Confirmatory Factor Analysis with nine psychological variables

It is important to distinguish between exploratory and confirmatory analysis. In an exploratory analysis, one wants to explore the empirical data to discover and detect characteristic features and interesting relationships without imposing any definite model on the data. An exploratory analysis may be structure generating, model generating, or hypothesis generating. In confirmatory analysis, on the other hand, one builds a model assumed to describe, explain, or account for the empirical data in terms of relatively few parameters. The model is based on *a priori* information about the data structure in the form of a specified theory or hypothesis, a given classificatory design for items or subtests according to objective features of content and format, know experimental conditions, or knowledge from previous studies based on extensive data.

Exploratory factor analysis is a technique often used to detect and assess latent sources of variation and covariation in observed measurements. It is widely recognized that exploratory factor analysis van be quite useful in the early stages of experimentation or test development. Thurstone's (1938) primary mental abilities, French's (1951) factors in aptitude and achievement tests and Guilford's (1956) structure of intelligence are good examples of this. The results of an exploratory analysis may have heuristic and suggestive value and may generate hypotheses which are capable of more objective testing by other multivariate methods. As more knowledge is gained about the nature of social and psychological measurements, however, exploratory factor analysis may not be a useful tool and may even become a hindrance.

Most studies are to some extent both exploratory and confirmatory since they involve some variables of known and other variables of unknown composition. The former should e chosen with great care in order that as much information as possible about the latter may be extracted. It is highly desirable that a hypothesis which has been suggested by mainly exploratory procedures should subsequently be confirmed, or disproved, by obtaining new data and subjecting these to more rigorous statistical techniques. Although LISREL is most useful in confirmatory studies, it can also be used to do exploratory analysis by means of a sequence of confirmatory analyses. It must be emphasized, however, that one must at least have a tentative theory or hypothesis to start out with.

The basic idea of factor analysis is the following. For a given set of response variables $x_1, x_2, \ldots, x_q$ one wants to find a set of underlying latent factors $\xi_1, \xi_2, \ldots, \xi_n$, fewer in number than the observed variables. These latent factors are supposed to account for the intercorrelations of the response variables in the sense that when the factors are partialed out from the observed variables, there should no longer remain any correlations between these. This leads to the model (see Jöreskog, 1979a)

$$ x_i = \lambda_{i1}\xi_1 + \lambda_{i2}\xi_2 + \ldots + \lambda_{in}\xi_n + \delta_i, $$

where δ_i, the unique part of x_i, is assumed to be uncorrelated with $\xi_1, \xi_2, \ldots, \xi_n$ and with δ_j for $j \neq i$. The unique part δ_i consists of two components: a specific factor s_i and a pure random measurement error e_i. These are indistinguishable, unless the measurements x_i are designed in such a way that they can be separately identified (panel designs and multitrait-multimethod designs). The term δ_i is often called the *measurement error* in x_i even though it is widely recognized that this term may also contain a specific factor as stated above. We shall continue this tradition and use this term in our documentation.

In a confirmatory factor analysis, the investigator has such knowledge about the factorial mature of the variables that he/she is able to specify that each measure x_i depends only on a few of the factors ξ_j. If x_i does not depend on ξ_j, $\lambda_{ij} = 0$. In a path diagram, this means that there is no one-way arrow from ξ_j to x_i. In many applications, the latent factor ξ_j represents a theoretical construct and the observed measures x_i are designed to be indicators of this construct. In this case there is only one non-zero λ_{ij} in each equation.

We shall illustrate confirmatory factor analysis by means of a detailed example. In particular, this example illustrates the assessment of model fit and the use of the model modification index.

Holzinger & Swineford (1930) collected data on twenty-six psychological tests administered tto 145 seventh- and eighth-grade children in the Grant-White school in Chicago. Nine of these tests were selected and for this example it was hypothesized that these measure three common factors: visual perception (P), verbal ability (V) and speed (S), such that the first three variables measure P, the next three measures V, and the last three measures S. The nine selected variables and their intercorrelations are given in the table below, followed by a path diagram of the model.

Broadly speaking, there are two basic problems that are important in the social and behavioral sciences. The first problem is concerned with the measurement properties – validities and reliabilities – of the measurement instruments. The second problem concerns the causal relationships among the variables and their relative explanatory power.

