

The assessment of invariance

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1. Introduction

In practice, many multivariate data sets are observations from several groups. Examples of these groups are genders, languages, political parties, countries, faculties, colleges, schools, etc. For these data sets, it is often of interest to determine whether or not the parameters of the structural equation model for the observed variables are invariant across the groups. The statistical methods for multiple group structural equation modeling may be used to determine whether or not these parameters are invariant across the groups.

LISREL (Jöreskog & Sörbom 2006) may be used to fit multiple group structural equation models to multiple group data. Traditional statistical methods such as Maximum Likelihood (ML), Robust Maximum Likelihood (RML), Weighted Least Squares (WLS), Diagonally Weighted Least Squares (DWLS), Generalized Least Squares (GLS) and Un-weighted Least Squares (ULS) are available for complete multiple group data while the Full Information Maximum Likelihood (FIML) method is available for incomplete multiple group data. The ML, RML, WLS, DWLS, GLS and ULS methods for multiple group structural equation modeling are described in Jöreskog & Sörbom (1999) while the FIML method is described in Du Toit & Du Toit (2001).

In this note, the FIML estimation method for incomplete data of LISREL is used to fit a measurement model to two multivariate data sets consisting of the simulated scores of a sample of boys and girls on six psychological tests. These data sets are described in the next section. The measurement model is described in Section 3. Thereafter, the measurement model is fitted to separately to the two data sets. In Section 4, we assess the invariance of the measurement model across gender. The multiple-group modeling feature of LISREL is used in Section 6 to cross-validate the measurement model across the two gender groups. In Section 7, the invariance of the factor loadings across the two gender groups is tested.

2. The data

The data are the simulated scores of 1250 girls and 1250 boys on six psychological tests. The raw data are listed in the LISREL System Files **MGGIRLS.LSF** and **MGBOYS.LSF** in the location **LISREL Examples\Multiple Groups\FIML for missing data** folder. The first couple of lines of these two files are shown in the following LSF windows respectively.

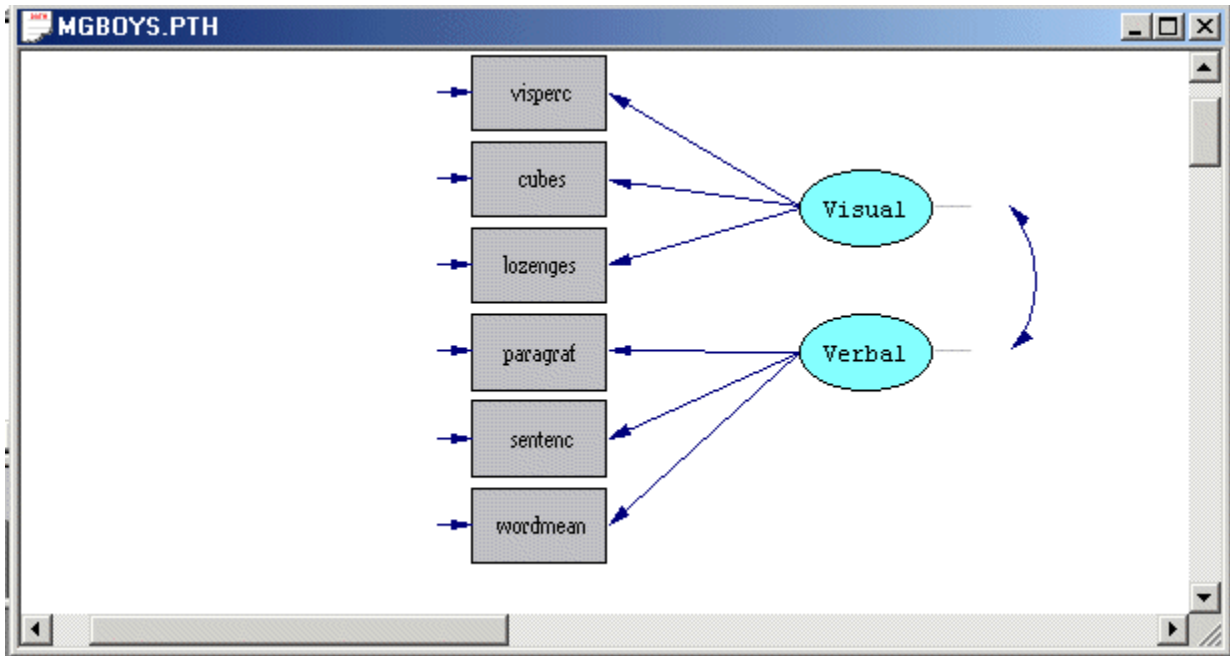
	visperc	cubes	ozenges	paragraf	sentenc	rordmeal
1	-0.71	-1.96	-6.87	-9.00	-0.24	7.60
2	-6.42	-8.64	-4.37	0.59	1.53	9.63
3	8.79	-2.15	10.21	2.29	2.81	8.92
4	9.72	-2.38	14.83	0.48	4.17	6.80
5	-9.00	0.65	-8.02	-9.00	-7.51	-12.61
6	-10.57	-9.00	-2.18	0.48	6.70	-9.00
7	-9.00	7.80	-9.00	-9.00	-1.22	5.38
8	-9.00	-2.14	5.24	-1.01	2.72	3.73
9	2.60	5.72	-0.64	-0.04	-1.32	-9.00
10	0.73	-7.94	-6.89	-9.00	-9.00	-27.54
11	-3.66	-4.20	3.51	6.26	9.25	12.35
12	-6.26	0.88	-14.83	4.86	0.88	12.27
13	6.53	-9.00	-8.80	-9.00	-9.00	-1.79
14	8.77	-3.62	-9.00	-3.47	-0.57	-9.00
15	-2.61	-6.94	1.41	-0.65	-5.81	-9.00
16	2.14	-3.61	3.60	5.11	3.33	6.69
17	-8.59	-5.92	-10.00	3.70	2.84	6.02
18	-7.98	-0.13	-7.12	-4.13	-7.88	-7.12
19	11.09	-3.32	6.42	-2.06	-1.24	-5.31
20	2.19	0.49	-1.88	5.21	-9.00	7.14

