



LISREL output

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1. Introduction

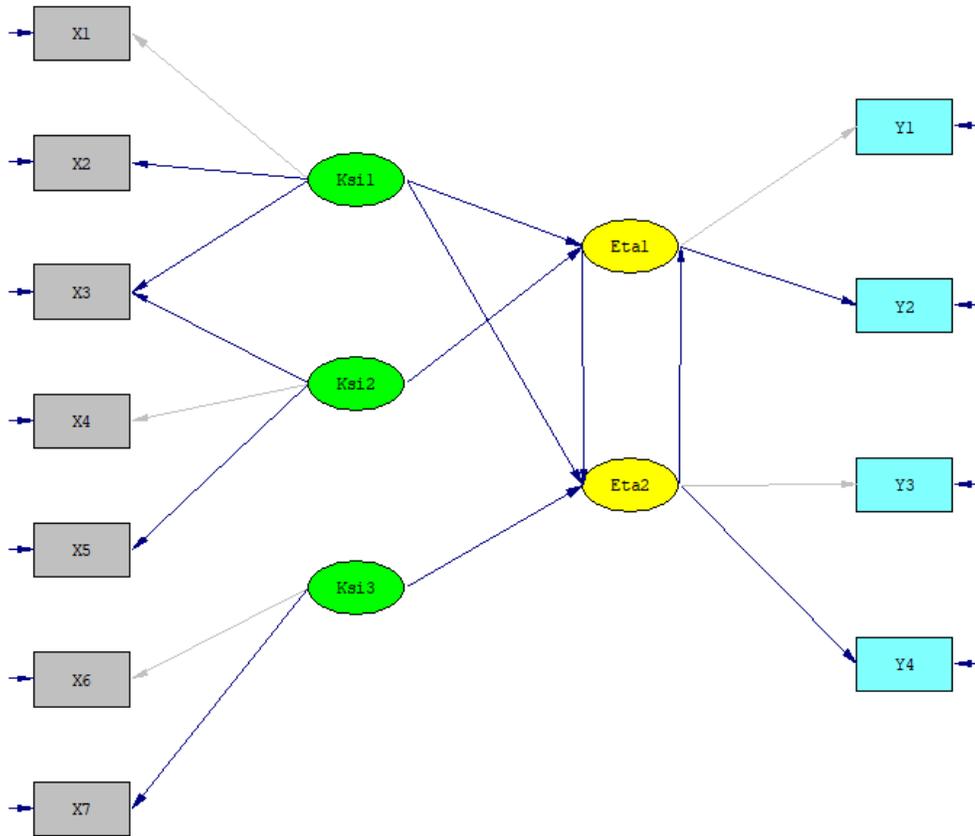
An important reason for considering output in LISREL format is that it is possible to obtain additional information in the output file that is not available in the SIMPLIS output. To obtain the output in LISREL format, include the line

```
LISREL Output
```

in the input file. The different kinds of optional output one can ask for are specified by two-character keywords on the same line, for example,

```
LISREL Output: RS MI SS SC EF
```

To illustrate the LISREL output and the various options available, we will use the hypothetical model shown below. This model includes most of the elements of LISREL modeling.



Since this is a hypothetical model, we use generic names Y1, Y2, Y3, Y4, X1, X2, ..., X7, to represent the observed variables and Eta1, Eta2, Ksi1, Ksi2, Ksi3, to represent the latent variables. If this were a real model, these names would be replaced by names associated with the content of the variables. It should be emphasized that in choosing the names of the variables, we have already classified each group of variables. However, this is not necessary; any names could have been chosen. The classification of the variables is performed by the program. A random sample of 100 observations of the observed variables has been generated artificially and the sample covariance matrix has been saved in the file **EX17.COV** in the **Simplis Examples** folder. The SIMPLIS input file for the hypothetical model is **EX17A.SPL**:

```
Hypothetical Model
Observed Variables: Y1-Y4 X1-X7
Covariance Matrix from file ex17.cov
```

```
Sample Size: 100
```

Latent Variables:

Eta1 Eta2 Ksi1-Ksi3

Relationships

Eta1 = Eta2 Ksi1 Ksi2

Eta2 = Eta1 Ksi1 Ksi3

Let the Errors of Eta1 and Eta2 Correlate

Y1 = 1*Eta1

Y2 = Eta1

Y3 = 1*Eta2

Y4 = Eta2

X1 = 1*Ksi1

X2 X3 = Ksi1

X4 = 1*Ksi2

X3 X5 = Ksi2

X6 = 1*Ksi3

X7 = Ksi3

LISREL Output: RS MI SC EF WP

Path Diagram

End of Problem

Note the following:

- On the line

Observed Variables: Y1-Y4 X1-X7

the observed variables Y1, Y2, Y3, Y4 can be defined as Y1-Y4. Similarly, for X1- X7.

- In this example we assume that the observed variables are measured in some well-defined units of measurements that we want to retain in the analysis. Consequently, we analyze the covariance matrix of the observed variables rather than the correlation matrix. Furthermore, by using observed variables as reference variables for the latent variables, these will also have units of measurements that are interpretable. The reference variables for the latent variables are defined in the lines containing the 1*. Thus, in the solution for this model, neither the observed nor the latent variables are standardized. However, the keyword SC on the LISREL Output line will produce a standardized solution as a by-product.

2. Classification of variables

LISREL classifies the observed and latent variables as follows:

- Eta (η) - variables: Eta1, Eta2.

These are the dependent latent variables. All latent variables appearing on the left side of the equal sign in the relationships are Eta-variables. In the path diagram, they are recognized as those variables in circles or ellipses that have one-way (unidirectional) arrows pointing to them.

- Ksi (ξ) -variables: Ksi1, Ksi2, Ksi3.

