \\ \mathbf{BE} &= \begin{bmatrix} 0 & 0 \\ * & 0 \end{bmatrix} & \mathbf{GA} &= \begin{bmatrix} 0 & * & 0 \\ * & 0 & * \end{bmatrix} & \mathbf{PS} &= \begin{bmatrix} * \\ * \end{bmatrix} (diag),\end{aligned}$$

where * means a free parameter to be estimated and all other values are fixed. As before, the fixed ones in Λ_y and Λ_x set the scales for the η 's and the ξ 's.

The LISREL command file (**EX56.LIS** in the **LISREL Examples** folder) is:

Modified Model for Performance and Satisfaction

References

Bagozzi, R.P. Performance and satisfaction in an industrial sales force: An examination of their antecedents and simultaneity. *Journal of Marketing*, 1980, 44, 65-77

Joreskog, K.G. and Sorbom, D. Recent developments in structural equation modeling. *Journal of Marketing Research*, 1982, 19, 404-416.

Da ni=8 no=122 mi=-999999.0

La

(8A8)

performmjbsatis1jbsatis2 achmot1 achmot2t-s s-e1t-s s-e2verbintm

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sd file=EX56.DAT

mo ny=3 nx=5 ne=2 nk=3 be=fub ps=di

le

perform jobsatis

lk

achmot 't-s s-e' 'verb int'

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fi te 1 td 5 ga 1 1 ga 2 2 ga 1 3

va 1 ly 1 1 ly 2 2 lx 1 1 lx 3 2 lx 5 3; va 1.998 td 5

ou se tv rs ef mi ss ad=off

The command file illustrates the following features:

- Upper case or lower case characters may be used freely on all lines,
- Several title lines are allowed, as long as they do not begin with DA. Here the exclamation mark (!) is used to avoid that problem. It indicates that the line is a title or comment.
- Blank lines are allowed between title lines as well as between command lines.
- Labels are enclosed with single quotes if they contain blank spaces.