Simulation study based on a linear growth curve model

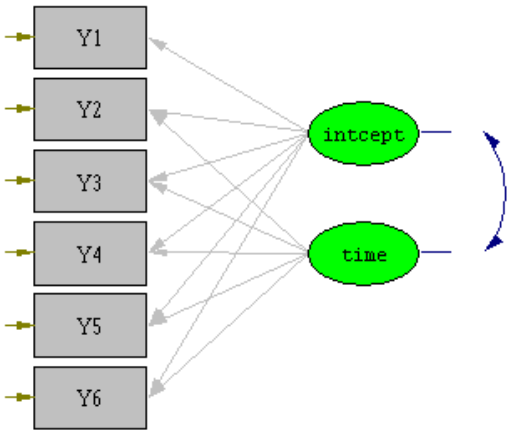
1. Introduction

A simulation study based on a linear growth model for continuous outcomes is discussed in this section. A structural equation model, mathematically equivalent to a multilevel model, is fiited to the data.

An unbalanced design, consisting of 500 univariate observations that are clustered within 100 level-two units, was used. Half of the level two units have four observations and the other half have six observations. The times of the observations are equally spaced starting at 0 and ending with 3 for the clusters with 4 observations and ending with 5 for the clusters with 6 observations. The linear growth model has random intercept and slope coefficients. Data is given in **simul.lsf** in the **Complex survey sample examples** folder.

2. The model

A path diagram of the model is shown below.



The LISREL model is

$$\mathbf{x} = \mathbf{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta}$$

where

$$\mathbf{\Lambda}_x = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix},$$

$$E(\boldsymbol{\xi}) = \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix}, \; Cov(\boldsymbol{\xi}) = \begin{bmatrix} \phi_{11} & \phi_{21} \\ \phi_{21} & \phi_{22} \end{bmatrix},$$

and $Cov(\boldsymbol{\delta}) = \sigma^2 \mathbf{I}$.

2.1 Setting up the analysis

The SIMPLIS syntax for fitting this growth model is as follows.

Simul.spl

```
Linear Growth Curve simulated data
Raw data from file SIMUL.LSF
Latent variables intcept time
Relationships
Y1 = 1*intcept  0*time
Y2 = 1*intcept  1*time
Y3 = 1*intcept  2*time
Y4 = 1*intcept  3*time
Y5 = 1*intcept  4*time
Y6 = 1*intcept  5*time
intcept time = CONST
Equal Error Variances: Y1 - Y6
Path diagram
Iterations = 240
LISREL OUTPUT ND = 3
Method of Estimation: Maximum Likelihood
End of Problem
```

The command intcept time = CONST indicates the estimation of the population intercept and slope coefficients. The command Equal Error Variances: Y1 – Y6 specifies a homogeneous error variance term on level-1 of the model.

2.2 Discussion of results

Results of the simulation study are summarized in Tables 1 and 2.

Table 1: Bias and Coverage in LISREL and Mplus

Parameter	True Value	Bias		Coverage	
		LISREL	Mplus	LISREL	Mplus
β_0	0.5	0.017	0.017	0.908	0.908
β_1	0.1	0.002	0.002	0.938	0.942
ϕ_{11}	1	-0.024	-0.024	0.845	0.848
ϕ_{22}	0.2	-0.006	-0.006	0.892	0.902
ϕ_{21}	0.3	-0.006	-0.006	0.936	0.940
σ^2	1	-0.008	-0.008	0.908	0.910

The bias and coverage produced by LISREL and Mplus are virtually identical.

As part of the simulation study, the unadjusted and adjusted χ^2 goodness-of-fit statistics for each of the 500 simulations were computed. The adjusted χ^2 was obtained by the multiplication of the unadjusted χ^2 with the χ^2 scale factor. The mean values and rejection rates of these statistics for the 500 samples are given in Table 2. The rejection rate (expressed as a percentage) denotes the number of times the χ^2 statistic exceeded $\chi^2_{21;0.05}$. The degrees of freedom, 21, is obtained as the number of non-duplicated elements of the covariance matrix plus the number of means minus the number of parameters estimated. Therefore $df = 21 + 6 - 6$.

Table 2: Mean values and rejection rates of χ^2 goodness-of-fit statistics

Unadjusted χ^2	**Adjusted χ^2**
32.98	22.49
32 %	9 %

The expected mean for a χ^2_{21} random variate is 21. The mean for the adjusted χ^2, which is higher than the expected mean, explains the rejection rate being higher than 5 %. This result implies that more research on the correction factor of the χ^2 test statistic under complex sampling is indicated.