Scientific Software International

Two-stage least squares

1. Introduction

Two-stage least-squares (TSLS) is particularly useful for estimating econometric models of the form

$$\mathbf{y} = \mathbf{B}\mathbf{y} + \mathbf{\Gamma}\mathbf{x} + \mathbf{u},$$

where $\mathbf{y} = (y_1, y_2, ..., y_p)$ is a set of endogenous or jointly dependent variables, $\mathbf{x} = (x_1, x_2, ..., x_q)$ is a set of exogenous or predetermined variables uncorrelated with the error terms $\mathbf{u} = (u_1, u_2, ..., u_p)$, and $\mathbf{B}$ and $\mathbf{\Gamma}$ are parameter matrices.

A typical feature of the above model is that not all y-variables and not all x-variables are included in each equation.

A necessary condition for identification of each equation is that, for every y-variable on the right side of the equation, there must be at least one x-variable excluded from that equation. There is also a sufficient condition for identification, the so-called rank condition, but this is often difficult to apply in practice. For further information on identification of interdependent systems, see, e.g., Goldberger (1964, pp. 313-318).

2. Klein's Model I of US Economy

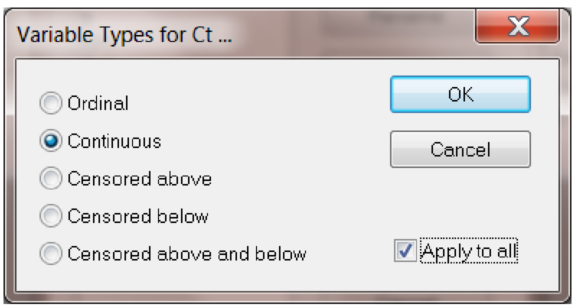
Klein's (1950) Model I is a classical econometric model that has been used extensively as a benchmark problem for studying econometric methods. It is an eight-equation system based on annual data for the United States in the period between the two World Wars. It is dynamic in the sense that elements of time play important roles in the model.

The data set, **klein.lsf** (**Prelis examples** folder), consists of the following 15 variables:

Ct	Aggregate Consumption
Pt_1	Total Profits, previous year
Wt*	Private Wage Bill
It	Net Investment
Kt_1	Capital Stock, previous year
Et_1	Total Production of Private Industry, previous year
Wt**	Government Wage Bill
Tt	Taxes
At	Time in Years from 1931
Pt	Total Profits
Kt	End-of-year Capital Stock
Et	Total Production of Private Industry
Wt	Total Wage Bill
Yt	Total Income
Gt	Government Non-Wage Expenditure

To estimate the consumption function, we use Ct as the y-variable, Pt, Pt_1 and Wt as x-variables and Wt**, Tt, Gt, At, Pt_1, Kt_1 and Et_1 as the z-variables. An intercept term in the equation can be estimated by introducing a variable denoted in this example by ONE, which is a constant equal to 1 for each year. When an intercept term is introduced, the moment matrix (MM) is used instead of the covariance matrix (CM).

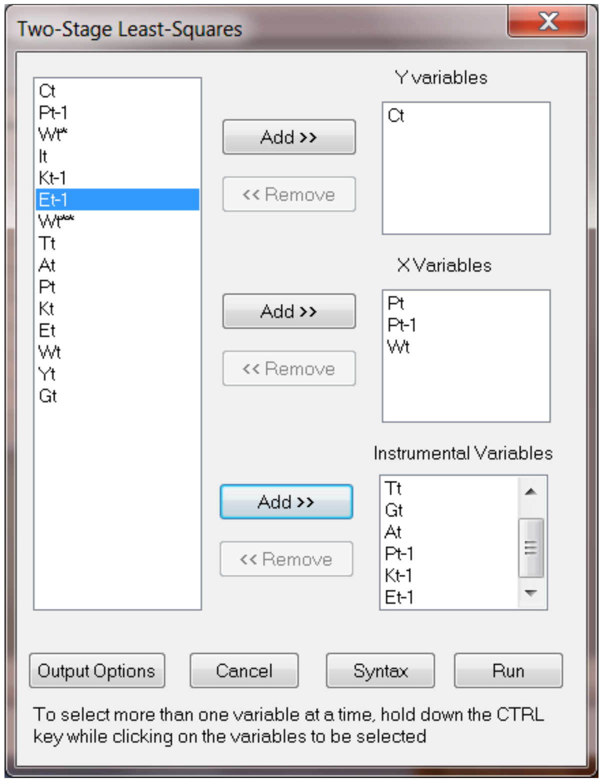
To change the variable type from the default of ordinal to continuous, select the variable Ct and click on **Variable Type** on the **Define Variables** dialog box to go to the **Variable Type for** dialog box.