These are the remaining latent variables in the model. In the path diagram they are recognized as those variables in circles or ellipses that do not have one-way (unidirectional) arrows pointing to them.

- Y-variables: Y1, Y2, Y3, Y4.

These are the observed variables which depend on Eta-variables.

- X-variables: X1, X2, X3, X4, X5, X6, X7

These are the observed variables which depend on Ksi-variables.

- Zeta (ζ) -variables: Zeta1, Zeta2.

These are the error terms in the structural equations, i.e., the error terms on Eta1 and Eta2.

- Epsilon (ε) -variables: Epsilon1, Epsilon2, Epsilon3, Epsilon4.

These are the measurement errors in the Y-variables. In the path diagram they are represented by one-way (unidirectional) arrows on the right side.

- Delta (δ) -variables: Delta1, Delta2, Delta3, Delta4, Delta5, Delta6, Delta7.

These are the measurement errors in the X-variables. In the path diagram they are represented by one-way (unidirectional) arrows on the left side.

3. Parameter matrices

Every one-way (unidirectional) arrow in the path diagram represents a parameter or coefficient. Depending on where the arrow is coming from or going to, these parameters have different names which correspond to Greek characters. In the following the mathematical Greek notation is given in parenthesis.

A path from an Eta-variable to another Eta -variables is called a BETA (β)-parameter, a path from a Ksi-variable to an Eta -variable is called a GAMMA (γ) -parameter, a path from an Eta -variable to a Y-variable is called a LAMBDA-Y ($\lambda^{(y)}$) -parameter, or LY-parameter, for short, and a path from a Ksi -variable to a X-variable is called a LAMBDA-X ($\lambda^{(x)}$) -parameter, or LX-parameter, for short.

Each parameter has two subscripts, the first being the index of the variable *to* which the path is going and the second being the index of the variable *from* which the path is coming. Thus, BETA(2,1) (β_{21}) is the parameter associated with the path from Eta1 (η_1) to Eta2 (η_2) and GAMMA(2,1) (γ_{21}) is the parameter associated with the path from Ksi1 (ξ_1) to Eta2 (η_2). In general, BETA(I,J) (β_{ij}) corresponds to the path from Eta-j (η_j) to Eta-i (η_i), and GAMMA(K,L) (λ_{kl}) corresponds to the path from Ksi-l (ξ_l) to Eta-k (η_k). Similarly, LY(2,1) ($\lambda_{21}^{(y)}$) represents the path from Eta1 (η_1) to Y2 and LX(3,1) ($\lambda_{31}^{(x)}$) represents the path from Ksi1 (ξ_1) to X3. In general, LY(I,J) ($\lambda_{ij}^{(y)}$) represents the path from Eta-j (η_j) to Y-i and LX(K,L) ($\lambda_{kl}^{(x)}$) represents the path from Ksi-l (ξ_l) to X-k.

The BETA parameters may be collected in a matrix:

$$\text{BETA} = \begin{bmatrix} 0 & \text{BETA}(1,2) \\ \text{BETA}(2,1) & 0 \end{bmatrix}$$

Similarly, the GAMMA parameters may be collected in a matrix

$$\text{GAMMA} = \begin{bmatrix} \text{GAMMA}(1,1) & \text{GAMMA}(1,2) & 0 \\ \text{GAMMA}(2,1) & 0 & \text{GAMMA}(2,3) \end{bmatrix}$$

The LY and LX parameters may also be collected in matrices

$$\text{LAMBDA - Y} = \begin{bmatrix} 1 & 0 \\ \text{LY}(2,1) & 0 \\ 0 & 1 \\ 0 & \text{LY}(4,2) \end{bmatrix}$$

$$\text{LAMBDA - X} = \begin{bmatrix} 1 & 0 & 0 \\ \text{LX}(2,1) & 0 & 0 \\ \text{LX}(3,1) & \text{LX}(3,2) & 0 \\ 0 & 1 & 0 \\ 0 & \text{LX}(5,2) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \text{LX}(7,3) \end{bmatrix}$$

Zero elements in these matrices correspond to non-existent paths in the path diagram. For example, since Eta1 (η_1) does not depend on Ksi3 (ξ_3), element GAMMA(1,3) (γ_{13}) should be zero. The ones in the matrices correspond to fixed ones according to the specification of the relationships in the input file. Note that the two indices i and j of a matrix element, which were previously defined to be the index of the “to-variable” and the “from-variable” respectively, now correspond to the row and column where the element is located within the matrix.

Using the Greek notation system, these matrices are written:

$$B = \begin{bmatrix} 0 & \beta_{12} \\ \beta_{21} & 0 \end{bmatrix} \quad \Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ \gamma_{21} & 0 & \gamma_{23} \end{bmatrix}$$

$$\Lambda_y = \begin{bmatrix} 1 & 0 \\ \lambda_{21}^{(y)} & 0 \\ 0 & 1 \\ 0 & \lambda_{42}^{(y)} \end{bmatrix} \quad \Lambda_x = \begin{bmatrix} 1 & 0 & 0 \\ \lambda_{21}^{(x)} & 0 & 0 \\ \lambda_{31}^{(x)} & \lambda_{32}^{(x)} & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_{52}^{(x)} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \lambda_{73}^{(x)} \end{bmatrix}$$

There are five additional parameter matrices, namely the covariance matrices of the Ksi-, Zeta-, Epsilon-, and Delta-variables and the covariance matrix between Delta- and Epsilon variables. These are called PHI (Φ), PSI (Ψ), THETA-EPS (Θ_ϵ), THETA-DELTA (Θ_δ), and THETA-DELTA-EPS ($\Theta_{\delta\epsilon}$), respectively. The elements of PHI are the variances and covariances of the latent independent variables. The diagonal elements of PSI, THETA-EPS, and THETA-DELTA are the variances of the various error terms, i.e., the variances of the Zeta-, Epsilon-, and Delta-variables, respectively. Off-diagonal elements in these matrices represent error covariances and correspond to two-way arrows in the path diagram. The elements in THETA-DELTA-EPS represent covariances between measurement errors in X- and Y-variables, i.e., between Delta- and Epsilon variables. In the hypothetical model, the only error covariance is PSI(2,1), the covariance between Zeta1 and Zeta2, the error terms on Eta1 and Eta2.