	visperc	cubes	ozenges	paragraf	sentenc	rordmeal
1	-0.72	-1.95	-7.69	-9.00	-9.00	6.71
2	-6.45	-8.07	-2.52	0.54	1.25	9.17
3	8.83	-3.73	11.51	2.46	2.89	9.31
4	-9.00	-4.13	16.95	1.23	4.65	8.29
5	-9.00	1.08	-9.41	-9.00	-6.49	-11.79
6	-10.63	-9.00	0.32	0.58	5.88	-9.00
7	-9.00	6.41	-9.00	-9.00	-0.04	-9.00
8	-9.00	-1.64	7.20	-0.43	2.98	4.68
9	2.61	5.62	-2.34	-0.26	-1.39	-9.00
10	0.73	-8.51	-6.62	-9.00	-9.00	-24.84
11	-3.68	-3.84	5.57	5.55	7.62	10.74
12	-6.29	1.97	-17.09	3.20	-0.97	8.66
13	6.57	-9.00	-11.94	-9.00	-9.00	-2.80
14	8.81	-5.28	-9.00	-2.74	0.14	-9.00
15	-2.62	-9.00	3.51	-0.24	-4.57	-9.00
16	2.15	-4.17	4.80	4.53	2.59	5.69
17	-8.64	-4.84	-9.55	2.73	1.48	3.89
18	-8.02	1.18	-7.44	-3.75	-6.72	-6.29
19	11.14	-5.34	6.93	-1.35	-0.32	-3.71
20	2.21	0.16	-2.64	4.17	-9.00	5.16

Note that the -9.000 entries represent missing values.

3. The measurement model

We consider six psychological tests for Junior High students. These six psychological tests are theoretically constructed to measure the visual perception and verbal ability of Junior High students. A path diagram for the corresponding measurement model for visual perception and verbal ability is shown in the figure below.



4. Separate analyses

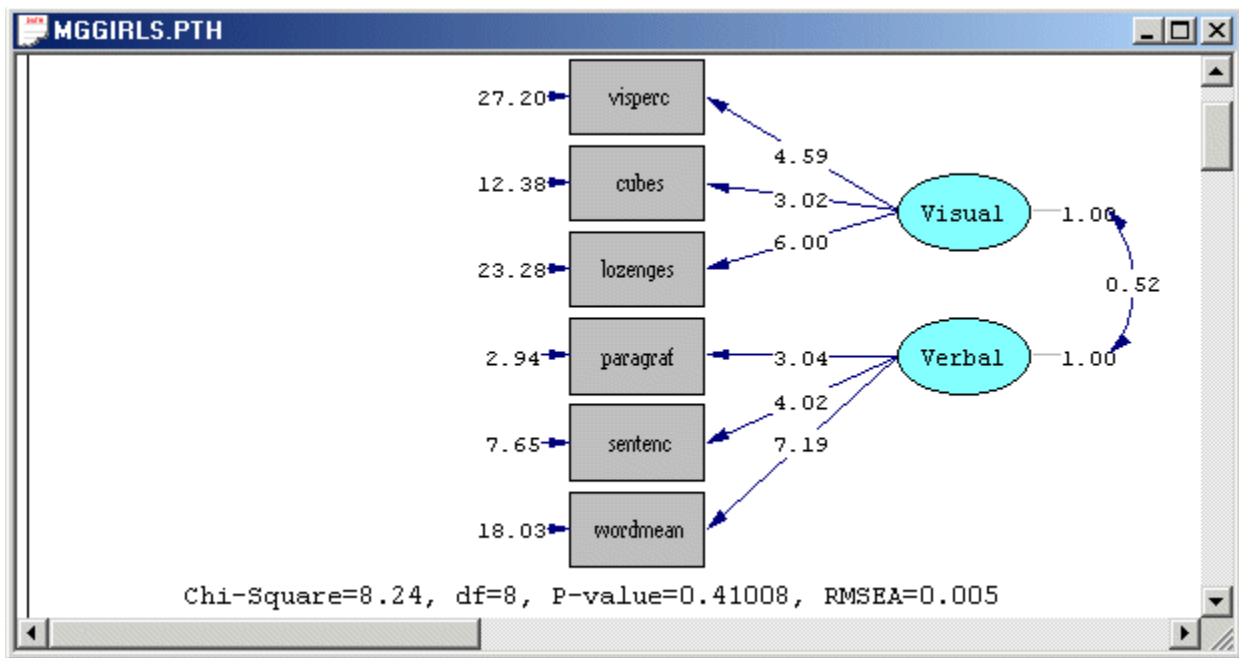
Fitting the model to the girl data:

Use the **Open** option on the **File** menu to load the **Open** dialog box. Browse for the file **MGGIRLS.SPL** and click on the **Open** button to open the following text editor window.

```
MGGIRLS.SPL
The measurement model for girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
Relationships
visperc cubes lozenges = Visual
paragraf sentenc wordmean = Verbal
LISREL Output: ND=3 SC
Path Diagram
End of Problem
```

- Line 1 specifies a descriptive title for the analysis.
- Line 2 specifies the raw data source.
- Line 3 specifies labels for the latent variables.
- Lines 4 to 6 specify the measurement model.
- Line 7 specifies that the results should be printed in terms of the LISREL model for the measurement model in Figure 1, requests results with three decimal places and the completely standardized solution.
- Line 8 requests a path diagram.
- Line 9 indicates no more SIMPLIS commands are to be processed.

Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



Fitting the measurement model to the boy data:

Select the **Close all** option on the **Windows** menu. Use the **Open** option on the **File** menu to load the **Open** dialog box. Browse for the file **MGBOYS.SPL** and click on the **Open** button to open the following text editor window.