Most theories and models in the social and behavioral sciences are formulated in terms of theoretical or hypothetical concepts, or constructs, or latent variables, which are not directly measurable or observable.

Table: Correlations matrix for nine psychological variables

VIS PERC	1.000								
CUBES	0.318	1.000							
LOZENGESL	0.436	0.419	1.000						
PAR COMP	0.335	0.234	0.323	1.000					
SEN COMP	0.304	0.157	0.283	0.722	1.000				
WORDMEAN	0.326	0.195	0.350	0.714	0.685	1.000			
ADDITION	0.116	0.057	0.056	0.203	0.246	0.170	1.000		
COUNTDOT	0.314	0.145	0.229	0.095	0.181	0.113	0.585	1.000	
S-C CAPS	0.489	0.239	0.361	0.309	0.345	0.280	0.408	0.512	1.000
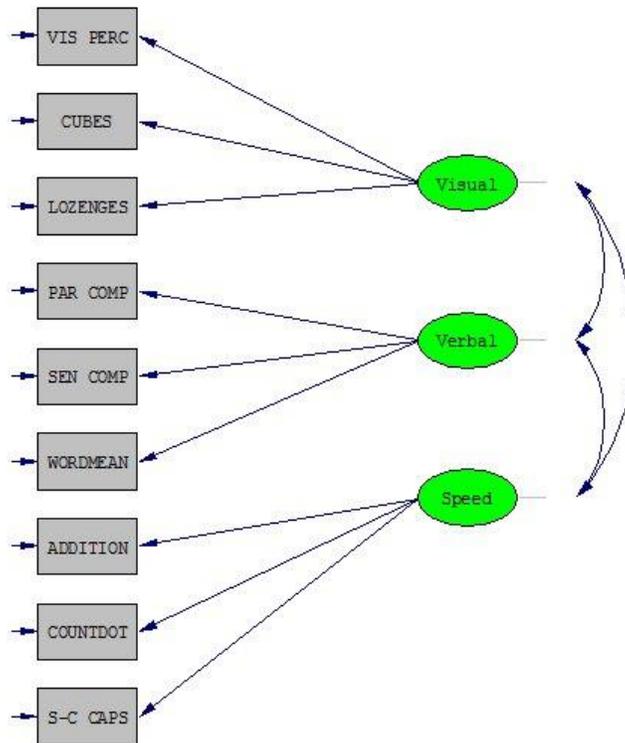
We want to examine the fit of the model implied by the stated hypothesis. If the model does not fit the data well, we want to suggest an alternative model that first the data better.

The model is very similar to the measurement model for Ability and Aspiration described in the previous example. The only differences are that there are nine observed variables instead of six and that there are three factors instead of two. However, as will e seen, the example will illustrate how the initial model can be evaluated and modified when it is found not to fit the data sufficiently well.

The SIMPLIS input (**EX5A.SPL** in the **Simplis Examples** folder) is almost a copy of the input file for **EX4A.SPL**:

```
Nine Psychological Variables - A Confirmatory Factor Analysis
Observed Variables
    'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
     ADDITION COUNTDOT 'S-C CAPS'
Correlation Matrix From File EX5.COR
Sample Size 145
Latent Variables: Visual Verbal Speed
Relationships:
    'VIS PERC' - LOZENGES = Visual
    'PAR COMP' - WORDMEAN = Verbal
     ADDITION - 'S-C CAPS' = Speed
Number of Decimals = 3
Wide Print
Print Residuals
Path Diagram
End of Problem
```

This time we specify the measurement model as relationships rather than as paths, as we did previously. The two new elements in this input file are:

```
Number of Decimals = 3
Wide Print
```

The first of these specifies that we wish to have the results in the output file given to three decimals. Since most users can only interpret at most two decimals, LISREL uses two decimals by default. This example illustrated how this default value can be overridden. The second of these lines is used to specify output with 132 characters per line. Otherwise, there will be 80 characters per line.