Using the Greek notational system, the first four of these matrices are:

$$\Phi = \begin{bmatrix} \phi_{11} & & \\ \phi_{21} & \phi_{22} & \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \quad \Psi = \begin{bmatrix} \psi_{11} & \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

$$\Theta_\epsilon = \text{diag}(\theta_{11}^{(\epsilon)}, \theta_{22}^{(\epsilon)}, \dots, \theta_{44}^{(\epsilon)}) \quad \Theta_\delta = \text{diag}(\theta_{11}^{(\delta)}, \theta_{22}^{(\delta)}, \dots, \theta_{77}^{(\delta)})$$

4. Parameter specifications

LISREL orders the parameter matrices as LAMBDA-Y, LAMBDA-X, BETA, GAMMA, PHI, PSI, THETA-EPS, THETA-DELTA-EPS, and THETA-DELTA, and the elements in these matrices row-wise. The parameter to be estimated are then labeled 1, 2, 3, etc. In the output file there is a section called PARAMETER SPECIFICATIONS where one can see how LISREL has classified the variables in the model and which parameters are to be estimated.

The table of parameter specifications consist of integer matrices corresponding to the parameter matrices. In each matrix an element is an integer equal to the index of the corresponding parameter in the sequence of independent parameters. Elements corresponding to fixed parameters are zero and elements constrained to be equal have the same index value. Parameter matrices which are entirely zero, i.e., which contain no parameters to be estimated, are omitted. In this example, THETA-DELTA-EPS is omitted because there are no covariances specified between Delta- and Epsilon-variables. For parameter matrices which are diagonal, only the diagonal elements are listed. In this example, THETA-EPS and THETA-DELTA are diagonal.

The parameter specifications for the hypothetical model look as follows:

Parameter Specifications

LAMBDA-Y

	Eta1 -----	Eta2 -----
Y1	0	0
Y2	1	0
Y3	0	0
Y4	0	2

LAMBDA-X

	Ksi1 -----	Ksi2 -----	Ksi3 -----
X1	0	0	0
X2	3	0	0
X3	4	5	0
X4	0	0	0
X5	0	6	0
X6	0	0	0
X7	0	0	7

BETA

	Eta1 -----	Eta2 -----
Eta1	0	8
Eta2	9	0

GAMMA

	Ksi1 -----	Ksi2 -----	Ksi3 -----
Eta1	10	11	0
Eta2	12	0	13

PHI

	Ksi1 -----	Ksi2 -----	Ksi3 -----
Ksi1	14		
Ksi2	15	16	
Ksi3	17	18	19

PSI

	Eta1	Eta2
Eta1	20	
Eta2	21	22

THETA-EPS

Y1	Y2	Y3	Y4
23	24	25	26

THETA-DELTA

X1	X2	X3	X4	X5	X6	X7
27	28	29	30	31	32	33

From this we can see that LISREL has interpreted the relationships in the model correctly, that the parameter BE(2,1), say, is the ninth parameter, and that there are 33 parameters to be estimated altogether.

5. LISREL estimates

In addition to the usual parameter estimates, LISREL gives standard errors of estimates, and *t*-values and squared multiple correlations for each equation in the model. The estimate joint covariance matrix is also given. The information is the same as in the SIMPLIS output, but the parameter estimates are given in terms of parameter matrices instead of as equations. Zero elements in the parameter matrices are indicated by - -. The parameter matrices appear in a section of the output file called LISREL ESTIMATES. They are called LISREL estimates regardless of what method of estimation was used to produce them, but the method used is specified in parentheses after the header LISREL ESTIMATES. For our example, the parameter estimates in LISREL format are:

LISREL Estimates (Maximum Likelihood)

LAMBDA-Y

	Eta1	Eta2
Y1	1.000	- -
Y2	0.921 (0.035) 26.124	- -
Y3	- -	1.000
Y4	- -	1.139 (0.029) 38.934

LAMBDA-X

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
X1	1.000	- -	- -
X2	1.291 (0.104) 12.397	- -	- -
X3	0.920 (0.123) 7.499	1.092 (0.117) 9.365	- -
X4	- -	1.000	- -
X5	- -	1.079 (0.084) 12.863	- -
X6	- -	- -	1.000
X7	- -	- -	1.437 (0.092) 15.557

BETA

	Eta1	Eta2
	-----	-----
Eta1	- -	0.538 (0.056) 9.573
Eta2	0.937 (0.177) 5.281	- -

GAMMA

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
Eta1	0.213 (0.153) 1.396	0.495 (0.147) 3.368	- -
Eta2	-1.223 (0.121) -10.102	- -	0.996 (0.151) 6.599

Covariance Matrix of ETA and KSI

	Eta1	Eta2	Ksi1	Ksi2	Ksi3
	-----	-----	-----	-----	-----
Eta1	2.957				
Eta2	3.115	4.719			
Ksi1	0.482	-0.217	0.974		
Ksi2	0.554	-0.312	0.788	1.117	
Ksi3	0.932	1.402	0.525	0.133	1.175

PHI

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
Ksi1	0.974 (0.187) 5.196		
Ksi2	0.788 (0.150) 5.239	1.117 (0.194) 5.753	
Ksi3	0.525 (0.133) 3.947	0.133 (0.125) 1.064	1.175 (0.202) 5.813