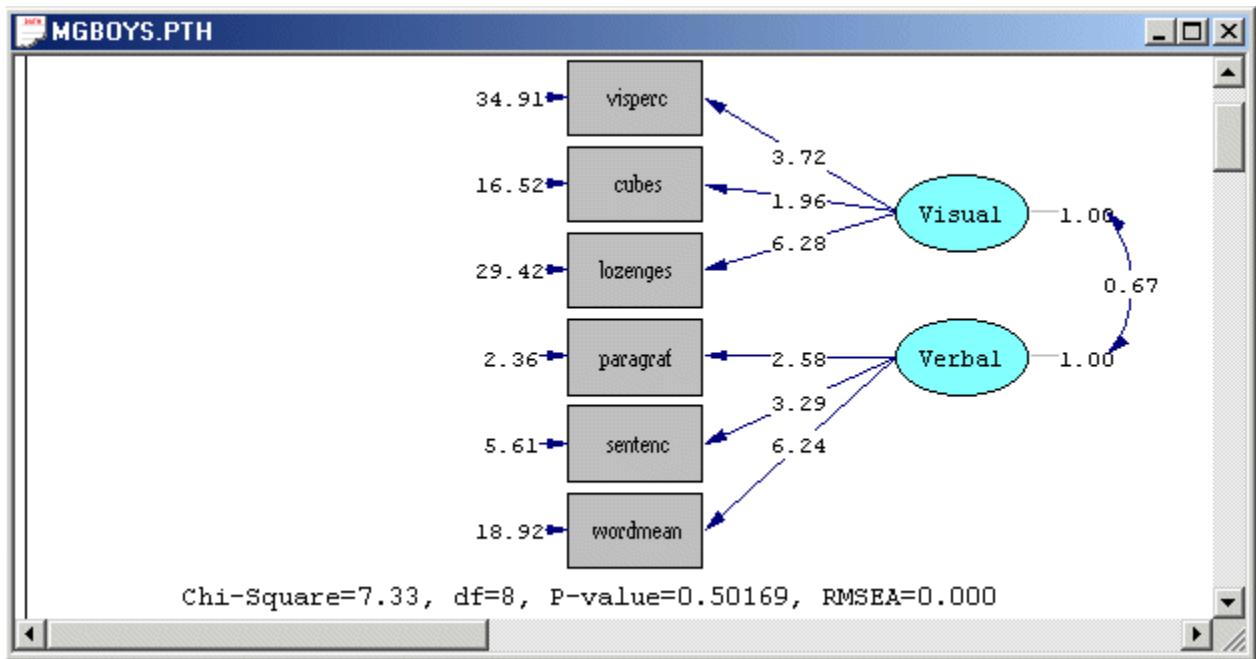
```

MGB0YS.SPL
The measurement model for girls
Raw Data from File MGB0YS.PSF
Latent Variables: Visual Verbal
Relationships
visperc cubes lozenges = Visual
paragraf sentenc wordmean = Verbal
LISREL Output: ND=3 SC
Path Diagram
End of Problem

```

- Line 1 specifies a descriptive title for the analysis.
- Line 2 specifies the raw data source.
- Line 3 specifies labels for the latent variables.
- Lines 4 to 6 specify the measurement model.
- Line 7 specifies that the results should be printed in terms of the LISREL model for the measurement model in Figure 1, requests results with three decimal places and the completely standardized solution.
- Line 8 requests a path diagram.
- Line 9 indicates no more SIMPLIS commands are to be processed.

Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



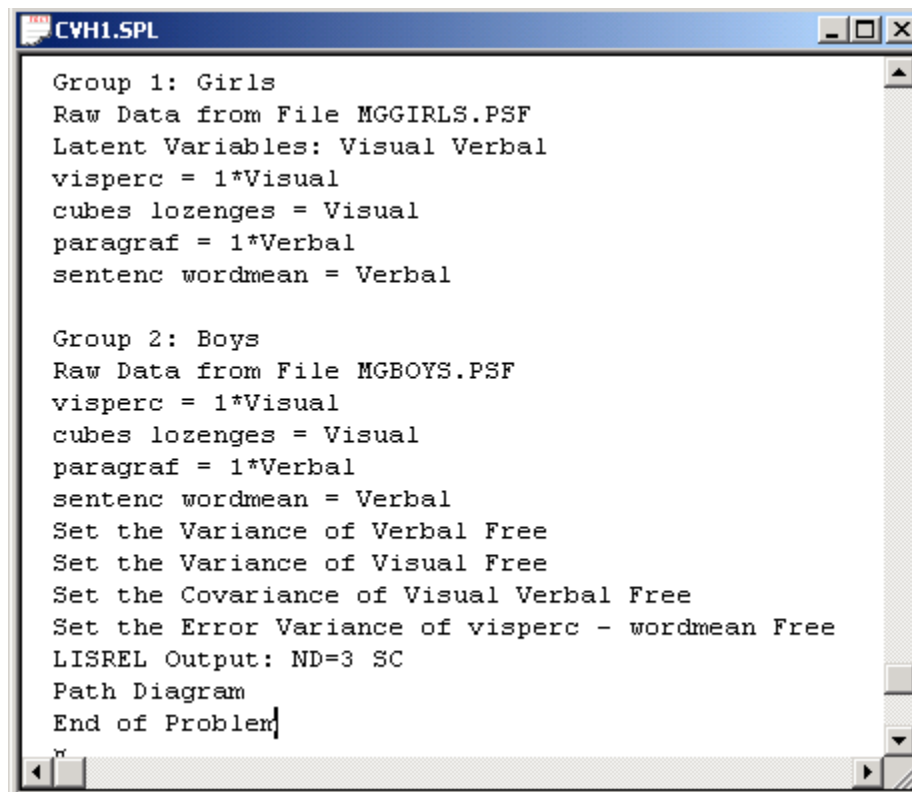
5. Assessing invariance

The invariance of a model is often of interest if the sample data consist of data from different groups such as males and females, different political parties, freshmen, sophomores, juniors and seniors, etc. In this section, we illustrate how LISREL can be used to assess various levels of invariance across groups.

Configural invariance

Configural invariance is achieved if the model of interest fits across the groups. Although the model is the same across groups, the unknown parameters of the model are assumed to be different across the groups. The multiple group (global) Chi-square test statistic for this multiple group model is used to assess configural invariance. The measurement model in Figure 1 will now be used to illustrate how the multiple group feature of LISREL may be used to assess the configural invariance of the measurement model in the first figure across gender.

Select the **Close all** option on the **Windows** menu. Use the **Open** option on the **File** menu to load the **Open** dialog box. Browse for the file **CVH1.SPL** and click on the **Open** button to open the following text editor window.