Check the **Apply to all** check box, then click **OK** to return to the **Define Variables** dialog box. On this dialog box, also click **OK**. The spreadsheet now contains the renamed variables. Use the **File, Save** option to save any changes made to the **.lsf** file.

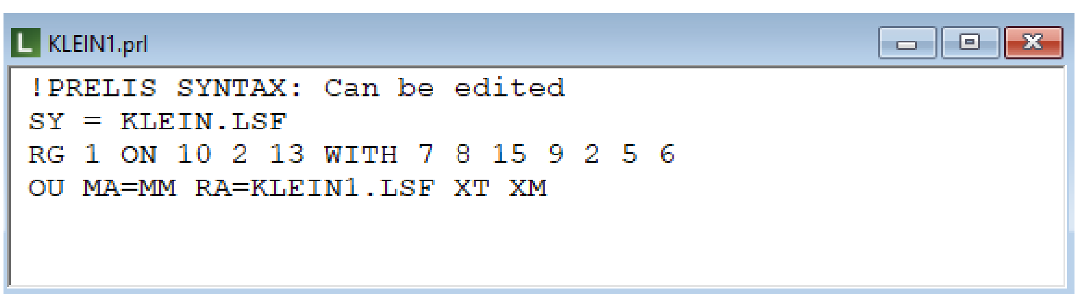
Next double click on the variable Intcept, then click on the equal sign and finally on the number 1. When done, click **OK** to generate the new spreadsheet with the variable Intcept added as the last column. Finally, select **Two-Stage Least-Squares** from the **Statistics** menu to obtain the **Two-Stage Least-Squares** dialog box.

Enter Ct as the Y-variable, Pt, Pt_1 and Wt as the X–variables and Wt**, Tt, Gt, At, Pt_1, Kt_1 and Et_1 as the instrumental variables.



Click on **Output Options** and select **Moments about zero** from the **Moment Matrix** drop-down list box. At this stage one could also save the data set under a different name. If the file extension is *.**lsf**, a LISREL system data file is created.

When done, click **OK** to return to the **Two-Stage Least-Squares** dialog box and then select **Run** to do the analysis or **Syntax** to view the newly created PRELIS syntax file (**klein.prl**).



A portion of the output, showing the estimated equations is given next.

KLEIN.OUT

```
 Estimated Equations

       Ct = 16.555 + 0.0173*Pt + 0.216*Pt-1 + 0.810*Wt + Error, R² = 0.977
 Standerr  (1.468)  (0.131)     (0.119)      (0.0447)
 t-values   11.277   0.132       1.814        18.111
 P-values   0.000    0.897       0.086        0.000

 Error Variance = 1.290

 Instrumental Variables: Wt** Tt Gt At Pt-1 Kt-1 Et-1

 The following chi-squares test the hypothesis that all
 regression coefficients are zero except the intercept.
 Variable       -2lnL  Chi-square   df  Covariates
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       Ct      60.501      78.955    3   Pt Pt-1 Wt

Analysis of Variance Table
   Regression d.f.    Residual d.f.           F  Covariates
   ---------------    -------------           -  ----------
      919.504    3      21.925   17     237.650  Pt Pt-1 Wt
```