PSI

	Eta1	Eta2
	-----	-----
Eta1	0.486 (0.126) 3.853	
Eta2	-0.069 (0.168) -0.410	0.133 (0.078) 1.707

Squared Multiple Correlations for Structural Equations

	Eta1	Eta2
	-----	-----
	0.434	0.997

NOTE: R² for Structural Equations are Hayduk's (2006) Blocked-Error R²

Reduced Form

	Ksi1	Ksi2	Ksi3
Eta1	-0.895 (0.428) -2.091	0.999 (0.342) 2.916	1.080 (0.224) 4.815
Eta2	-2.062 (0.517) -3.987	0.936 (0.403) 2.321	2.008 (0.268) 7.491

Squared Multiple Correlations for Reduced Form

Eta1	Eta2
0.381	0.629

THETA-EPS

Y1	Y2	Y3	Y4
0.247 (0.053) 4.687	0.123 (0.038) 3.256	0.136 (0.041) 3.342	0.197 (0.054) 3.621

Squared Multiple Correlations for Y - Variables

Y1	Y2	Y3	Y4
0.923	0.953	0.972	0.969

THETA-DELTA

X1	X2	X3	X4	X5	X6	X7
0.389 (0.064) 6.125	0.336 (0.067) 5.026	0.063 (0.051) 1.235	0.259 (0.050) 5.162	0.440 (0.075) 5.905	0.247 (0.053) 4.619	0.259 (0.091) 2.841

Squared Multiple Correlations for X - Variables

X1	X2	X3	X4	X5	X6	X7
0.715	0.829	0.983	0.812	0.747	0.826	0.903

It may be instructive to run the same input file without the line

```
LISREL Output: RS MI SC EF WP
```

To establish the fact that the two output files give the same information about the LISREL solution. The information is just displayed in different ways. As in the SIMPLIS output, the standard errors and the t -values are given below the parameter estimates.

6. Goodness-of-fit statistics

LISREL gives many measures of the goodness-of-fit of the whole model. All fit measures are functions of chi-square, the fit measure appearing on the first line. These fit measures appear in a section of the output called GOODNESS OF FIT STATISTICS. For the hypothetical model, these measures are listed in the output file as:

Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	33
Maximum Likelihood Ratio Chi-Square (C1)	29.394 (P = 0.6473)
Browne's (1984) ADF Chi-Square (C2_NT)	27.247 (P = 0.7488)
Estimated Non-centrality Parameter (NCP)	0.0
90 Percent Confidence Interval for NCP	(0.0 ; 12.673)
Minimum Fit Function Value	0.294
Population Discrepancy Function Value (F0)	0.0
90 Percent Confidence Interval for F0	(0.0 ; 0.127)
Root Mean Square Error of Approximation (RMSEA)	0.0
90 Percent Confidence Interval for RMSEA	(0.0 ; 0.0620)
P-Value for Test of Close Fit (RMSEA < 0.05)	0.893
Expected Cross-Validation Index (ECVI)	0.990
90 Percent Confidence Interval for ECVI	(0.990 ; 1.117)
ECVI for Saturated Model	1.320
ECVI for Independence Model	14.785
Chi-Square for Independence Model (55 df)	1456.516
Normed Fit Index (NFI)	0.980
Non-Normed Fit Index (NNFI)	1.004
Parsimony Normed Fit Index (PNFI)	0.588
Comparative Fit Index (CFI)	1.000
Incremental Fit Index (IFI)	1.003
Relative Fit Index (RFI)	0.966
Critical N (CN)	185.484

Root Mean Square Residual (RMR)	0.0652
Standardized RMR	0.0266
Goodness of Fit Index (GFI)	0.953
Adjusted Goodness of Fit Index (AGFI)	0.906
Parsimony Goodness of Fit Index (PGFI)	0.476

These statistics illustrate the situation of a small random sample for a population in which the specified model holds exactly. In situations of large samples of real data or of random samples from populations in which the specified model does not hold, these statistics may look quite different.

7. Fitted and Standardized Residuals

A residual is an observed minus a fitted covariance (variance). A standardized residual is a residual divided by its estimated standard error. There are such residuals for every pair of observed variables. If the variances of the variables vary considerably from one variable to another, it is rather difficult to know whether a fitted residual should be considered large or small. Standardized residuals, on the other hand, are independent of the units of measurement of the variables and provides a “statistical” metric for judging the size of a residual.

The fitted and standardized residuals for the hypothetical model are typical for the situation when the data fits the model well. In the output file they look like this:

Fitted Residuals

	Y1	Y2	Y3	Y4	X1	X2	X3	X4	X5	X6
Y1	0.000									
Y2	0.000	0.000								
Y3	0.083	0.007	0.000							
Y4	-0.002	-0.064	0.000	0.000						
X1	-0.153	-0.073	-0.140	-0.224	0.000					
X2	-0.063	0.019	-0.036	-0.017	0.013	0.000				
X3	-0.042	0.054	0.051	0.023	-0.015	0.007	0.000			
X4	-0.086	-0.054	-0.126	-0.184	0.000	0.025	0.008	0.000		
X5	-0.096	-0.011	-0.027	-0.042	-0.013	-0.028	-0.009	-0.017	0.000	
X6	0.118	0.102	0.014	0.118	-0.051	0.016	0.026	-0.062	-0.040	0.000
X7	-0.079	-0.079	-0.091	0.016	-0.068	-0.067	0.014	-0.056	-0.045	0.000