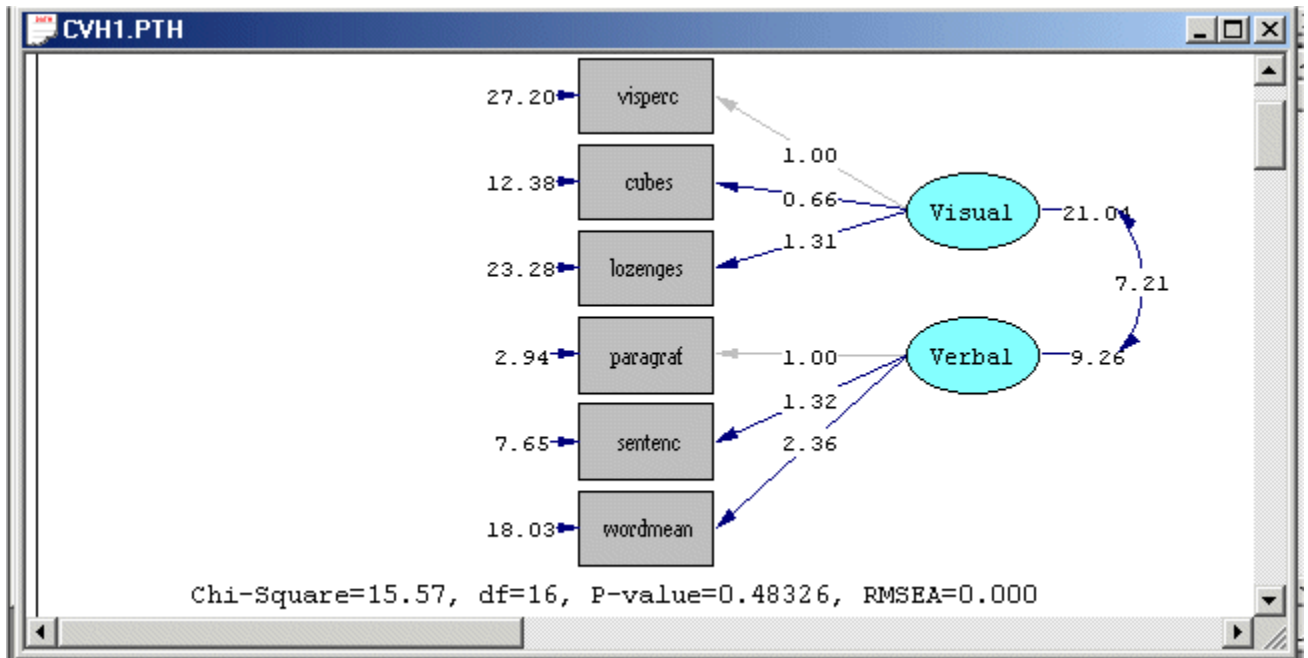
```
CVH1.SPL
Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal

Group 2: Boys
Raw Data from File MGBOYS.PSF
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal
Set the Variance of Verbal Free
Set the Variance of Visual Free
Set the Covariance of Visual Verbal Free
Set the Error Variance of visperc - wordmean Free
LISREL Output: ND=3 SC
Path Diagram
End of Problem
```

- Line 1 specifies the first group as girls.
- Line 2 specifies the location and name of the raw data file for girls.
- Line 3 specifies descriptive labels for the two latent variables.
- Lines 4-7 specify the measurement model for girls.
- Line 9 specifies the second group as boys.
- Line 10 specifies the location and name of the raw data file for boys.

- Lines 11-18 specify the measurement model for boys. Lines 15-18 specify that the variance and covariance parameters of the model are different across the two groups.
- Line 19 requests the results in terms of the parameter matrices of the LISREL model for the measurement model in Figure 1. In addition, 3 decimal places (ND=3) and the completely standardized solutions (SC) are specified.
- Line 20 requests a path diagram (PTH) file.
- Line 21 indicates the end of the SIMPLIS commands to be processed.

Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



The large *P*-value for the Chi-square test statistic value implies that the data supports the configural invariance of the measurement model in across boys and girls.

Weak invariance

Weak invariance for a measurement model implies that the model is the same across the groups and that the factor loadings are identical across the groups. Weak convergence is then achieved if this multiple group model fits the data. In the case of the measurement model shown in the figure, weak convergence across gender is assessed as follows.

Select the **Close all** option on the **Windows** menu. Use the **Open** option on the **File** menu to load the **Open** dialog box. Browse for the file **IFLH0.SPL** and click on the **Open** button to open the following text editor window.

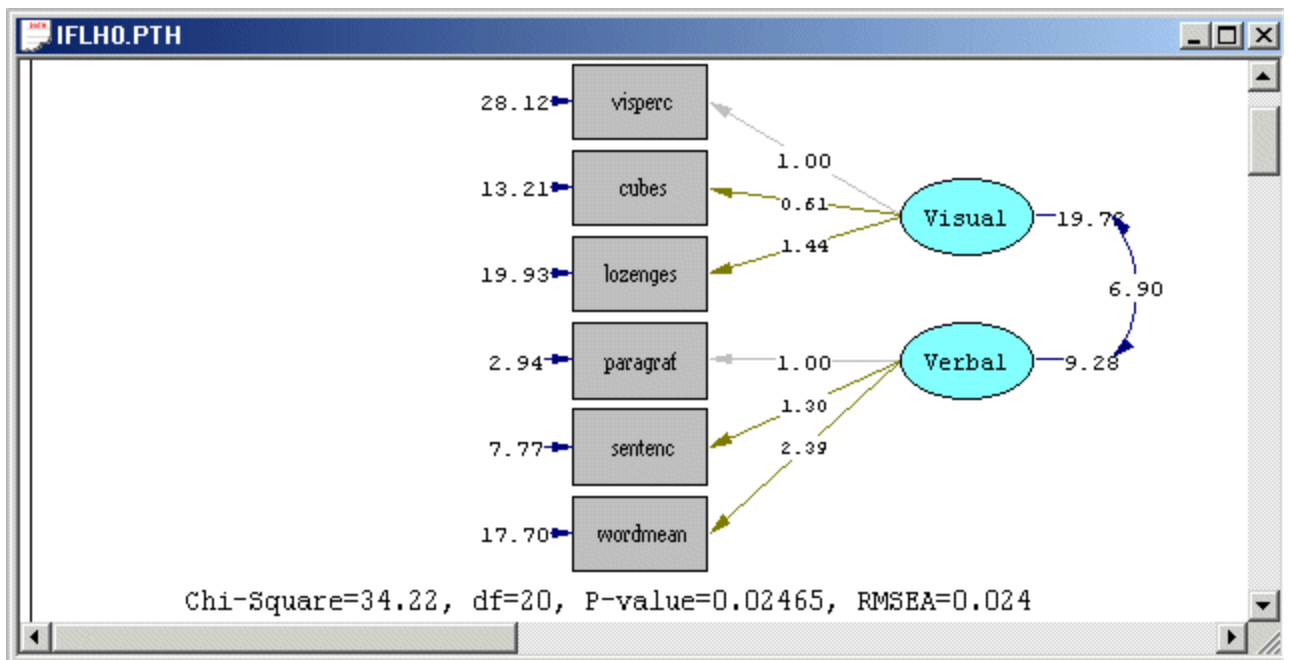
```
IFLHO.SPL

Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal

Group 2: Boys
Raw Data from File MGBBOYS.PSF
Set the Variance of Verbal Free
Set the Variance of Visual Free
Set the Covariance of Visual Verbal Free
Set the Error Variance of visperc - wordmean Free
LISREL Output: ND=3 SC
Path Diagram
End of Problem
```

- Line 1 specifies the first group as girls.
- Line 2 specifies the location and name of the raw data file for girls.
- Line 3 specifies descriptive labels for the two latent variables.
- Lines 4-7 specify the measurement model for girls.
- Line 8 specifies the second group as boys.
- Line 9 specifies the location and name of the raw data file for boys.
- Lines 10-13 specify that the variance and covariance parameters of the model are different across the two groups.
- Line 14 requests the results in terms of the parameter matrices of the LISREL model for the measurement model in Figure 1. In addition, 3 decimal places (ND=3) and the completely standardized solutions (SC) are specified.
- Line 15 requests a path diagram (PTH) file.
- Line 16 indicates the end of the SIMPLIS commands to be processed.

Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.

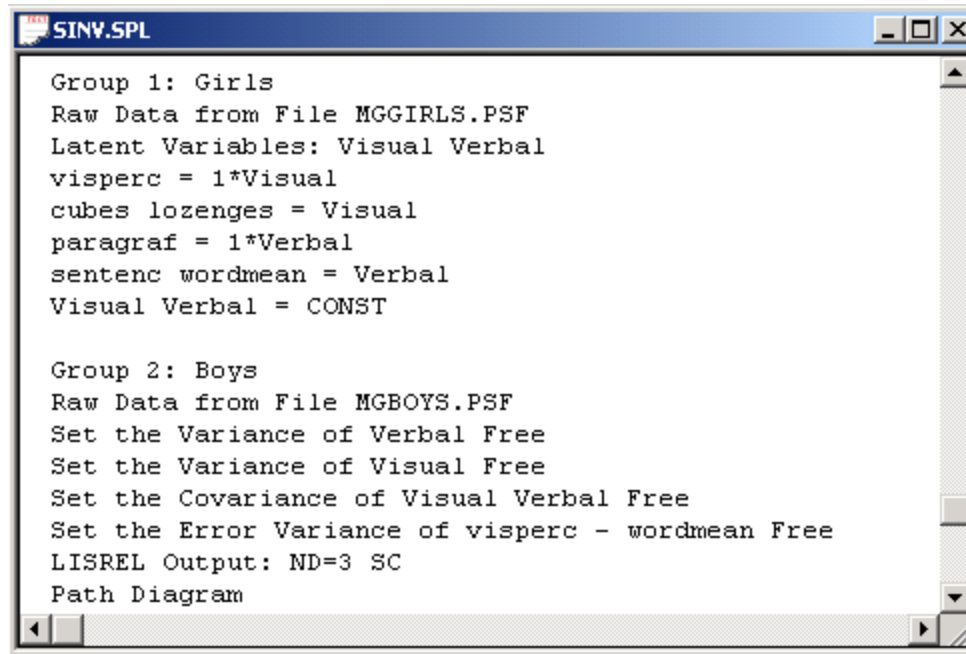


The small *P*-value suggests that there is sufficient evidence that weak invariance does not hold for the measurement model in Figure 1 across gender.

Strong invariance

The measurement model in Figure 1 is said to have strong invariance if the model is the same with identical factor loadings and means across boys and girls. This is assessed as follows.

Select the **Close all** option on the **Windows** menu. Use the **Open** option on the **File** menu to load the **Open** dialog box. Browse for the file **SINV.SPL** and click on the **Open** button to open the following text editor window.

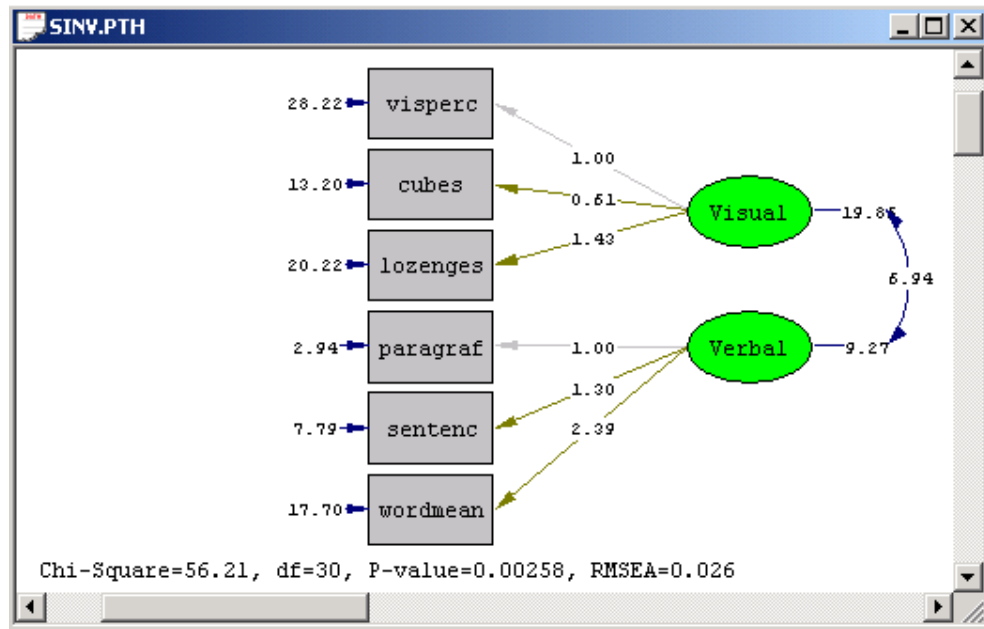


```
Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal
Visual Verbal = CONST

Group 2: Boys
Raw Data from File MGBBOYS.PSF
Set the Variance of Verbal Free
Set the Variance of Visual Free
Set the Covariance of Visual Verbal Free
Set the Error Variance of visperc - wordmean Free
LISREL Output: ND=3 SC
Path Diagram
```

- Line 1 specifies the first group as girls.
- Line 2 specifies the location and name of the raw data file for girls.
- Line 3 specifies descriptive labels for the two latent variables.
- Lines 4-7 specify the measurement model for girls.
- Line 8 specifies the latent variable (factor) means for girls.
- Line 9 specifies the second group as boys.
- Line 10 specifies the location and name of the raw data file for boys.
- Lines 11-14 specify that the variance and covariance parameters of the model are different across the two groups.
- Line 15 requests the results in terms of the parameter matrices of the LISREL model for the measurement model in Figure 1. In addition, 3 decimal places (ND=3) and the completely standardized solutions (SC) are specified.
- Line 16 requests a path diagram (PTH) file.
- Line 17 indicates the end of the SIMPLIS commands to be processed.

Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



The small *P*-value suggests that there is sufficient evidence that the measurement model does not exhibit strong invariance across the two genders.

Complete invariance

A model is completely invariant across groups if the model is the same across the groups and all the parameters are identical across the groups. We will now demonstrate how to assess complete invariance for the measurement model in Figure 1 with LISREL.

Select the **Close all** option on the **Windows** menu. Use the **Open** option on the **File** menu to load the **Open** dialog box. Browse for the file **CINV.SPL** and click on the **Open** button to open the following text editor window.

```

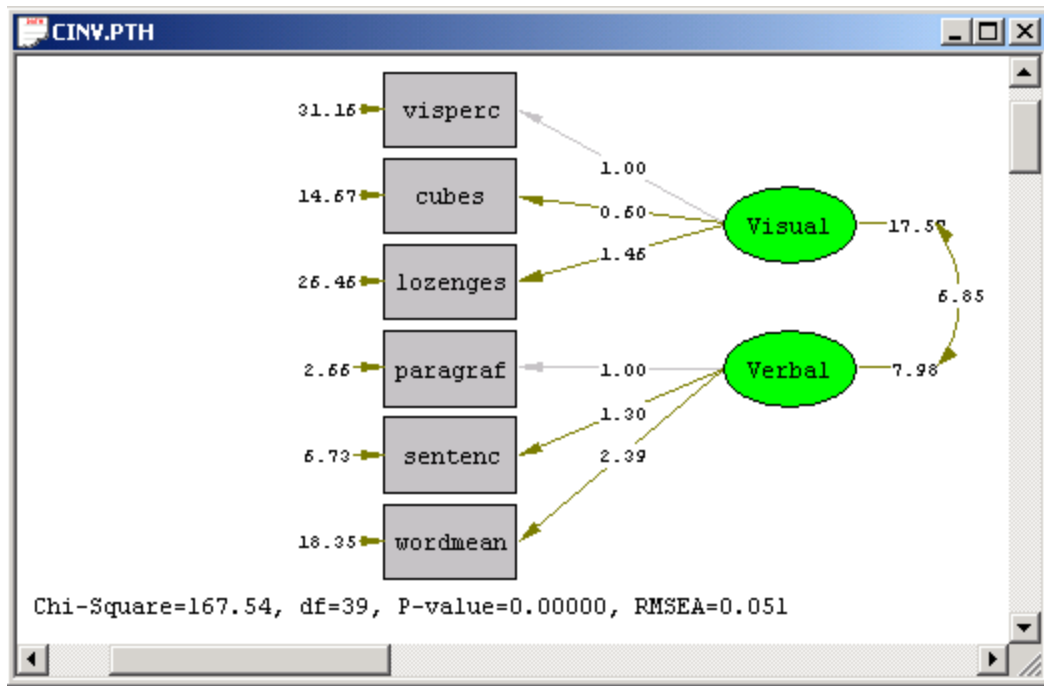
CINV.SPL

Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal
Visual Verbal = CONST

Group 2: Boys
Raw Data from File MGBOYS.PSF
LISREL Output: ND=3 SC
Path Diagram
End of Problem

```

Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



The small *P*-value suggests that there is sufficient evidence that the measurement model in Figure 1 is not completely invariant across gender.

6. Cross validation

The cross validation of a measurement model refers to the ability of the model to be invariant across two or more random samples from the same population. The measurement model in Figure 1 will be used to illustrate how the multiple group feature of LISREL may be used to assess the cross validation of a measurement model. This assessment consists of testing the null hypothesis (*H*₀) that the measurement model is identical across gender against the alternative hypothesis (*H*₁) that the model is not identical across gender. A Chi-square difference test is used to test *H*₀ and *H*₁.

Fitting the multiple group model under *H*₁

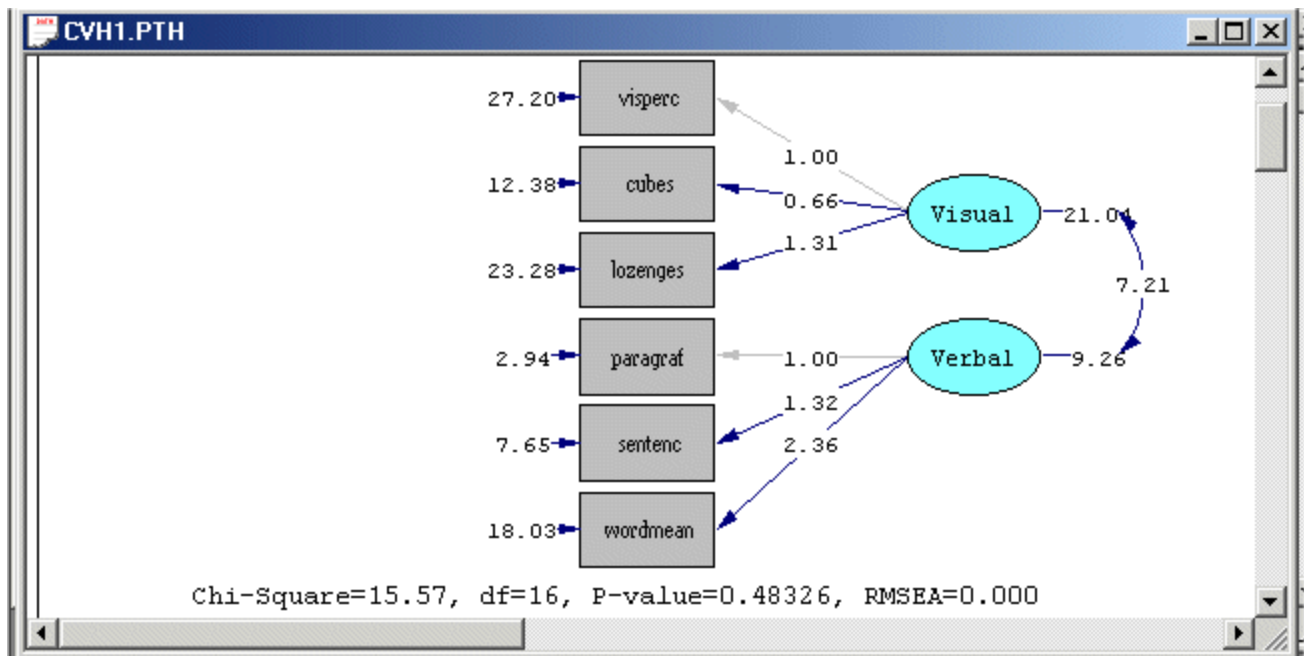
Use the **New** option on the **File** menu of the root window to load the **New** dialog box. Select the **Syntax Only** option from the list box on the **New** dialog box to open the **SYNTAX1** text editor window. Enter the following commands into the **SYNTAX1** text editor window.

```
CVH1.SPL
Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal

Group 2: Boys
Raw Data from File MGBBOYS.PSF
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal
Set the Variance of Verbal Free
Set the Variance of Visual Free
Set the Covariance of Visual Verbal Free
Set the Error Variance of visperc - wordmean Free
LISREL Output: ND=3 SC
Path Diagram
End of Problem
```

