Fit statistics

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LISREL can compute two different sets of standard errors of parameter estimates and up to four different chi-squares for testing overall fit of the model. These new standard errors and chi-squares can be obtained for single group problems as well as multiple group problems and for covariance structure models in which only covariance matrices are analyzed, as well as for mean and covariance structure models, where both means and covariance matrices are analyzed.

Which set of standard errors and which chi-squares will be obtained depends on whether an asymptotic covariance matrix is provided or not and which method of estimation (ULS, GLS, ML, WLS, DWLS) is used to fit the model. The asymptotic covariance matrix is a consistent estimate of *N* times the asymptotic covariance matrix of the matrix being analyzed. This is computed by PRELIS and saved in a binary file which is read by LISREL, as shown in the examples here.

1. Standard Errors

If no asymptotic covariance matrix is provided, standard errors are estimated under multivariate normality. Otherwise, if an asymptotic covariance matrix is provided, standard errors are estimated under non-normality.

2. Chi-squares

The four different chi-squares are denoted C1, C2, C3, C4. Which ones are obtained in different situations is seen in the following table, where • means obtained and ◦ means not obtained (* means that C2 will be obtained only if the asymptotic variances are provided.)

Asymptotic Covariance Matrix not Provided

	ULS	GLS	ML	WLS	DWLS
C1	◦	•	•	◦	◦
C2	•	•	•	◦	*
C3	◦	◦	◦	◦	◦
C4	◦	◦	◦	◦	◦

Asymptotic Covariance Matrix Provided

	ULS	GLS	ML	WLS	DWLS
C1	◦	•	•	•	◦
C2	•	•	•	◦	•
C3	•	•	•	◦	•
C4	•	•	•	◦	•

- C1 is $N - 1$ times the minimum value of the fit function.
- C2 is $N - 1$ times the minimum of the WLS fit function using a weight matrix estimated under multivariate normality.
- C3 is the Satorra-Bentler scaled chi-square statistic (Satorra & Bentler, 1988, equation 4.1) or its generalization to mean and covariance structures and multiple groups.
- C4 is computed from equation (2.20a) in Browne (1984) using the asymptotic covariance matrix provided, and in more general cases, from equation (30) in Satorra (1993).

Under multivariate normality of the observed variables, C1 and C2, whenever provided, are asymptotically equivalent and have an asymptotic chisquare distribution if the model holds exactly and an asymptotic non-central chi-square distribution if the model holds approximately. The same holds for C4 under the more general assumption that the observed variables have a multivariate distribution with finite moments up to order four. C3 is a correction to C2 which makes C3 have the correct asymptotic mean even under non-normality. This correction is applied to C2, not to C1.

Even though C2 and C4 are correct asymptotic chi-squares under normality and non-normality, respectively, it does not mean that these are the “best” chi-squares in small and moderate samples. By Monte Carlo simulations, Hu, Bentler, & Kano (1992) found that, for a certain type and size of model, C3 performed better overall over a number of different sample sizes and degrees of non-normality. Further studies are needed to determine the relative advantages and disadvantages of these and other chi-square statistics in small and moderate samples, under different types and sizes of models, and under different distributions of the observed variables, see also Yuan & Bentler (1997).

3. LISREL implementation

If no asymptotic covariance matrix is provided and no method of estimation is specified, LISREL will use ML by default and estimate standard errors and C1 and C2 assuming that the observed variables are multivariate normal. If an asymptotic covariance matrix is provided and no method of estimation is specified, LISREL will use WLS (ADF) by default and estimate standard errors and C1, under non-normality.

In both cases, any other method of estimation than the default may be specified, in which case, the standard errors and chi-squares are estimated under non-normality (C3 and C4) or normality (C1 and C2), depending on whether an asymptotic covariance matrix is provided or not. For ULS and DWLS, C1 does not have an asymptotic chi-square distribution, so for these methods, C1 is not given.

In an input file in the SIMPLIS command language, the method of estimation may be specified as (here exemplified by ULS)

```
Method: Unweighted Least Squares
```

or by

```
Options: ULS or by LISREL Output: ULS
```

In an input file in the LISREL command language, simply write ME = ULS on the OU command.

An asymptotic covariance matrix may be provided by writing

```
Asymptotic Covariance Matrix from File [filename]
```

in a SIMPLIS command file, or by writing AC = *[filename]* in a LISREL command file.

