Path analysis with latent variables: stability of alienation

Path analysis with directly observed variables was discussed in a previous example. It is also possible to consider path analysis for latent variables. In its most general form there is a structural equation system for a set of latent variables classified as dependent or independent. In most applications, the system is recursive, but models with non-recursive systems have also been proposed. Recursive systems are considered in the following two examples, and a non-recursive system for latent variables after that.

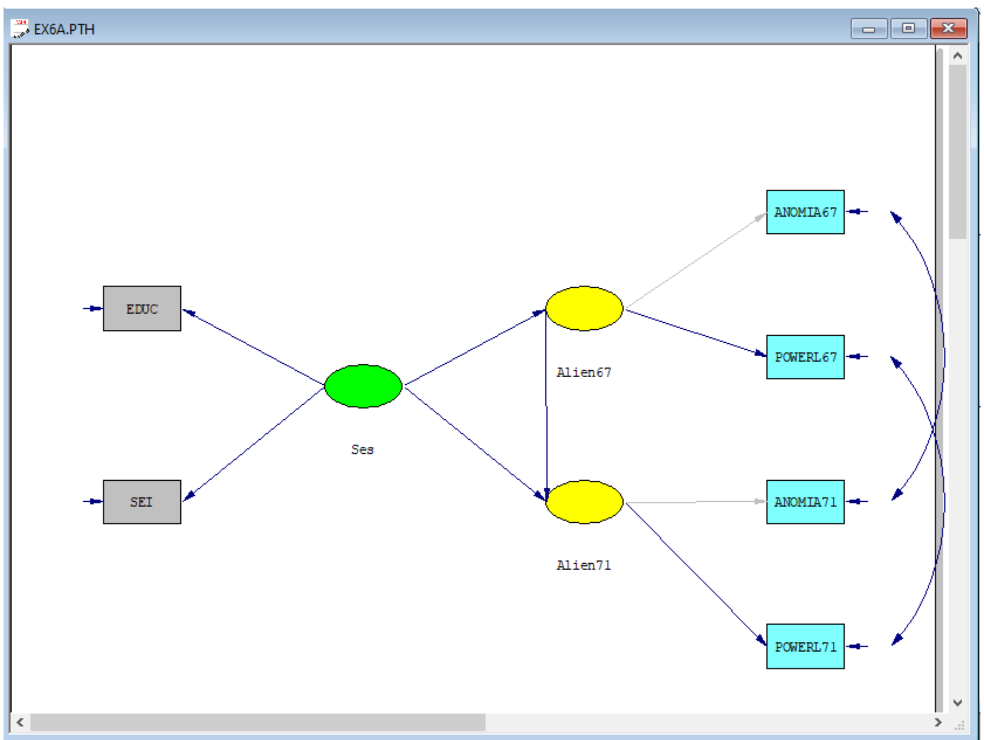
Recursive models are particularly useful for analyzing data from longitudinal studies in psychology, education, and sociology. In the sociological literature, there have been a number of articles concerned with the specification of models incorporating causation and measurement errors, and analysis from data from panel studies; see Bohrnstedt (1969), Heise (1969, 1070) and Duncan (1969, 1972). Jöreskog and Sörbom (1976, 1977, 1985), Jöreskog (1979b), Jagodzinski & Kühnel (1988), among others, discuss statistical models and methods for analysis of longitudinal data.

The characteristic feature of a longitudinal research design is that the same measurement instruments are used on the same people at two or more occasions. The purpose of a longitudinal or panel study is to assess the changes that occur between the occasions, and to attribute these changes to certain background characteristics and events existing or occurring before the first occasion and/or to various treatments and developments that occur after the first occasion. Often, when the same variables are used repeatedly, there is a tendency for the measurement errors in these variables to correlate over time because of specific factors, memory or other retest effects. Hence there is a need to consider models with correlated measurement errors.

Wheaton, *et al.*, (1977) report on a study concerned with the stability over time of attitudes such as alienation, and the relation to background variables such as education and occupation. Data on attitude scales were collected from 932 persons in two rural regions in Illinois at three points in time: 1966, 1967, and 1971. The variables used for the present examples are the Anomia subscale and the Powerlessness subscale, taken to be indicators of Alienation. This example uses data from 1967 and 1971 only. The background variables are the respondent's education (years of schooling completed) and Duncan's Socioeconomic Index (SEI). These are taken to be indicators of the respondent's socioeconomic status (Ses). The sample covariance matrix of the six observed variables is given in Table 1 below.

Table 1: Covariance matrix for stability of alienation

ANOMIA67	11.834					
POWERL67	6.947	9.364				
ANOMIA71	6.819	5.091	12.532			
POWERL71	4.783	5.028	7.495	9.986		
EDUC	-3.839	-3.889	-3.841	-3.625	9.610	
SEI	-2.190	-1.883	-2.175	-1.878	3.552	4.503



The model to be considered here is shown in the figure below. We specify the error terms of ANOMIA and POWERL to be correlated over time to take specific factors into account. The four one-way arrows on the right side represent the measurement errors in ANOMIA67, POWERL67, ANOMIA71, and POWERL71, respectively. The two-way arrows on the right side indicate that some of these measurement errors are correlated. The covariance between the two error terms for each variable can be interpreted as a specific error variance. For other models for the same data, see Jöreskog & Sörbom (1989, pp. 170-171).