- Line 1 specifies the first group as girls.
- Line 2 specifies the location and name of the raw data file for girls.
- Line 3 specifies descriptive labels for the two latent variables.
- Lines 4-7 specify the measurement model for girls.
- Line 9 specifies the second group as boys.
- Line 10 specifies the location and name of the raw data file for boys.
- Lines 11-18 specify the measurement model for boys. Lines 15-18 specify that the variance and covariance parameters of the model are different across the two groups.
- Line 19 requests the results in terms of the parameter matrices of the LISREL model for the measurement model in Figure 1. In addition, 3 decimal places (ND=3) and the completely standardized solutions (SC) are specified.
- Line 20 requests a path diagram (PTH) file.
- Line 21 indicates the end of the SIMPLIS commands to be processed.

Use the **Save As** option on the **File** menu to load the **Save As** dialog box. Enter the name **CVH1.SPL** in the **File name** string field. Click on the **Save** button to create the text editor window for **CVH1.SPL**. Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



Fitting the multiple group model under H0

Use the **Open** option on the **File** menu to open the SIMPLIS syntax file **CVH1.SPL** in a text editor window. Delete lines 11-18. Use the **Save As** option on the **File** menu to load the **Save As** dialog box. Enter the name **CVH0.SPL** in the **File name** string field. Click on the **Save** button to produce the following text editor window.

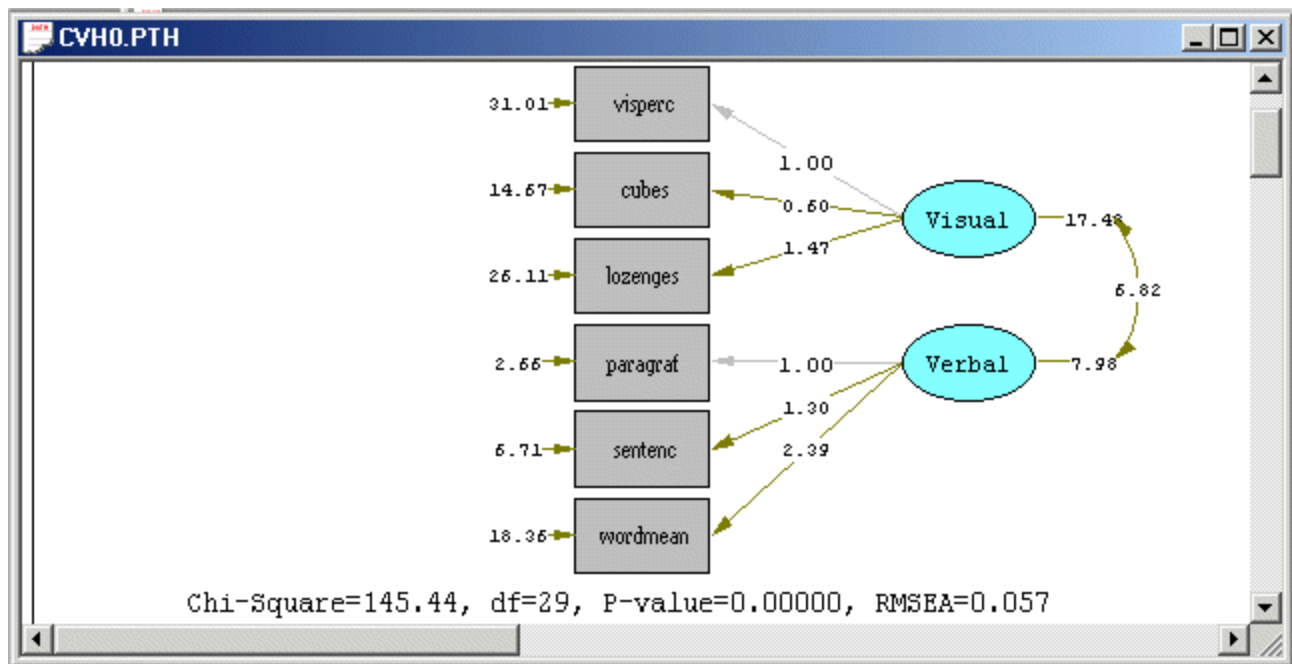
```

CVH0.SPL
Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables
Visual Verbal
Relationships
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal

Group 2: Boys
Raw Data from File MGBBOYS.PSF
LISREL Output: ND=3 SC
Path Diagram
End of Problem

```

Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



Performing the Chi-square difference test

A Chi-square difference test is used to assess the cross validation of the measurement model. In other words, a Chi-square difference test is used to test the null and alternative hypotheses. The test statistic value for the Chi-square difference test is merely the difference between the goodness-of-fit Chi-square test statistic values of the multiple group measurement models under the null and the alternative hypotheses. The associated degrees of freedom are merely the difference between the degrees of freedom of the multiple group measurement models under the null and the alternative hypotheses. The Chi-square difference test results for the measurement model for visual perception and verbal ability are summarized in the MS-Excel workbook **CV.XLS**. The contents of this file are shown below.