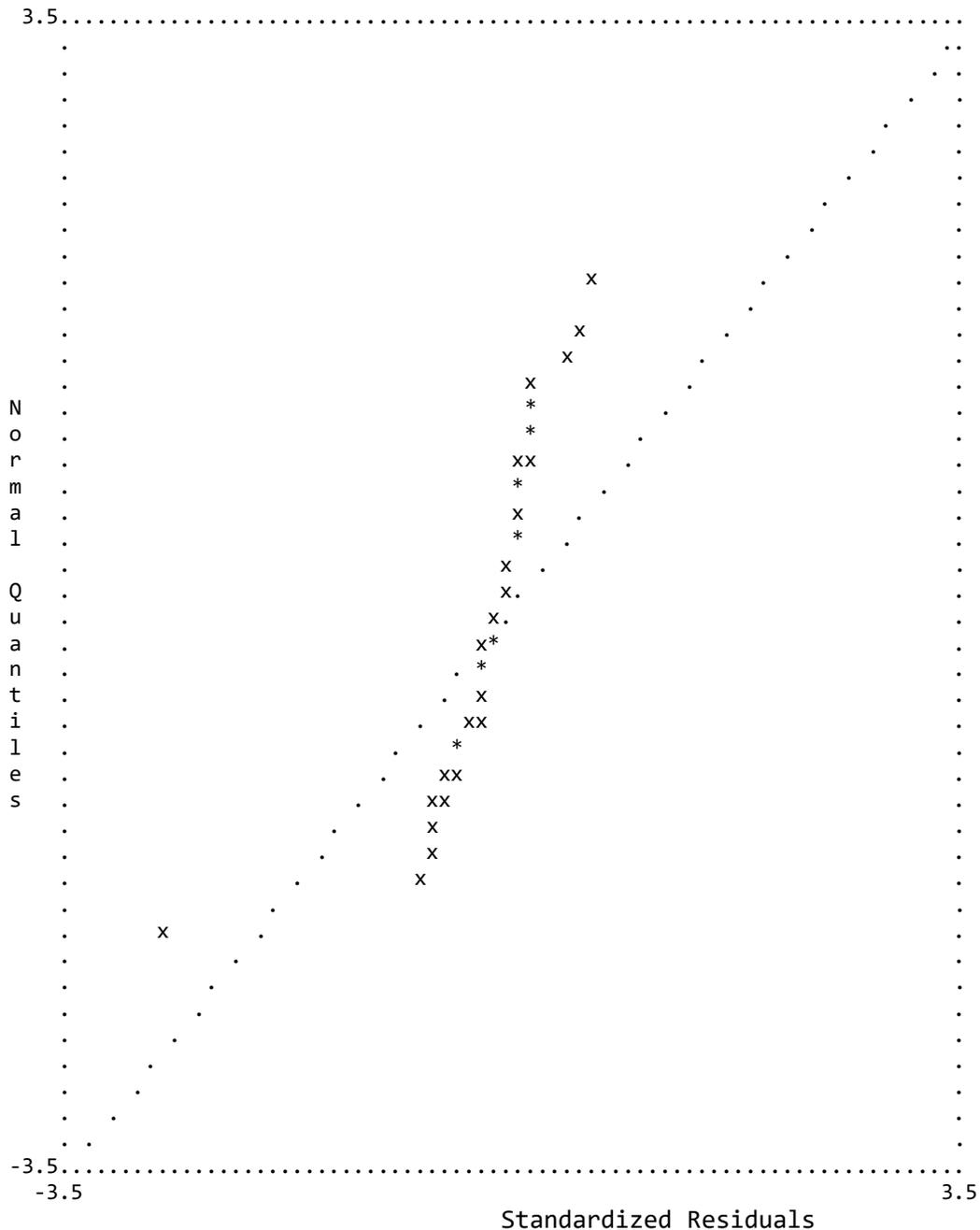
Fitted Residuals

	X7
X7	0.000

Summary Statistics for Fitted Residuals

Smallest Fitted Residual =	-0.224
Median Fitted Residual =	-0.001
Largest Fitted Residual =	0.118

Qplot of Standardized Residuals



8. Modification indices

A modification index (Sörbom, 1989) is given for each fixed and constrained parameter in the model. Each such modification index measures how much the chi-square is expected to decrease if this particular parameter is set free and the model is reestimated. Thus, the modification index is approximately equal to the difference in chi-square between two models in which one parameter is fixed or constrained in one model and free in the other model, all other parameters estimated in both models. The largest modification index tells which parameter to set free to improve the fit maximally.

Associated with each modification index is an estimated change of the parameter. This measures how much the parameter is expected to change, in the positive or negative direction, if set free. In addition, the expected change is given in the metric when the latent variables are standardized and in the metric. When both the observed and the latent variables are standardized, as suggested by Kaplan (1989). For the hypothetical model, the modification indices and the expected changes are:

Modification Indices and Expected Change

Modification Indices for LAMBDA-Y

	Eta1	Eta2
	-----	-----
Y1	- -	0.990
Y2	- -	0.990
Y3	2.115	- -
Y4	2.115	- -

Expected Change for LAMBDA-Y

	Eta1	Eta2
	-----	-----
Y1	- -	0.058
Y2	- -	-0.053
Y3	0.089	- -
Y4	-0.102	- -

Standardized Expected Change for LAMBDA-Y

	Eta1	Eta2
	-----	-----
Y1	- -	0.126
Y2	- -	-0.116
Y3	0.154	- -
Y4	-0.175	- -

Completely Standardized Expected Change for LAMBDA-Y

	Eta1	Eta2
	-----	-----
Y1	- -	0.070
Y2	- -	-0.072
Y3	0.070	- -
Y4	-0.070	- -

Modification Indices for LAMBDA-X

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
X1	- -	0.108	1.049
X2	- -	0.108	0.342
X3	- -	- -	3.376
X4	0.304	- -	1.373
X5	0.304	- -	0.241
X6	0.208	0.001	- -
X7	0.208	0.001	- -

Expected Change for LAMBDA-X

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
X1	- -	-0.044	-0.082
X2	- -	0.057	-0.054
X3	- -	- -	0.148
X4	0.082	- -	-0.077