Look at the output file obtained for this model. Apart from the estimated relationships and the factor correlation matrix, which all look reasonable, many goodness-of-fit statistics are given. For the moment we will only use the chi-square:

```
Degrees of Freedom for (C1)-(C2)                    24
Maximum Likelihood Ratio Chi-Square (C1)            52.991 (P = 0.0006)
```

A chi-square of 52.991 with 24 degrees of freedom indicates that the model does not fit the data well. How should the model be modified to fit the data better? A very powerful tool for answering this question is the modification index. There is a modification index for each fixed parameter in the model, i.e., for every path that is missing in the path diagram. For each such path, the modification index is an estimate or predictor

of the decrease in chi-square that will be obtained if that particular path is introduced in the model. LISREL lists all the large modification indices as follows:

```
        The Modification Indices Suggest to Add the
 Path to  from      Decrease in Chi-Square    New Estimate
ADDITION  Visual           10.5                  -0.37
COUNTDOT  Verbal           10.1                  -0.28
S-C CAPS  Visual           24.7                   0.57
S-C CAPS  Verbal           10.0                   0.26

The Modification Indices Suggest to Add an Error Covariance
 Between     and     Decrease in Chi-Square    New Estimate
COUNTDOT  ADDITION         25.1                   0.62
S-C CAPS  VIS PERC          9.1                   0.18
S-C CAPS  COUNTDOT          8.3                  -0.38
```

The largest modification index is 24.7 for the path from Visual to S-C CAPS. This indicates that we can expect a large decrease in chi-square if we include this path in the model. Therefore, *if we can interpret this path substantively* (see below), we can modify the model by adding this path and running the modified model. It is also predicted that the new path will be 0.57.

The fact that the model is misspecified can also be seen from the standardized residuals in the output file:

```
        Standardized Residuals

          VIS PERC      CUBES    LOZENGES    PAR COMP    SEN COMP    WORDMEAN
ADDITION  COUNTDOT    S-C CAPS
          --------    --------    --------    --------    --------    --------    ---
-----    --------    --------
 VIS PERC    0.000
    CUBES   -0.312      0.000
 LOZENGES   -0.266      0.772      0.000
 PAR COMP    0.220     -0.097      0.009      0.000
 SEN COMP    0.012     -0.878     -0.672      0.027      0.000
 WORDMEAN    0.334     -0.412      1.158     -0.020     -0.004      0.000
 ADDITION   -1.623     -1.586     -2.186      0.268      0.854     -0.040
0.000
 COUNTDOT    0.532     -0.802     -0.650     -1.525     -0.434     -1.108
0.602      0.000
 S-C CAPS    3.363      0.736      1.466      1.644      2.159      1.604      -
0.491     -0.595      0.000
```

This shows a large standardized residual of 3.363 between VIS PERC and S-C CAPS, indicating that these two variables correlate more than the model accounts for. Although this shows where the lack of fit is, it does not tell how the model should be modified to fit the data better. From this point of view, modification indices are often more useful than standardized residuals for detecting specification errors in the model.

To run the modified model, change the line (see file **EX5B.SPL**)

```
   'VIS PERC' - LOZENGES = Visual
```

to

```
'VIS PERC' - LOZENGES 'S-C CAPS' = Visual
```

The chi-square for the modified model is:

```
Degrees of Freedom for (C1)-(C2)                    23
 Maximum Likelihood Ratio Chi-Square (C1)           29.062 (P = 0.1783)
```

This indicates that the fit of the modified model is acceptable. Note that the reduction in chi-square is 52.991 - 29.062 = 23.929 which is roughly the same as what the modification index precited. Also note that the estimated loading of S-C CAPS on Visual is 0.46 which is a little smaller than the prediction. The -t-value is 5.16. Hence this loading is "significant". Thus, S-C CAPS is not a pure measure of Speed, but rather a composite measure of both Visual and Speed.

The substantive interpretation of the results of these analyses may be as follows. The first factor is "visual perception" as represented by the first three variables containing spatial problems with geometrical configurations. The third factor is a "speed" factor supposed to measure the ability to perform very simple tasks quickly and accurately. However, unlike the two measures "Addition" and "Counting dots", which are purely numerical, variable nine "Straight-curved capitals" requires the ability to distinguish between capital letters which contain curved parts (like P) from those which contain only straight lines (like L), and to do this quickly and accurately. It is therefore conceivable that "straight-curved capitals" contains a component correlated with "visual perception" as represented in this data and also that it contains a component of "speed". Thus, the variable "Straight-curved capitals" is a composite measure unlike all the other measures in this example, which are all pure measures.