To set up this model for SIMPLIS is straightforward as shown in the following input file (**EX6A.SPL** in the **Simplis Examples** folder).

```
EX6A.SPL

Stability of Alienation
Observed Variables
  ANOMIA67  POWERL67  ANOMIA71  POWERL71  EDUC  SEI
Covariance Matrix
  11.834
   6.947   9.364
   6.819   5.091   12.532
   4.783   5.028   7.495   9.986
  -3.839  -3.889  -3.841  -3.625   9.610
  -2.190  -1.883  -2.175  -1.878   3.552  4.503
Sample Size 932
Latent Variables  Alien67 Alien71 Ses
Relationships
  ANOMIA67 POWERL67 = Alien67
  ANOMIA71 POWERL71 = Alien71
  EDUC SEI = Ses

  Alien67 = Ses
  Alien71 = Alien67 Ses

Let the Errors of ANOMIA67 and ANOMIA71 Correlate
Let the Errors of POWERL67 and POWERL71 Correlate
Path Diagram
End of Problem
```

The model is specified in terms of relationships. The first three lines specify the relationships between the observed and the latent variables. The last two lines specify the structural relationships. For example,

```
ANOMIA71 POWERL71 = Alien71
```

means that the observed variables ANOMIA71 and POWERL71 depend on the latent variable Alien71, i.e., that ANOMIA71 and POWERL71 are indicators of Alien71. The line

```
Alien71 = Alien67 Ses
```

means that the latent variable Alien71 depends on the two latent variables Alien67 and Ses. This is one of the two structural relationships.

One can specify the model in terms of its paths instead of its relationships:

```
Paths
      Alien67 -> ANOMIA67 POWERL67
      Alien71 -> ANOMIA71 POWERL71
      Ses -> EDUC SEI
      Alien67 -> Alien71
      Ses -> Alien67 Alien71
```

The output reveals that the model fits very well. Chi-square is 4.74 with 4 degrees of freedom. The two structural equations are estimated as

```
  Alien67 =  - 0.563*Ses, Errorvar.= 0.683  , R² = 0.317
 Standerr     (0.0466)              (0.0659)
 Z-values     -12.092                10.359
 P-values      0.000                 0.000

  Alien71 = 0.567*Alien67 - 0.208*Ses, Errorvar.= 0.503  , R² = 0.497
 Standerr  (0.0477)        (0.0459)              (0.0498)
 Z-values   11.897         -4.518                 10.104
 P-values   0.000           0.000                 0.000
```

The estimate covariance matrix of all the latent variables is also given in the output file:

```
             Alien67    Alien71        Ses
            --------   --------   --------
  Alien67      1.000
  Alien71      0.684      1.000
      Ses     -0.563     -0.527      1.000
```

Since the latent variables are standardized in this solution, this is a correlation matrix. The error covariances appear in the output file as

```
Error Covariance for ANOMIA71 and ANOMIA67 = 1.625
                                            (0.314)
                                             5.174

 Error Covariance for POWERL71 and POWERL67 = 0.339
                                            (0.261)
                                             1.297
```

The solution just presented is in terms of standardized latent variables. LISREL automatically standardizes all latent variables unless some other units of measurement are specified. In this example, when a covariance matrix is analyzed and the units of measurement are the same at the two occasions, it would be more meaningful to assign units of measurement to the latent variables in relation to the observed variables. This will make the two paths from Ses directly comparable.

For this purpose, the relationships should be specified as (see **EX6B.SPL**):

```
ANOMIA67 = 1*Alien67
   POWERL67 = Alien67
```

```
ANOMIA71 = 1*Alien71
POWERL71 = Alien71
EDUC = 1*Ses
SEI = Ses

Alien67 = Ses
Alien71 = Alien67 Ses
```

The l* in the first measurement relation specifies a fixed coefficient of 1 in the relationship between ANOMIA67 and ALIEN67. The effect of this is to fix the unit of measurement in Alien67 in relation to the unit in the observed variable ANOMIA67. Similarly, in the third relationship, the unit of measurement in Alein71 is fixed in relation to the unit in the observed variable ANOMIA71. Since ANOMIA67 and ANOMIA71are measured in the same units, this puts Alien67 and Alien71 on the same scale. The fifth relationship specifies Ses to be on the same scale as EDUC.

When the model is estimated with this new set of scales, the results are as follows:

```
 Alien67 =  - 0.575*Ses, Errorvar.= 4.847 , R² = 0.317
Standerr     (0.0564)             (0.468)
Z-values     -10.195               10.359
P-values      0.000                0.000

 Alien71 = 0.607*Alien67 - 0.227*Ses, Errorvar.= 4.087 , R² = 0.497
Standerr  (0.0510)       (0.0523)             (0.405)
Z-values   11.897        -4.334                10.104
P-values   0.000          0.000                0.000
     Covariance Matrix of Latent Variables

            Alien67    Alien71       Ses
           --------   --------   --------
 Alien67      7.097
 Alien71      5.196      8.130
     Ses     -3.913     -3.919      6.805
```

It should be emphasized that the two solutions presented here simply represent the same model with the latent variables in different units. The two solutions are equivalent in the sense of goodness-of-fit to the data.

The effect of Ses on Alienation is negative and larger in 1967 than in 1971, as should be expected. The covariance between the measurement errors in POWERL67 and POWERL71 is not significant. Thus, whereas the specific variance in the ANOMIA measures is rather large, there is no evidence of a specific variance in the POWERLESSNESS measure.