X5	-0.089	- -	-0.038
X6	0.034	-0.001	- -
X7	-0.048	0.002	- -

Standardized Expected Change for LAMBDA-X

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
X1	- -	-0.047	-0.089
X2	- -	0.061	-0.058
X3	- -	- -	0.161
X4	0.081	- -	-0.083
X5	-0.087	- -	-0.041
X6	0.033	-0.001	- -
X7	-0.048	0.002	- -

Completely Standardized Expected Change for LAMBDA-X

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
X1	- -	-0.040	-0.076
X2	- -	0.043	-0.042
X3	- -	- -	0.082
X4	0.069	- -	-0.071
X5	-0.066	- -	-0.031
X6	0.028	-0.001	- -
X7	-0.029	0.001	- -

No Non-Zero Modification Indices for BETA

No Non-Zero Modification Indices for GAMMA

No Non-Zero Modification Indices for PHI

No Non-Zero Modification Indices for PSI

Modification Indices for THETA-EPS

	Y1	Y2	Y3	Y4
	-----	-----	-----	-----
Y1	- -			
Y2	- -	- -		
Y3	0.529	0.001	- -	
Y4	0.550	0.002	- -	- -

Expected Change for THETA-EPS

	Y1	Y2	Y3	Y4
	-----	-----	-----	-----
Y1	- -			
Y2	- -	- -		
Y3	0.024	-0.001	- -	
Y4	-0.028	0.002	- -	- -

Completely Standardized Expected Change for THETA-EPS

	Y1	Y2	Y3	Y4
	-----	-----	-----	-----
Y1	- -			
Y2	- -	- -		
Y3	0.006	0.000	- -	
Y4	-0.006	0.000	- -	- -

Modification Indices for THETA-DELTA-EPS

	Y1	Y2	Y3	Y4
	-----	-----	-----	-----
X1	0.134	0.495	2.039	4.211
X2	0.242	0.097	0.349	0.623
X3	0.004	0.057	0.013	0.054
X4	0.907	0.028	0.036	0.367
X5	1.025	0.104	0.110	0.712
X6	0.947	1.116	2.403	0.166
X7	0.288	1.696	0.015	0.907

Expected Change for THETA-DELTA-EPS

	Y1	Y2	Y3	Y4
	-----	-----	-----	-----
X1	-0.015	0.023	0.048	-0.080
X2	-0.020	0.011	-0.021	0.031
X3	0.002	-0.007	0.003	0.008
X4	0.032	-0.005	-0.005	-0.020
X5	-0.043	0.011	-0.012	0.035
X6	0.034	0.032	-0.048	0.015
X7	-0.025	-0.052	0.005	0.046

Completely Standardized Expected Change for THETA-DELTA-EPS

	Y1	Y2	Y3	Y4
	-----	-----	-----	-----
X1	-0.007	0.012	0.019	-0.027
X2	-0.008	0.005	-0.007	0.009
X3	0.001	-0.002	0.001	0.002
X4	0.015	-0.002	-0.002	-0.007
X5	-0.018	0.005	-0.004	0.010
X6	0.016	0.017	-0.018	0.005
X7	-0.008	-0.020	0.001	0.011

Modification Indices for THETA-DELTA

	X1	X2	X3	X4	X5	X6	X7
	-----	-----	-----	-----	-----	-----	-----
X1	- -						
X2	0.341	- -					
X3	0.572	0.016	- -				
X4	0.000	0.091	0.183	- -			
X5	0.136	0.287	2.014	1.160	- -		
X6	0.459	0.806	0.079	0.586	0.161	- -	
X7	0.605	1.926	0.522	0.503	0.049	- -	- -

Expected Change for THETA-DELTA

	X1	X2	X3	X4	X5	X6	X7
X1	- -						
X2	0.034	- -					
X3	-0.034	0.007	- -				
X4	-0.001	0.013	-0.032	- -			
X5	0.018	-0.027	0.109	-0.081	- -		
X6	-0.027	0.038	-0.010	-0.026	-0.017	- -	
X7	0.041	-0.079	0.032	0.031	-0.012	- -	- -

Completely Standardized Expected Change for THETA-DELTA

	X1	X2	X3	X4	X5	X6	X7
X1	- -						
X2	0.021	- -					
X3	-0.015	0.003	- -				
X4	0.000	0.008	-0.014	- -			
X5	0.012	-0.015	0.042	-0.052	- -		
X6	-0.020	0.023	-0.004	-0.018	-0.011	- -	
X7	0.021	-0.034	0.010	0.016	-0.006	- -	- -

Maximum Modification Index is 4.21 for Element (1, 4) of THETA DELTA-EPSILON

9. Standardized solutions

The latent variables in the LISREL solution are generally standardized. In this example, however, neither the latent nor the observed variables are standardized. Nevertheless, it is possible to obtain standardized solutions as a by-product after the original solution has been obtained. There are two kinds of standardized solutions: SS (Standardized Solution), in which the latent variables are scaled to have variances equal to one and the *observed variables are still in the original metric* and SC (Standardized Completely), in which both observed and latent variables are standardized.

These standardized solutions can only be computed after the original solution has been estimated. To obtain these solutions, one must first obtain diagonal matrices of estimated standard deviations of the latent variables (SS) and the observed variables (SC).

The standard deviations of the latent variables are obtained from the joint covariance matrix of the Eta- and Ksi-variables which is given in the section LISREL ESTIMATES:

Covariance Matrix of ETA and KSI

	Eta1	Eta2	Ksi1	Ksi2	Ksi3
Eta1	2.957				
Eta2	3.115	4.719			
Ksi1	0.482	-0.217	0.974		
Ksi2	0.554	-0.312	0.788	1.117	
Ksi3	0.932	1.402	0.525	0.133	1.175

The standard deviations of the latent variables are the square roots of the diagonal elements of this matrix.

