Scientific Software International

Multi-sample analyses using path diagrams

1   Introduction

In the previous sections we showed how LISREL models may be specified and estimated by drawing the appropriate path diagram from which the SIMPLIS or LISREL syntax is created. The examples used were based on data from a single sample. The path diagram can, however, also be used to create syntax for models based on data from several samples simultaneously, according to a multiple-group LISREL model with some or all parameters constrained to be equal over groups.

Consider a set of G mutually exclusive groups of individuals. It is assumed that a number of variables have been measured on a number of individuals from each of these populations. This approach is particularly useful in comparing a number of treatment and control groups regardless of whether individuals have been assigned to the groups randomly or not.

Any LISREL model may be specified and fitted for each group of data. However, LISREL assumes by default that the models are identical over groups, i.e., all relationships and all parameters are the same in each group. Thus, only differences between groups need to be specified.

2 Testing Equality of Factor Structures

Table 1 below gives sample covariance matrices for two samples ($N_1 = 865$, $N_2 = 900$, respectively) of candidates who took the Scholastic Aptitude Test in January 1971. The four measures are, in order, VERBAL40, a 40-item verbal aptitude section, VERBAL50, a separately timed 50-item verbal aptitude section, MATH35, a 35-item math aptitude section, and MATH25, a separately timed 25-item math aptitude section.

Table 1: Covariance Matrices for SAT Verbal and Math Sections

Covariance Matrix for Group 1

Tests	VERBAL40	VERBAL50	MATH35	MATH25
VERBAL40	63.382			
VERBAL50	70.984	110.237		
MATH35	41.710	52.747	60.584	
MATH25	30.218	37.489	36.392	32.295

Covariance Matrix for Group 2

	VERBAL40	VERBAL50	MATH35	MATH25
Tests				
VERBAL40	67.898			
VERBAL50	72.301	107.330		
MATH35	40.549	55.347	63.203	
MATH25	28.976	38.896	39.261	35.403

The data are used here to illustrate how one can test equality of factor loadings and factor correlations in a confirmatory factor analysis model, while allowing the error variances to be different.

We regard VERBAL40 and VERBAL50 as indicators of a latent variable Verbal and MATH35 and MATH25 as indicators of a latent variable Math. The model we consider is shown in the figure below.
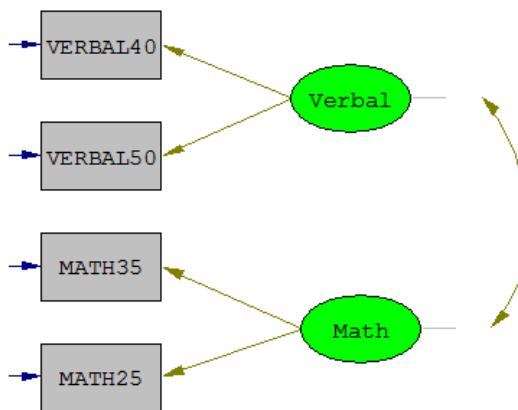
There are three sets of parameters in the model:

- o   the four factor loadings corresponding to the paths from Verbal and Math to the observed variables,
- o   the correlation between Verbal and Math, and
- o   the four error variances of the observed variables.

We begin by constructing a data file **pdex10.cov** containing the two covariance matrices. In free format, each covariance matrix can be written on one line.

pdex10.cov

```
63.382 70.984 110.237 41.710 52.747 60.584 30.218 37.489 36.392 32.295
67.898 72.301 107.330 40.549 55.347 63.203 28.976 38.896 39.261 35.403
```

We assume that all parameters are the same in both groups. The syntax file **pdex10a.spl** (see the **Simplis Examples** folder) is:

Group 1: Testing Equality Of Factor Structures Model A: Factor Loadings, Factor Correlation, Error Variances Invariant
Observed Variables: VERBAL40 VERBAL50 MATH35 MATH25
Covariance Matrix from File PDEX10.COV
Sample Size = 865
Latent Variables: Verbal Math
Relationships: VERBAL40 VERBAL50 = Verbal
MATH35 MATH25 = Math
Group 2: Testing Equality Of Factor Structures

Covariance Matrix from File PDEX10.COV
Sample Size = 900
Set the error variances of VERBAL40-MATH25 free
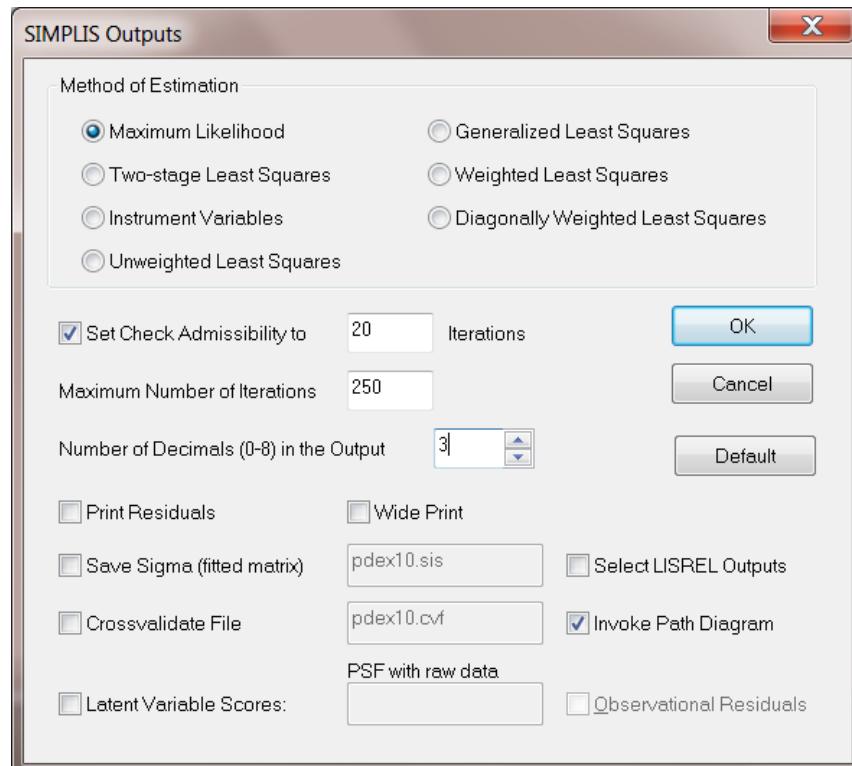End of Problem

In the syntax above, the data in the covariance matrices and the sample sizes are different. The names of the variables, observed as well as latent, are the same, and the model is the same.

Select the **New** option from the **File** menu and select **Path Diagram** in the **New** dialog box. Click **OK** when done. Enter the name **pdex10.pth**. When done, click **Save**.

It is possible to control specific SIMPLIS default features such as **Maximum Number of Iterations** or **Method of Estimation**. From the **Output** menu, select **SIMPLIS Outputs**.

| Output	Window	Help
SIMPLIS Outputs		
LISREL Outputs		▶
Fit Indices		Ctrl+F
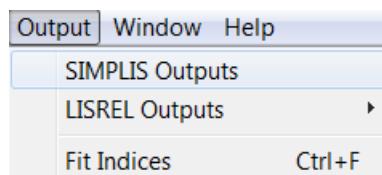
The **SIMPLIS Outputs** dialog box will appear. Customize this dialog box according to your preferences, for example, by changing the **Number of Decimals in the Output** option to 4 and by making sure that the **Invoke path diagram** option is selected. Click **OK** when done.