4. Other Fit Statistics

In LISREL there are many fit statistics other than the chi-square and its associated degrees of freedom and *p*-value. Many of these depend on chi-square explicitly or implicitly, such as NCP and RMSEA and their confidence limits. As there are now several chi-squares available, a decision has been made to base these other fit statistics on C2 in case of normality and C3 in case of non-normality. This makes a difference compared to previous versions of LISREL, where C1 was used as a basis for computing the other fit statistics. The reason for choosing C2 and C3 is that these are available for all methods except WLS.

5. GF File

The GF file is a file containing all the fit statistics including all the lower and upper confidence limits. It is described in Jöreskog & Sörbom (1996a, pp. 194–195). The numbers in the GF file are given in the same order as in the output file. The GF file is useful only in simulation studies (Bootstrap or Monte Carlo) when one wants to study the distribution of these fit statistics over a number of replicates. The GF file is obtained by writing `Options: GF=[filename]` in a SIMPLIS command file or by writing GF=*[filename]* on the OU command in a LISREL command file.

There is a short version of the GF file listing only the degrees of freedom, the chi-square, and the *p*-value for chi-square. This will be obtained if the keyword XI is also specified.

The GF file contains four chi-squares and their corresponding *p*-values. These are given in the order C1, P1, C2, P2, C3, P3, C4, P4. Whenever a quantity has not been computed, a zero entry will appear in the GF file.

6. Example

Two examples are given, one single group example, where the model fits the data well and one multi-group example where the model does not fit the data. This gives some "feeling" for how close the different chi-squares can be in these different situations.

Holzinger & Swineford (1939) collected data on twenty-six psychological tests administered to seventh- and eighth-grade children in two schools in Chicago: the Pasteur School and the Grant–White School. Six of these tests are selected for this example. The six tests are (with the original variable number in parenthesis):

VIS PERC	Visual Perception (V1)
CUBES	Cubes (V2)
LOZENGES	Lozenges (V4)
PAR COMP	Paragraph Comprehension (V6)
SEN COMP	Sentence Completion (V7)
WORDMEAN	Word meaning (V9)

The raw data on these six variables, in free format, is in the file **SPV.RAW**, where the first 156 cases constitute the Pasteur school sample and the last 145 cases constitute the Grant-White school sample.

The following stacked PRELIS input file computes the mean vector (ME), covariance matrix (CM), and asymptotic covariance matrix (AC) for each school and saves these in files with suffices ME, CM, and ACC, respectively (**SPV.PRL**). All files for this example can be found in the **SIMPLIS Examples** folder.

```
Pasteur School
Data Ninputvars=6
Rawdata=SPV.RAW Rewind
Labels
'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
Continuous All
SCases Case < 157
Output Matrix=CMatrix ME=SPVPA.ME CM=SPVPA.CM AC=SPVPA.ACC

Grant-White School
Data Ninputvars=6
Rawdata=SPV.RAW
Labels
'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
Continuous All
SCases Case > 156
Output Matrix=CMatrix ME=SPVGW.ME CM=SPVGW.CM AC=SPVGW.ACC
```

The output file reveals some non-normality in both schools, in particular in variables LOZENGES and WORDMEAN:

```
Total Sample Size(N) =    156

 Univariate Summary Statistics for Continuous Variables

 Variable     Mean  St. Dev.  Skewness  Kurtosis  Minimum Freq.  Maximum Freq.
 --------     ----  --------  --------  --------  ------- -----  ------- -----
 VIS PERC   29.647     7.110    -0.378     0.736    4.000     1   45.000     2
    CUBES   23.936     4.921     0.695     0.255   14.000     1   37.000     3
 LOZENGES   19.897     9.311     0.156    -1.074    2.000     1   36.000     5
 PAR COMP    8.468     3.457     0.221    -0.052    0.000     1   18.000     1
```

SEN COMP	15.981	5.244	-0.132	-0.839	4.000	1	27.000	1
WORDMEAN	13.455	6.932	1.009	2.034	1.000	2	43.000	1

Test of Univariate Normality for Continuous Variables

	Skewness		Kurtosis		Skewness and Kurtosis	
Variable	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
VIS PERC	-1.934	0.053	1.682	0.093	6.568	0.037
CUBES	3.360	0.001	0.782	0.434	11.900	0.003
LOZENGES	0.818	0.414	-5.966	0.000	36.257	0.000
PAR COMP	1.153	0.249	0.018	0.985	1.329	0.514
SEN COMP	-0.691	0.489	-3.585	0.000	13.332	0.001
WORDMEAN	4.555	0.000	3.207	0.001	31.028	0.000