The standard deviations of the observed variables are the square roots of the diagonal elements of the fitted covariance matrix. For the hypothetical model this is:

Fitted Covariance Matrix

Y1	Y2	Y3	Y4	X1	X2	X3	X4	X5	X6
Y1	3.204								
Y2	2.722	2.629							
Y3	3.115	2.868	4.855						
Y4	3.547	3.266	5.373	6.315					
X1	0.482	0.444	-0.217	-0.247	1.363				
X2	0.622	0.573	-0.280	-0.318	1.258	1.960			
X3	1.048	0.965	-0.540	-0.614	1.757	2.269	3.803		
X4	0.554	0.510	-0.312	-0.355	0.788	1.018	1.945	1.376	
X5	0.598	0.550	-0.336	-0.383	0.851	1.098	2.099	1.206	1.741
X6	0.932	0.858	1.402	1.596	0.525	0.678	0.629	0.133	0.144
1.422									
X7	1.339	1.233	2.014	2.293	0.754	0.974	0.903	0.192	0.207
1.688									

Fitted Covariance Matrix

	X7
X7	2.684

The standardized solutions SS and SC are computed by applying these standard deviations or their reciprocals as scale factors in the rows and columns of the estimated parameter matrices of the LISREL solution. For formulas, see Jöreskog & Sörbom (1989, pp. 38-39). Note that the standard deviations of the observed variables are obtained from the fitted covariance matrix rather than the observed covariance matrix. For the hypothetical model, the completely standardized solution (SC) is:

Completely Standardized Solution

LAMBDA-Y			
	Eta1	Eta2	
Y1	0.961	- -	
Y2	0.976	- -	
Y3	- -	0.986	
Y4	- -	0.984	
LAMBDA-X			
	Ksi1	Ksi2	Ksi3
X1	0.845	- -	- -
X2	0.910	- -	- -
X3	0.465	0.592	- -
X4	- -	0.901	- -
X5	- -	0.864	- -
X6	- -	- -	0.909
X7	- -	- -	0.950

BETA

	Eta1	Eta2
Eta1	- -	0.679
Eta2	0.742	- -

GAMMA

	Ksi1	Ksi2	Ksi3
Eta1	0.123	0.304	- -
Eta2	-0.556	- -	0.497

Correlation Matrix of ETA and KSI

	Eta1	Eta2	Ksi1	Ksi2	Ksi3
Eta1	1.000				
Eta2	0.834	1.000			
Ksi1	0.284	-0.101	1.000		
Ksi2	0.305	-0.136	0.756	1.000	
Ksi3	0.500	0.595	0.491	0.117	1.000

PSI

	Eta1	Eta2
Eta1	0.164	
Eta2	-0.018	0.028

THETA-EPS

Y1	Y2	Y3	Y4
0.077	0.047	0.028	0.031

THETA-DELTA

X1	X2	X3	X4	X5	X6	X7
0.285	0.171	0.017	0.188	0.253	0.174	0.097

10. Direct, indirect, and total effects (EF)

It can be seen in the path diagram that there are both direct and indirect effects of Ksi1 on Eta2. For example, in addition to the direct effect GAMMA(2,1) of Ksi1 on Eta2, there is an indirect effect BETA(2,1) x GAMMA(2,1) mediated by Eta1. And even though there is no direct effect of Ksi3 on Eta1, there is a similar indirect effect BETA(1,2) x GAMMA(2,3) mediated by Eta2.

There are usually no direct effects of an Eta-variable on itself, i.e., all diagonal elements of the BETA matrix are zero. Nevertheless, there may be a total effect of an Eta-variable on itself. How can this be? This can only occur in non-recursive models and can best be understood by defining a cycle. A cycle is a causal chain going from one Eta -variable, passing over some other Eta-variables and returning to the original Eta -variable. For example, one cycle for Eta1 consist of one path to Eta2 and a return to Eta1. Writing β_{21} for BETA(2,1) and β_{12} for BETA(1,2), the effect of one cycle on Eta1 is $\beta_{21}\beta_{12}$.

After two cycles the effect will be $\beta_{21}^2\beta_{12}^2$, after three cycles $\beta_{21}^3\beta_{12}^3$, etc. The total effect on Eta1 will be the sum of the infinite geometric series

$$\beta_{21}\beta_{12} + \beta_{21}^2\beta_{12}^2 + \beta_{21}^3\beta_{12}^3 \dots$$

which is $\beta_{21}\beta_{12} / (1 - \beta_{21}\beta_{12})$ if $|\beta_{21}\beta_{12}| < 1$.

A necessary and sufficient condition for convergence of the series is that all the eigenvalues of the BETA matrix are within the unit circle. In general, the eigenvalues of a BETA matrix are complex numbers somewhat difficult to compute. However, a sufficient condition for convergence is that the largest eigenvalue of \mathbf{BB}' is less than one. This is very easy to verify. The program prints the largest eigenvalue of \mathbf{BB}' under the name of STABILITY INDEX. For a fuller discussion of indirect and total effects, see Bollen (1987, 1989a).

The indirect and total effects will be computed and printed if requested by the EF keyword on the LISREL Output line. Standard errors of indirect and total effects can also be computed (Sobel, 1982). In LISREL, these are automatically included in the LISREL output, together with the corresponding *t*-values.