SIMPLIS Outputs

Method of Estimation
- (●) Maximum Likelihood
- ( ) Two-stage Least Squares
- ( ) Instrument Variables
- ( ) Unweighted Least Squares
- ( ) Generalized Least Squares
- ( ) Weighted Least Squares
- ( ) Diagonally Weighted Least Squares

[✔] Set Check Admissibility to   20   Iterations

Maximum Number of Iterations   250

Number of Decimals (0-8) in the Output   3

[ ] Print Residuals   [ ] Wide Print

[ ] Save Sigma (fitted matrix)   pdex10.sis   [ ] Select LISREL Outputs

[ ] Crossvalidate File   pdex10.cvf   [✔] Invoke Path Diagram

PSF with raw data

[ ] Latent Variable Scores: [ ] Observational Residuals

OK   Cancel   Default

Before drawing the path diagram, select **Toolbars** from the **View** menu and ensure that the items shown below are selected. These are: **Toolbar**, **Status Bar**, **Typebar**, **Variables**, **Drawing Bar**. Optionally, check the **Grid Lines** option.
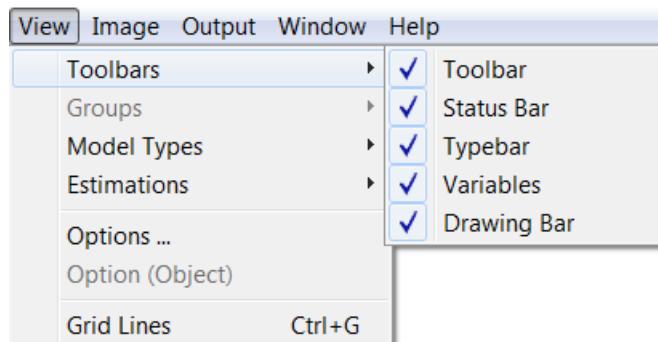
Select the **Title and Comments** option from the **Setup** menu to obtain the **Titles and Comments** dialog box.
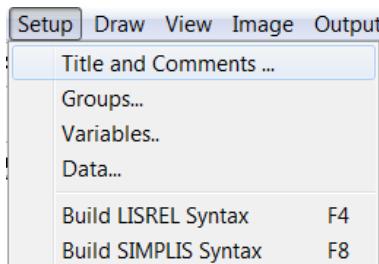

The first item on the **Setup** dialog box is the **Title** for the problem. Provision is also made for any additional **Comments** that the user may wish to enter. After typing in the title and (optionally) the comments, click **Next** to go to the **Group Names** dialog box.
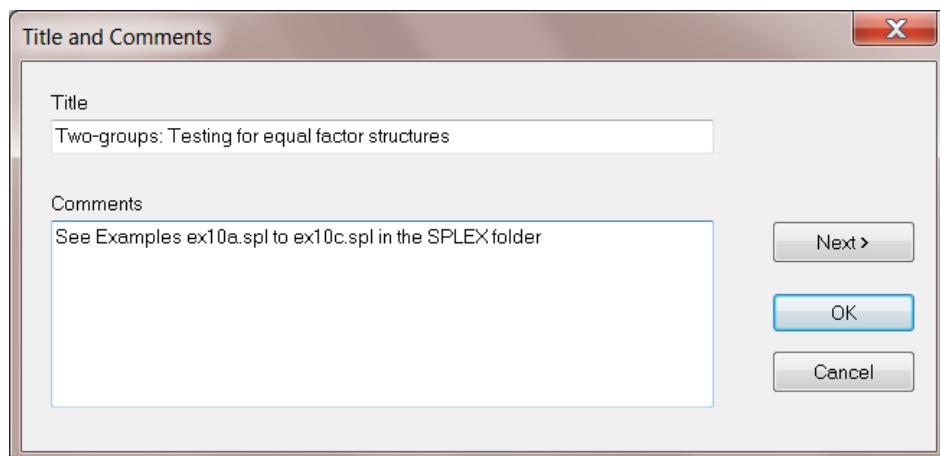