Relative Multivariate Kurtosis = 1.050

Test of Multivariate Normality for Continuous Variables

Skewness			Kurtosis			Skewness and Kurtosis	
Value	Z-Score	P-Value	Value	Z-Score	P-Value	Chi-Square	P-Value
5.141	5.403	0.000	50.416	1.779	0.075	32.357	0.000

Total Sample Size(N) = 145

Univariate Summary Statistics for Continuous Variables

Variable	Mean	St. Dev.	Skewness	Kurtosis	Minimum	Freq.	Maximum	Freq.
VIS PERC	29.579	6.914	-0.119	-0.046	11.000	1	51.000	1
CUBES	24.800	4.445	0.239	0.872	9.000	1	37.000	2
LOZENGES	15.966	8.317	0.623	-0.454	3.000	2	36.000	1
PAR COMP	9.952	3.375	0.405	0.252	1.000	1	19.000	1
SEN COMP	18.848	4.649	-0.550	0.221	4.000	1	28.000	1
WORDMEAN	17.283	7.947	0.729	0.233	2.000	1	41.000	1

Test of Univariate Normality for Continuous Variables

	Skewness		Kurtosis		Skewness and Kurtosis	
Variable	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
VIS PERC	-0.604	0.546	0.045	0.964	0.367	0.833
CUBES	1.202	0.229	1.843	0.065	4.842	0.089
LOZENGES	2.958	0.003	-1.320	0.187	10.491	0.005
PAR COMP	1.995	0.046	0.761	0.447	4.559	0.102
SEN COMP	-2.646	0.008	0.693	0.489	7.483	0.024
WORDMEAN	3.385	0.001	0.720	0.472	11.977	0.003

Relative Multivariate Kurtosis = 1.064

Test of Multivariate Normality for Continuous Variables

Skewness Kurtosis Skewness and Kurtosis

```
 Value  Z-Score P-Value     Value  Z-Score P-Value      Chi-Square P-Value
------  ------- -------   -------  ------- -------      ---------- -------
 3.974    3.190   0.001    51.058    2.041   0.041          14.339   0.001
```

Since the sample sizes are rather small, it may not be a good idea to use the WLS (ADF) method; it may be better to take non-normality into account by means of C3 or C4.

The following SIMPLIS input file (**SPV1.SPL**) estimates and tests a confirmatory factor analysis model with two correlated factors: Visual (Visual Perception) and Verbal (Verbal Ability), where the first three variables are indicators of Visual and the last three variables are indicators of Verbal. The model is estimated by maximum likelihood but standard errors are estimated under non-normality.

```
Estimating and Testing a Confirmatory Factor Analysis Model
on Grant-White School
Observed Variables: 'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
Covariance Matrix from File SPVGW.CM
! Asymptotic Covariance Matrix from File SPVGW.ACC
Sample Size: 145
Latent Variables: Visual Verbal
Relationships:
'VIS PERC' - LOZENGES Visual
'PAR COMP' - WORDMEAN Verbal
Method: Maximum Likelihood
Path Diagram
End of Problem
```

First run this model assuming normality, *i.e.*, without the line

```
Asymptotic Covariance Matrix from File SPVGW.ACC
```

This can be done by putting an exclamation sign ! in front of the word 'Asymptotic.'

The following solution is obtained:

```
LISREL Estimates (Maximum Likelihood)

         Measurement Equations

 VIS PERC = 4.369*Visual, Errorvar.= 28.709, R² = 0.399
 Standerr  (0.645)                  (4.788)
 Z-values   6.775                    5.996
 P-values   0.000                    0.000

    CUBES = 2.369*Visual, Errorvar.= 14.148, R² = 0.284
 Standerr  (0.414)                  (1.998)
 Z-values   5.715                    7.080
 P-values   0.000                    0.000

 LOZENGES = 6.087*Visual, Errorvar.= 32.116, R² = 0.536
 Standerr  (0.787)                  (7.347)
 Z-values   7.736                    4.371
 P-values   0.000                    0.000
```

```
 PAR COMP = 2.930*Verbal, Errorvar.= 2.810 , R² = 0.753
 Standerr  (0.237)                  (0.590)
 Z-values   12.350                   4.759
 P-values   0.000                    0.000

 SEN COMP = 3.834*Verbal, Errorvar.= 6.918 , R² = 0.680
 Standerr  (0.334)                  (1.176)
 Z-values   11.491                   5.880
 P-values   0.000                    0.000

 WORDMEAN = 6.583*Verbal, Errorvar.= 19.828, R² = 0.686
 Standerr  (0.569)                  (3.420)
 Z-values   11.563                   5.798
 P-values   0.000                    0.000


         Correlation Matrix of Independent Variables

              Visual     Verbal
            --------   --------
   Visual      1.000

   Verbal      0.533      1.000
             (0.085)
               6.252
```