For the hypothetical model, some of the estimated indirect and total effects and their standard errors are:

Total and Indirect Effects

Total Effects of KSI on ETA

	Ksi1	Ksi2	Ksi3
Eta1	-0.895 (0.426) -2.102	0.999 (0.341) 2.931	1.080 (0.223) 4.839
Eta2	-2.062 (0.515) -4.007	0.936 (0.401) 2.333	2.008 (0.267) 7.529

Indirect Effects of KSI on ETA

	Ksi1	Ksi2	Ksi3
Eta1	-1.109 (0.338) -3.281	0.503 (0.238) 2.112	1.080 (0.223) 4.839
Eta2	-0.839 (0.464) -1.808	0.936 (0.401) 2.333	1.012 (0.308) 3.289

Total Effects of ETA on ETA

	Eta1	Eta2
Eta1	1.016 (0.408) 2.489	1.084 (0.280) 3.866

Eta2	1.890	1.016
	(0.714)	(0.408)
	2.649	2.489

Largest Eigenvalue of B*B' (Stability Index) is 0.879

Indirect Effects of ETA on ETA

	Eta1	Eta2
	-----	-----
Eta1	1.016	0.546
	(0.408)	(0.246)
	2.489	2.224
Eta2	0.953	1.016
	(0.545)	(0.408)
	1.747	2.489

Total Effects of ETA on Y

	Eta1	Eta2
	-----	-----
Y1	2.016	1.084
	(0.408)	(0.280)
	4.938	3.866
Y2	1.856	0.998
	(0.383)	(0.257)
	4.852	3.877
Y3	1.890	2.016
	(0.714)	(0.408)
	2.649	4.938
Y4	2.152	2.296
	(0.813)	(0.469)
	2.648	4.899

Indirect Effects of ETA on Y

	Eta1	Eta2
	-----	-----
Y1	1.016	1.084
	(0.408)	(0.280)
	2.489	3.866
Y2	0.936	0.998
	(0.378)	(0.257)
	2.478	3.877
Y3	1.890	1.016
	(0.714)	(0.408)
	2.649	2.489

Y4	2.152	1.157
	(0.813)	(0.466)
	2.648	2.484

Total Effects of KSI on Y

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
Y1	-0.895	0.999	1.080
	(0.426)	(0.341)	(0.223)
	-2.102	2.931	4.839
Y2	-0.824	0.919	0.994
	(0.392)	(0.313)	(0.205)
	-2.103	2.936	4.861
Y3	-2.062	0.936	2.008
	(0.515)	(0.401)	(0.267)
	-4.007	2.333	7.529
Y4	-2.348	1.066	2.287
	(0.586)	(0.457)	(0.304)
	-4.006	2.333	7.521

Since there are no direct effects of KSI on Y, the indirect effects are equal to the total effects; hence only the total effects are given.

If SS or SC appears on the LISREL output line, the corresponding standardized effects will also be obtained in the output file. These are the effects that would be obtained if the latent (SS) and the observed variables (SC) were standardized. For our example, the completely standardized effects are:

Standardized Total and Indirect Effects

Standardized Total Effects of KSI on ETA

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
Eta1	-0.514	0.614	0.681
Eta2	-0.937	0.455	1.002

Standardized Indirect Effects of KSI on ETA

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
Eta1	-0.636	0.309	0.681
Eta2	-0.381	0.455	0.505

Standardized Total Effects of ETA on ETA

	Eta1	Eta2
	-----	-----
Eta1	1.016	1.370
Eta2	1.496	1.016

Standardized Indirect Effects of ETA on ETA

	Eta1	Eta2
	-----	-----
Eta1	1.016	0.690
Eta2	0.754	1.016

Standardized Total Effects of ETA on Y

	Eta1	Eta2
	-----	-----
Y1	3.467	2.355
Y2	3.192	2.168
Y3	3.250	4.380
Y4	3.700	4.987

Completely Standardized Total Effects of ETA on Y

	Eta1	Eta2
	-----	-----
Y1	1.937	1.316
Y2	1.969	1.337
Y3	1.475	1.988
Y4	1.473	1.985

Standardized Indirect Effects of ETA on Y

	Eta1	Eta2
	-----	-----
Y1	1.747	2.355
Y2	1.609	2.168
Y3	3.250	2.207
Y4	3.700	2.514

Completely Standardized Indirect Effects of ETA on Y

	Eta1	Eta2
	-----	-----
Y1	0.976	1.316
Y2	0.992	1.337
Y3	1.475	1.002
Y4	1.473	1.000

Standardized Total Effects of KSI on Y

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
Y1	-0.884	1.055	1.170
Y2	-0.814	0.972	1.078
Y3	-2.035	0.989	2.177
Y4	-2.317	1.127	2.479

Completely Standardized Total Effects of KSI on Y

	Ksi1	Ksi2	Ksi3
	-----	-----	-----
Y1	-0.494	0.590	0.654
Y2	-0.502	0.599	0.665
Y3	-0.924	0.449	0.988

Y4 -0.922 0.448 0.986

11. Estimating the standardized solution directly

It is instructive to estimate the standardized solution directly and compare the unstandardized and standardized solutions. To estimate completely standardized solution, one must analyze a correlation matrix and omit the I* in the relationships which specify that the latent variables are to be scaled in the metric of observed reference variables. The following input file will estimate the completely standardized solution directly (see **EX17B.SPL**):

```
Hypothetical Model
Observed Variables: Y1-Y4 X1-X7
Correlation Matrix from File EX17.COR
Sample Size: 100
Latent Variables: Eta1 Eta2 Ksi1-Ksi3
Relationships
  Eta1 = Eta2 Ksi1 Ksi2
  Eta2 = Eta1 Ksi1 Ksi3
Let the Errors of Eta1 and Eta2 Correlate

Y1 - Y2 = Eta1
Y3 - Y4 = Eta2

X1 - X3 = Ksi1
X3 - X5 = Ksi2
X6 - X7 = Ksi3

LISREL Output: RS MI SC EF WP
End of Problem
```

Verify that the following quantities are the same in the output files for **EX17A.SPL** and **EX17B.SPL**:

- All goodness-of-fit statistics
- Standardized residuals and Q-plot
- Modification indices
- Completely standardized expected changes
- Completely standardized solution
- Completely standardized total effects