Next we enter a description for each group. Group labels can be inserted by clicking on the first string field, entering the label for the first group and then using the down arrow on the computer keyboard to create the next group's string field. When done, click **Next** to go to the **Labels** dialog box.
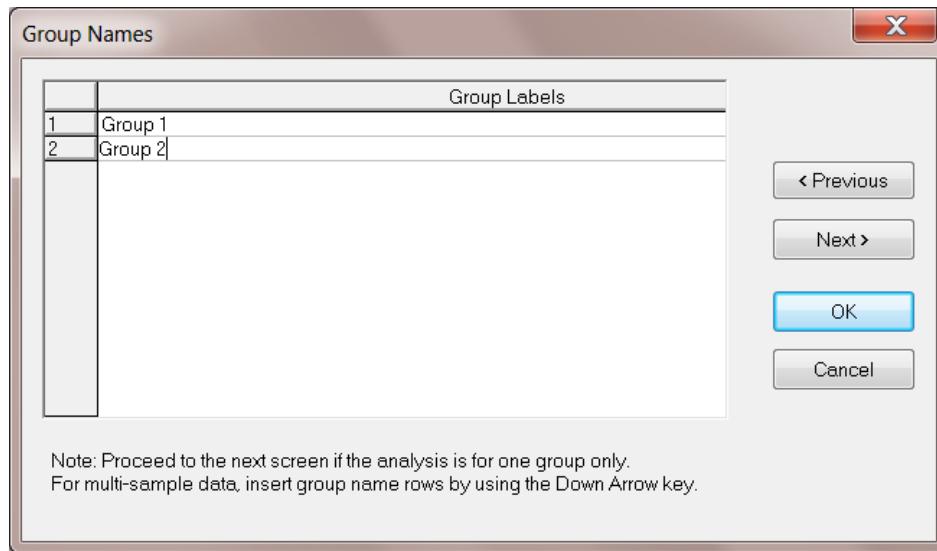
The default number of variables shown on the **Labels** dialog box is 3, these being CONST, VAR 1 and VAR 2. Move the mouse pointer to the **Observed Variables** box and click in the string field of CONST. Rename this variable to VERBAL40. Press the "down arrow" on the keyboard to move to the second observed variable string field and enter the label VERBAL50. Proceed in a similar way to enter the labels