This solution gives the chi-squares C1 and C2 as follows:

```
Goodness-of-Fit Statistics

 Degrees of Freedom for (C1)-(C2)                       8
 Maximum Likelihood Ratio Chi-Square (C1)               3.663 (P = 0.8862)
 Browne's (1984) ADF Chi-Square (C2_NT)                 3.729 (P = 0.8807)
```

Next run the problem under non-normality (**SPV2.SPL**), *i.e.*, including the line:

```
   Asymptotic Covariance Matrix from File SPVGW.ACC
```

This gives the following solution, where the standard errors have been estimated under non-normality:

```
LISREL Estimates (Robust Maximum Likelihood)

         Measurement Equations


 VIS PERC = 4.369*Visual, Errorvar.= 28.709, R² = 0.399
 Standerr  (0.721)                  (6.328)
 Z-values   6.056                    4.537
 P-values   0.000                    0.000

    CUBES = 2.369*Visual, Errorvar.= 14.148, R² = 0.284
 Standerr  (0.370)                  (2.273)
 Z-values   6.401                    6.224
 P-values   0.000                    0.000
```

```
 LOZENGES = 6.087*Visual, Errorvar.= 32.116, R² = 0.536
 Standerr  (0.795)                  (8.134)
 Z-values   7.659                    3.948
 P-values   0.000                    0.000

 PAR COMP = 2.930*Verbal, Errorvar.= 2.810 , R² = 0.753
 Standerr  (0.252)                  (0.595)
 Z-values   11.618                   4.719
 P-values   0.000                    0.000

 SEN COMP = 3.834*Verbal, Errorvar.= 6.918 , R² = 0.680
 Standerr  (0.333)                  (1.187)
 Z-values   11.510                   5.828
 P-values   0.000                    0.000

 WORDMEAN = 6.583*Verbal, Errorvar.= 19.828, R² = 0.686
 Standerr  (0.576)                  (3.739)
 Z-values   11.423                   5.304
 P-values   0.000                    0.000


        Correlation Matrix of Independent Variables

             Visual     Verbal
           --------   --------
   Visual     1.000

   Verbal     0.533      1.000
            (0.093)
              5.759
```

In this case, all four chi-squares C1, C2, C3, C4 are given in the output in that order:

```
Goodness-of-Fit Statistics

 Degrees of Freedom for (C1)-(C3),C(5)                8
 Maximum Likelihood Ratio Chi-Square (C1)             3.663 (P = 0.8862)
 Browne's (1984) ADF Chi-Square (C2_NT)               3.729 (P = 0.8807)
 Browne's (1984) ADF Chi-Square (C2_NNT)              4.149 (P = 0.8434)
 Satorra-Bentler (1988) Scaled Chi-Square (C3)        3.895 (P = 0.8665)
 Satorra-Bentler (1988) Adjusted Chi-Square (C4)      3.458 (P = 0.8473)
 Degrees of Freedom for C4                             7.103
 Chi-Square Scaled and Shifted (C5)                   4.132 (P = 0.8450)
 P-Value of C1 under Non-Normality                              = 0.8522

 Estimated Non-centrality Parameter (NCP)             0.0
 90 Percent Confidence Interval for NCP               (0.0 ; 2.456)

 Minimum Fit Function Value                           0.0253
 Population Discrepancy Function Value (F0)           0.0
 90 Percent Confidence Interval for F0                (0.0 ; 0.0169)
 Root Mean Square Error of Approximation (RMSEA)      0.0
 90 Percent Confidence Interval for RMSEA             (0.0 ; 0.0460)
 P-Value for Test of Close Fit (RMSEA < 0.05)         0.957
```

```
Expected Cross-Validation Index (ECVI)                    0.234
90 Percent Confidence Interval for ECVI                   (0.234 ; 0.251)
ECVI for Saturated Model                                  0.290
ECVI for Independence Model                               2.314

Chi-Square for Independence Model (15 df)               323.559

Normed Fit Index (NFI)                                    0.988
Non-Normed Fit Index (NNFI)                               1.025
Parsimony Normed Fit Index (PNFI)                         0.527
Comparative Fit Index (CFI)                               1.000
Incremental Fit Index (IFI)                               1.013
Relative Fit Index (RFI)                                  0.977

Critical N (CN)                                         743.819


Root Mean Square Residual (RMR)                           0.912
Standardized RMR                                          0.0271
Goodness of Fit Index (GFI)                               0.992
Adjusted Goodness of Fit Index (AGFI)                     0.978
Parsimony Goodness of Fit Index (PGFI)                    0.378
```

