



Estimation of means of latent variables: nine psychological variables with factor means

As a third example of estimating factor means, we consider a large example involving nine-observed variables, three latent variables and four groups. Altogether, this example involves 57 parameters to be estimated.

Sörbom (1974) used nine selected variables from the classical study of Holzinger & Swineford (1939) to illustrate his methodology. The nine variables were selected to measure three latent variables: Space, Verbal and Memory. The groups consist of eighth-grade children from two schools in Chicago: The Pasteur and Grant-White schools. The children from each school were divided into two groups according to whether they scored above or below the median on a speeded addition test. Thus the groups are:

1. Pasteur Low ($N_1 = 77$)
2. Pasteur High ($N_2 = 79$)
3. Grant-White Low ($N_3 = 74$)
4. Grant- White High ($N_4 = 71$)

The variables, correlations, standard deviations, and means are given in the tables below. The standard deviations have been scaled so that a weighted average of the within group covariance matrices is a correlation matrix.

Table 1: Nine psychological variables - correlations*Group 1 above main diagonal; Group2 below main diagonal*

	1	2	3	4	5	6	7	8	9
Visual Perception	-	.32	.48	.28	.26	.40	.42	.12	.23
Cubes	.24	-	.33	.01	.01	.26	.32	.05	-.04
Paper Form Board	.23	.22	-	.06	.01	.10	.22	.03	.01
General Information	.32	.05	.23	-	.75	.60	.15	-.08	-.05
Sentence Completion	.35	.23	.18	.68	-	.63	.07	.06	.10
Word Classification	.36	.10	.11	.59	.66	-	.36	.19	.24
Figure Recognition	.22	.01	-.07	.09	.11	.12	-	.29	.19
Object-Number	-.02	.01	-.13	.05	.08	.03	.19	-	.38
Number-Figure	.09	-.14	-.06	.16	.02	.12	.15	.29	-

Group 3 above main diagonal; Group 4 below main diagonal

	1	2	3	4	5	6	7	8	9
Visual Perception	-	.34	.41	.38	.40	.42	.35	.16	.35
Cubes	.32	-	.21	.32	.16	.13	.27	.01	.27
Paper Form Board	.34	.18	-	.31	.24	.35	.30	.09	.09
General Information	.31	.24	.31	-	.69	.55	.17	.31	.34
Sentence Completion	.22	.16	.29	.62	-	.65	.20	.30	.27
Word Classification	.27	.20	.32	.57	.61	-	.31	.34	.278
Figure Recognition	.48	.31	.32	.18	.20	.29	-	.31	.38
Object-Number	.20	.01	.15	.06	.19	.15	.36	-	.38
Number-Figure	.42	.28	.40	.40	.07	.18	.35	.44	-

Table 2: Nine psychological variables - means and standard deviations

	<i>Standard deviations</i>				<i>Means</i>			
	1	2	3	4	1	2	3	4
Visual Perception	1.06	0.96	0.95	1.03	4.20	4.30	4.29	4.21
Cubes	1.20	0.86	1.03	0.86	5.25	5.03	5.32	5.33
Paper Form Board	1.02	0.99	0.92	1.06	4.96	5.06	5.02	5.09
General Information	1.03	0.96	0.99	1.01	2.98	3.41	3.72	4.15
Sentence Completion	1.08	1.06	0.96	0.91	3.20	3.38	3.78	3.88
Word Classification	0.99	1.01	0.95	1.05	4.45	4.76	5.17	5.59
Figure Recognition	1.17	1.01	0.81	0.98	13.42	13.62	13.70	13.72
Object-Number	1.00	1.10	0.83	1.04	1.74	2.14	1.30	1.78
Number-Figure	1.04	1.00	0.88	1.07	2.10	2.16	1.87	2.44

The nine labels are stored in the file **EX14.LAB**. Note that the labels must be enclosed within single quotes if they contain blank spaces.

'VIS PERC' CUBES 'PAP FORM' 'GEN INGO' 'SENT COM' WORDCLAS 'FIG REC' 'OBJ NUM'
'NUM FIG'