MATH35, MATH 25 and the labels Verbal and Math for the latent variables. Click **Next** when done to go to the **Data** dialog box.
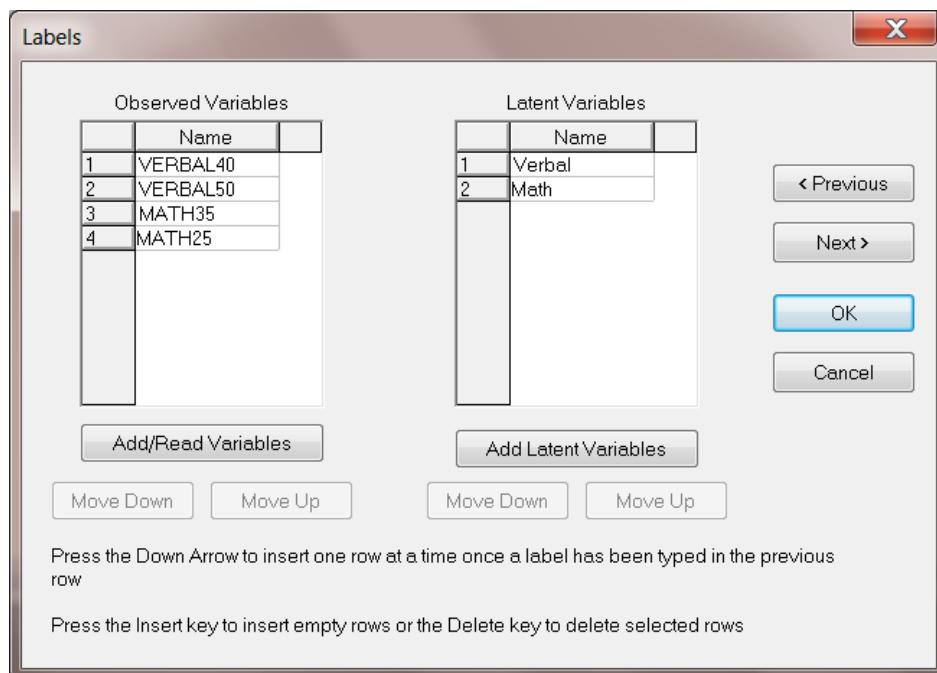
From the **Data** dialog box, select **Covariances** from the **Statistics from:** drop-down list box and also **Covariances** from the **Matrix to be analyzed** drop-down list box. Enter 865 in the **Number of Observations** string field. For **File type** select **External ASCII Data**.
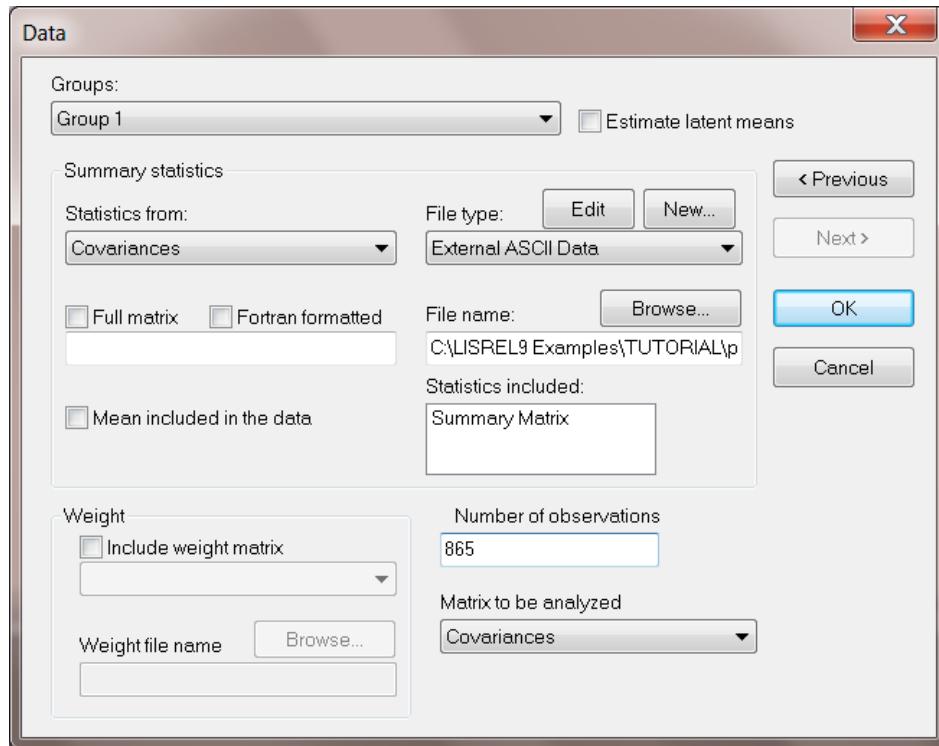
Click the **Browse** button to select the path and name of the file that contains the values of the covariance matrices of the 2 groups. For the present example, the file is **pdex10.cov**, which was created earlier in this section. Click **Open** when done.