This may be compared with what was obtained under normality. Note that the parameter estimates are the same (they have been estimated by ML in both cases) but the standard errors and *t*-values are slightly different. This is the effect of non-normality. Also note, that the two chi-squares C1 and C2 are the same as in the previous case.

The following table shows the various chi-square values that are obtained with different methods, assuming the asymptotic covariance matrix is provided. The degrees of freedom is 8.

	ULS	GLS	ML	WLS	DWLS
C1	◦	3.43	3.64	4.13	◦
C2	3.70	3.87	3.70	◦	3.66
C3	4.03	3.97	3.94	◦	3.85
C4	4.13	4.12	4.12	◦	4.07

Continuing the example, the following input file (**SPV3.SPL**) estimates the same confirmatory factor analysis model simultaneously in both schools assuming that the intercepts and the factor loadings are the same in the two groups and estimates the mean difference in the latent variables between schools, see Sörbom (1974).

```
Group: Grant-White
Estimating Latent Mean Difference between Two Schools
Observed Variables: 'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
Means from File SPVGW.ME
Covariance Matrix from File SPVGW.CM
Asymptotic Covariance Matrix from File SPVGW.ACC
Sample Size: 145
Latent Variables: Visual Verbal
Relationships:
'VIS PERC' - LOZENGES = CONST Visual
```

```
'PAR COMP' - WORDMEAN = CONST Verbal
'VIS PERC' = 1*Visual
'PAR COMP' = 1*Verbal
Method: Maximum Likelihood

Group: Pasteur
Estimating Latent Mean Difference between Two Schools
Means from File SPVPA.ME
Covariance Matrix from File SPVPA.CM
Asymptotic Covariance Matrix from File SPVPA.ACC
Sample Size: 156
Relationships:
Visual - Verbal = CONST
Set the Variances of Visual - Verbal Free
Set the Covariance of Visual - Verbal Free
Set the Error Variances of 'VIS PERC' - WORDMEAN Free
Path Diagram
End of Problem
```

The corresponding chi-squares are now given in the output as:

```
   Global Goodness-of-Fit Statistics

 Degrees of Freedom for (C1)-(C3),C(5)                    24
 Maximum Likelihood Ratio Chi-Square (C1)                 63.599 (P = 0.0000)
 Browne's (1984) ADF Chi-Square (C2_NT)                   39.244 (P = 0.0257)
 Browne's (1984) ADF Chi-Square (C2_NNT)                  41.262 (P = 0.0156)
 Satorra-Bentler (1988) Scaled Chi-Square (C3)            24.505 (P = 0.4331)
 Satorra-Bentler (1988) Adjusted Chi-Square (C4)          30.184 (P = 0.0768)
 Degrees of Freedom for C4                                20.483
```

It is clear that this model does not fit the data. Further study of the output reveals that the intercept for LOZENGES is probably different in the two schools. Even though the model does not fit, the output suggests that Grant–White School students are ahead of the Pasteur School students in verbal ability. This agrees with the results found and interpretations made by Jöreskog & Sörbom (1996c, pp. 75–76).

The following table shows the various chi-square values that are obtained with different methods, assuming the asymptotic covariance matrices are provided. The degrees of freedom is 24.

	ULS	GLS	ML	WLS	DWLS
C1	◦	61.35	63.39	960.63	◦
C2	54.81	74.74	60.27	◦	58.87
C3	61.88	72.25	66.35	◦	61.48
C4	71.81	76.03	71.45	◦	67.30

The solutions for WLS and DWLS are non-admissible. This shows that using asymptotic variances and covariances estimated from a small sample can do more harm than good if used with WLS or DWLS. In such cases it is probably better to use any of the other methods and correct for non-normality with C3 or C4.