All the remaining data are stored in the file **EX14.DAT** in the order: correlations for group 1, standard deviations for group 1, means for group 1, correlations for group 2, standard deviations for group 2, etc. In this data file, we use FORTRAN formats to record the data. Each group of data begins with a format line. After the last right parenthesis in the format, one can write any text, e.g., to identify the particular part of the data for which the format is intended. After the format the data follows. The file **EX14.DAT** looks like this:

```
(21F3.2/24F3.2) Group 1: Pasteur Low: Correlations
100 32100 48 33100 28 01 06100 26 01 01 75100 40 26 10 60 63100
 42 32 22 15 07 36100 12 05 03-08 06 19 29100 23-04 01-05 10 24 19 38100
(9F3.2) Group 1: Pasteur Low: Standard deviations
106120102103108 99117100104
(9F4.2) Group 1: Pasteur Low: Means
 420 525 496 298 320 4451342 174 210
(21F3.2/24F3.2) Group 2: Pasteur High: Correlations
100 24100 23 22100 32 05 23100 35 23 18 68100 36 10 11 59 66100
 22 01-07 09 11 12100-02-01-13 05 08 03 19100 09-14-06 16 02 12 15 29100
(9F3.2) Group 2: Pasteur High: Standard deviations
 96 86 99 96106101101110100
(9F4.2) Group 2: Pasteur High: Means
 430 503 506 341 338 4761362 214 216
(21F3.2/24F3.2) Group 3: Grant-White Low: Correlations
100 34100 41 21100 38 32 31100 40 16 24 69100 42 13 35 55 65100
 35 27 30 17 20 31100 16 01 09 31 30 34 31100 35 27 09 34 27 27 38 38100
(9F3.2) Group 3: Grant-White Low: Standard deviations
 95103 92 99 96 95 81 83 88
(9F4.2) Group 3: Grant-White Low: Means
 429 532 502 372 378 5171370 130 187
(21F3.2/24F3.2) Group 4: Grant-White High: Correlations
100 32100 34 18100 31 24 31100 22 16 29 62100 27 20 32 57 61100
 48 31 32 18 20 29100 20 01 15 06 19 15 36100 42 28 40 11 07 18 35 44100
(9F3.2) Group 4: Grant-White High: Standard deviations
103 86106101 91105 98104107
(9F4.2) Group 4: Grant-White High: Means
 421 533 509 415 388 5591372 178 244
```

The model is similar to the fifth example in this set, but in addition to factor loadings, factor variances and covariances, and error variances, we are now also estimating intercept terms in the measurement relationships and means of the factors. The intercept terms, the loadings, and the error variances are assumed to be invariant over groups. The observed variables VIS PERC, GEN INFO, and FIG REC are used as reference variables for the latent variables. Consequently, their loadings are fixed at 1. The means of the latent variables are assumed to be zero in group 1 (Pasteur Low). The input file is **EX14.SPL**. These files can all be found in the **Simplis Examples** folder.

```
Group Pasteur Low
Observed Variables from File EX14.LAB
Sample size 77
Covariance Matrix from File EX14.DAT
Standard Deviations from File EX14.DAT
Means from File EX14.DAT
```

Latent Variables: Space Verbal Memory

Relationships:

'VIS PERC' - 'PAP FORM' = CONST Space
'GEN INFO' - 'WORDCLAS' = CONST Verbal
'FIG REC' - 'NUM FIG' = CONST Memory
'VIS PERC' = 1*Space
'GEN INFO' = 1*Verbal
'FIG REC' = 1*Memory

Group Pasteur High

Sample Size 79

Covariance Matrix from File EX14.DAT

Standard Deviations from File EX14.DAT

Means from File EX14.DAT

Relationships:

Space Verbal Memory = CONST

Set the Variances of Space - Memory Free

Set the Covariances of Space - Memory Free

Group Grant-White Low

Sample Size 74

Covariance Matrix from File EX14.DAT

Standard Deviations from File EX14.DAT

means from File EX14.DAT

Relationships:

Space Verbal Memory = CONST

Set the Variances of Space - Memory Free

Set the Covariances of Space - Memory Free

Group Grant-White High

Sample Size 71

Covariance Matrix from File EX14.DAT

Standard Deviations from File EX14.DAT

Means from File EX14.DAT

Relationships:

Space Verbal Memory = CONST

Set the Variances of Space - Memory Free

Set the Covariances of Space - Memory Free

Path Diagram

End of Problem

The maximum likelihood solution is shown in the table below. Scaled factor means have been computed such that the weighted mean (weighted by sample size) over the groups is zero for each factor.

Table 3: maximum likelihood estimates for nine psychological variables with factor means

Factor loadings

Test	Space	Verbal	Memory	Error Variance	Intercept
Visual Perception	1.00	0	0	0.48	4.20
Cubes	0.58	0	0	0.82	5.20
Paper Form Board	0.71	0	0	0.73	5.00
General Information	0	1.00	0	0.34	3.05
Sentence Completion	0	0.94	0	0.34	3.08
Word Classification	0	0.95	0	0.43	4.50
Figure Recognition	0	0	1.00	0.78	13.55
Object-Number	0	0	1.18	0.76	1.68
Number-Figure	0	0	1.31	0.63	2.06

Factor covariance matrices

School	Low level			High level		
Pasteur	0.71			0.37		
	0.26	0.76		0.30	0.70	
	0.20	0.09	0.25	0.01	0.08	0.20
Grant-White	0.46			0.53		
	0.38	0.63		0.32	0.63	
	0.19	0.20	0.13	0.33	0.13	0.29

Factor means

Scaled factor means

	Space	Verbal	Memory	Space	Verbal	Memory
Pasteur Low	0.00	0.00	0.00	-0.05	-0.49	-0.06
Pasteur High	-0.05	0.32	0.17	0.00	-0.17	0.12
Grant-White Low	0.09	0.70	-0.14	0.04	0.21	-0.20
Grant-White High	0.06	1.02	0.20	0.01	0.52	0.13

Spatial ability does not differentiate between the groups. For verbal ability, there is a difference between schools, with students from Grant-White being superior. This reflects the fact that the Pasteur school enrolls children of factory workers, a large percentage of whom were foreign-born and the Grant-White schools enrolls children in a middle-class suburban area (Meredith, 1964). With regard to the memory factor, both groups of the Pasteur school seem superior to the groups of the Grant-White school, although the difference between the high groups is small.