From the **Data** dialog box, select the second group from the **Groups:** drop-down list box. Proceed as outlined above to enter the number of observations (900), file type and matrix to be analyzed.

3 Drawing the Path Diagram

We now proceed with the actual drawing of the path diagram. Start by clicking on the VERBAL40 label under the **Observed** variables portion of the **Variables** window. Hold the mouse button down and drag the label to the draw area. A rectangular-shaped object will appear on this part of the screen when the mouse button is released as shown below. Repeat this procedure until all observed variables are dragged to the drawing area as shown in the next image.

Note also that:

- The observed variables are assumed to be X (or independent) variables unless appropriate check boxes under the **Y**-column are checked.
- The latent variables are assumed to be Ksi (or independent) variables unless appropriate check boxes under the **Eta**-column are checked.

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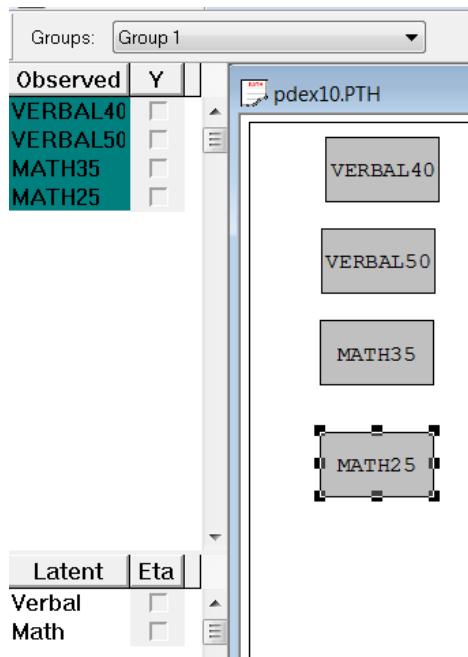
Proceed by dragging the latent variables, Verbal and Math, to the draw area of the path window. Note that the latent variables are represented by elliptically shaped objects.

Finally, arrows can be drawn pointing from the latent variables to the observed variables. To accomplish this, click on the single-headed arrow of the **Draw** toolbar (seen on the right of the picture given below). As an alternative, use the **Draw** menu on the main menu bar. The related drawing tool options may be selected from this menu.

Once the single-headed arrow (or **One-way Path** option) has been selected, move the mouse pointer to within one of the elliptically shaped objects. With the left mouse button held down, "drag" the arrow to within a rectangular-shaped object. When the colors of both objects change (see diagram below) release the mouse button.
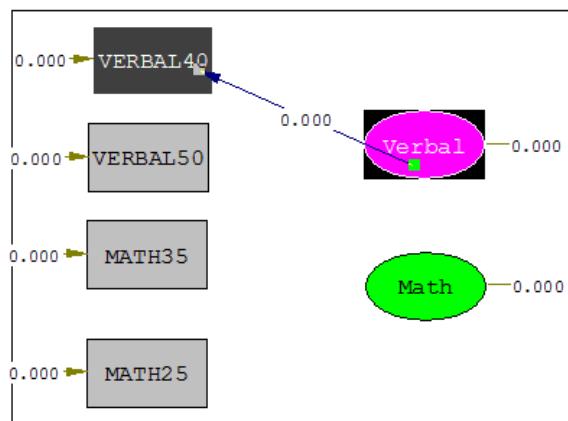
Proceed in a similar way to graphically display the relationships between the observed and latent variables:
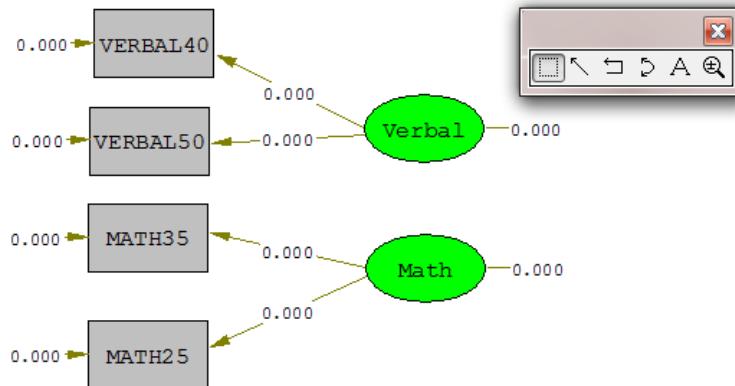
Select **Build SIMPLIS Syntax** from the **Setup** menu. SIMPLIS syntax will be built from the path diagram and is stored in a system file named **pdex10.spj** shown below.


Group 1: Testing Equality Of Factor Structures Model A: Factor Loadings, Factor
SYSTEM FILE from file 'C:\LISREL Examples\SIMPLIS examples\pdex10a.DSF'
Sample Size = 865
Latent Variables Verbal Math
Relationships
VERBAL40 = Verbal
VERBAL50 = Verbal
MATH35 = Math
MATH25 = Math
Path Diagram

Group 2: Testing Equality Of Factor Structures
SYSTEM FILE from file 'C:\LISREL Examples\SIMPLIS examples\pdex10a.DSF'
Sample Size = 900
Latent Variables Verbal Math
Relationships
Set the Error Variance of VERBAL40 Free
Set the Error Variance of VERBAL50 Free
Set the Error Variance of MATH35 Free
Set the Error Variance of MATH25 Free

Click the **Run LISREL** icon button to fit the multi-sample model to the data. The parameter estimates and χ^2 goodness-of-fit index are shown on the path diagram displayed below:

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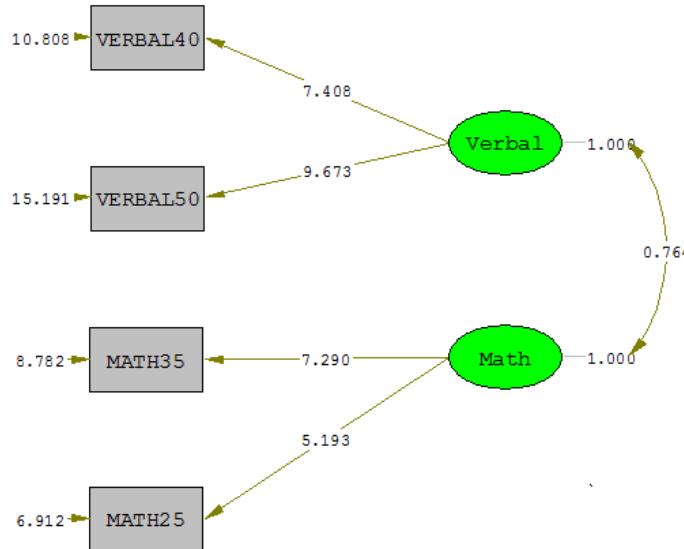
Chi-Square=33.78, df=11, P-value=0.00039, RMSEA=0.048

The values shown on the path diagram are the parameter estimates. This is the default selection. It is, however, also possible to view the t-values, modification indices or expected changes by selecting the **View** menu on the main menu bar. Alternatively, use the drop-down list by clicking in the **Estimates:** pane as shown below. To view t-values, select the **T-values** option. The figure below shows how this is done and also indicates the t-values on the path diagram.
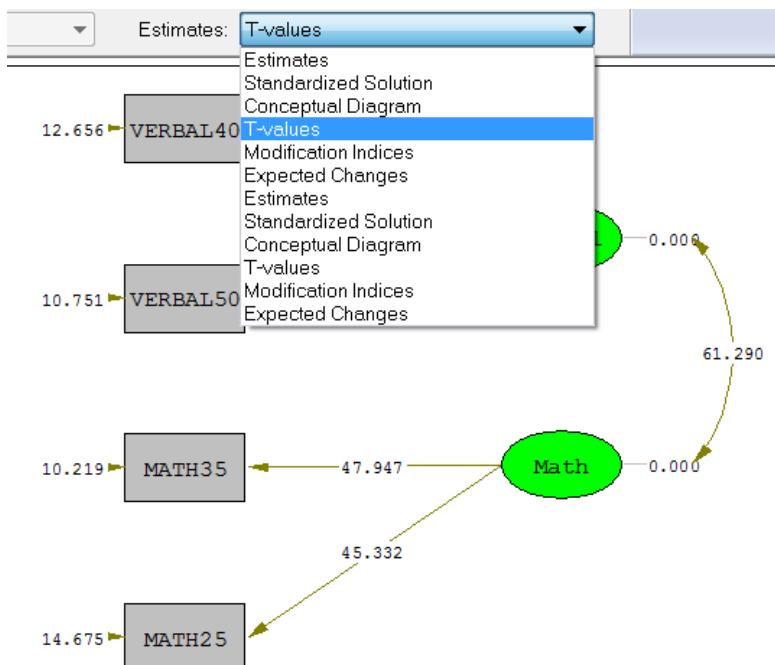
To view the corresponding t-values for the second group, select group 2 on the menu bar. The resultant changes in the path diagram are shown in the next image.

In the example above, the error variances, factor loadings and factor covariance are constrained to be equal (invariant) across the two groups. To test for invariance of factor loadings and covariance only, the error variances can be freed as follows. Make sure that the path diagram is displayed, and then select Group 2 from the **Groups:** text box. Right-click in the error variance of VERBAL40 and click the **Free** option. Repeat this procedure for the remaining three error variances, and then select the **Setup, Build SIMPLIS Syntax** option.
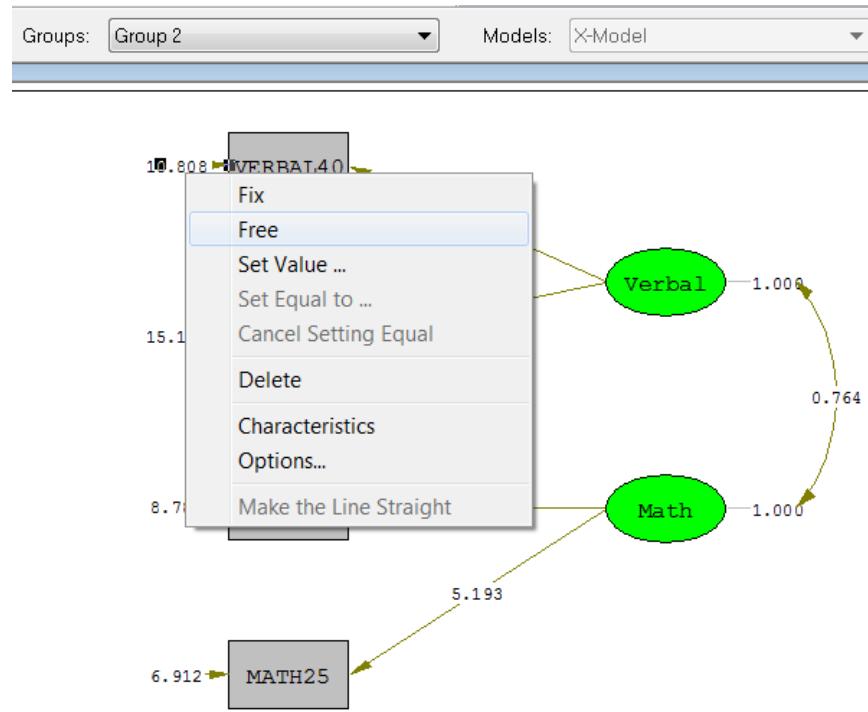
The actions described above result in a new SIMPLIS project file that contains the syntax required to free the error variances across groups.

When done, click the **Run LISREL** icon. The output file reveals two solutions, one for each group. The value of χ^2 is reported only after the second group.

In a multi-sample analysis, the χ^2 is a measure of fit of all models in all groups, and, in general, this χ^2 cannot be decomposed into a χ^2 for each group separately. For our example, χ^2 is 10.87 with 7 degrees of freedom (p = 0.14), so that the model does fit the data.