	A	B	C	D
1	HYPOTHESIS	CHISQ	DF	P-VALUE
2	EQUAL (H0)	145.442	29	1.88015E-17
3	UNEQUAL (H1)	15.571	16	0.483281241
4	DIFFERENCE	129.871	13	2.21864E-21

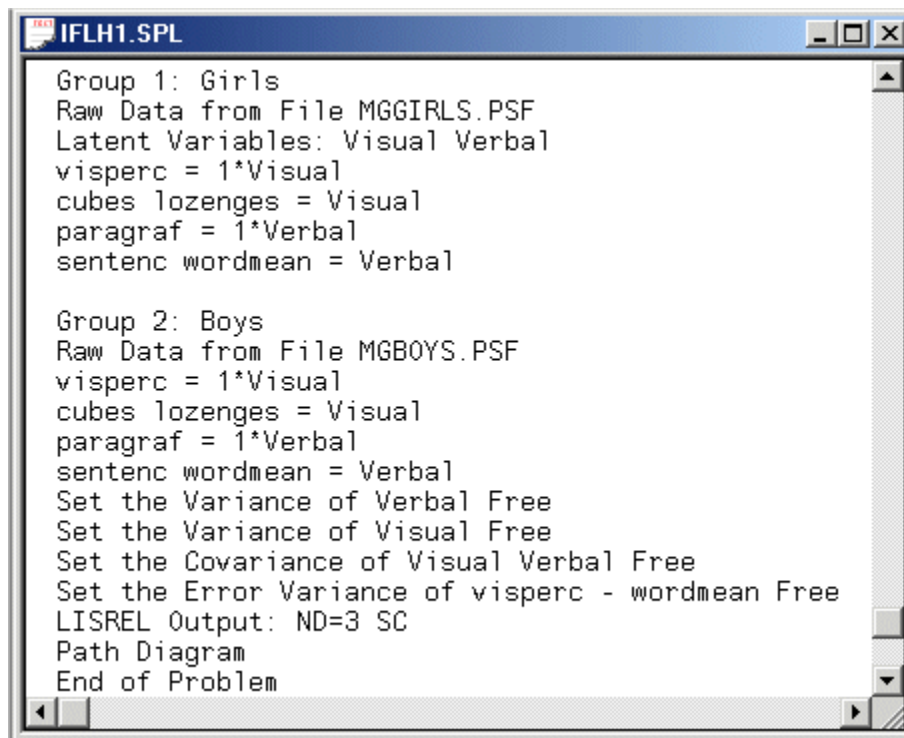
The small *P*-value suggests that there is sufficient evidence that the null hypothesis should be rejected. In other words, the cross validation of the measurement model for the six psychological tests across gender is not supported by the data of the two samples.

7. Invariance of factor loadings

The measurement model in our first figure will now be used to illustrate how the multiple group feature of LISREL may be used to assess whether or not the factor loadings of the measurement model are invariant across the 2 gender groups. This assessment consists of testing the null hypothesis (H0) that the factor loadings are identical across gender against the alternative hypothesis (H1) that the factor loadings are not identical across gender. A Chi-square difference test is used to test H0 and H1.

Fitting the multiple group model under H1

Use the **New** option on the **File** menu of the root window to load the **New** dialog box. Select the **Syntax Only** option from the list box on the **New** dialog box to open the **SYNTAX1** text editor window. Enter the following commands into the **SYNTAX1** text editor window.



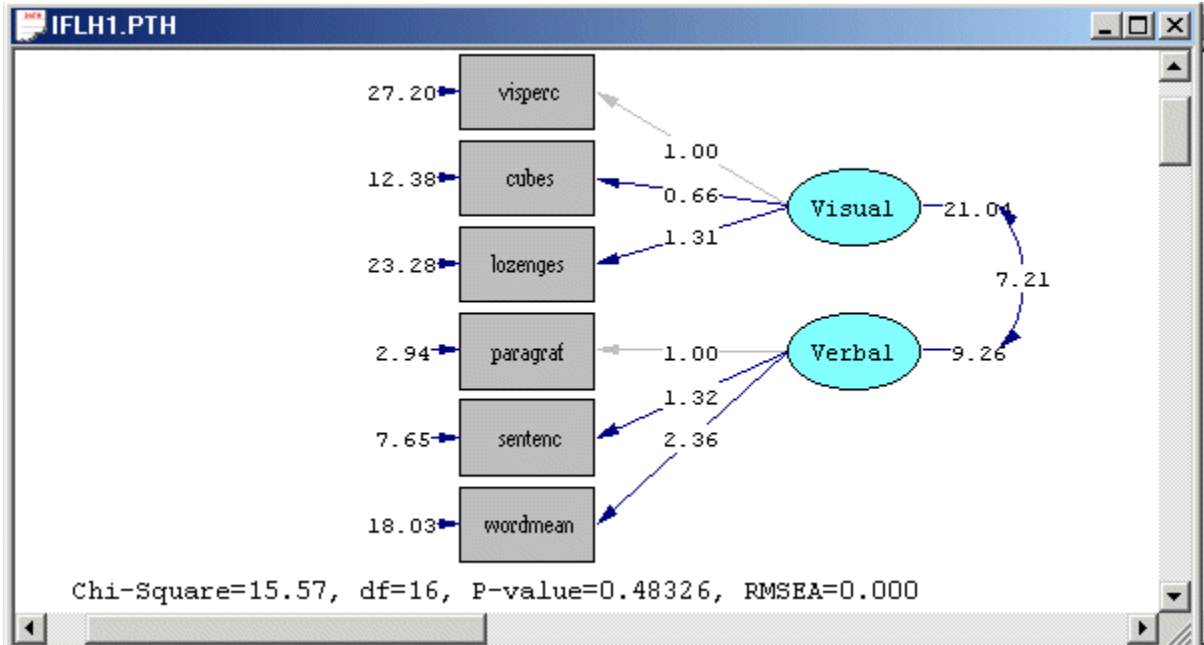
```
IFLH1.SPL
Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal

Group 2: Boys
Raw Data from File MGBBOYS.PSF
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal
Set the Variance of Verbal Free
Set the Variance of Visual Free
Set the Covariance of Visual Verbal Free
Set the Error Variance of visperc - wordmean Free
LISREL Output: ND=3 SC
Path Diagram
End of Problem
```

- Line 1 specifies the first group as girls.
- Line 2 specifies the location and name of the raw data file for girls.
- Line 3 specifies descriptive labels for the two latent variables.
- Lines 4-7 specify the measurement model for girls.
- Line 9 specifies the second group as boys.
- Line 10 specifies the location and name of the raw data file for boys.
- Lines 11-18 specify the measurement model for boys. Lines 15-18 specify that the variance and covariance parameters of the model are different across the two groups.
- Line 19 requests the results in terms of the parameter matrices of the LISREL model for the measurement model in Figure 1. In addition, 3 decimal places (ND=3) and the completely standardized solutions (SC) are specified.
- Line 20 requests a path diagram (PTH) file.

- Line 21 indicates the end of the SIMPLIS commands to be processed.

Use the **Save As** option on the **File** menu to load the **Save As** dialog box. Enter the name **IFLH1.SPL** in the **File name** string field. Click on the **Save** button to create the text editor window for **IFLH1.SPL**. Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



Fitting the multiple group model under H0

Use the **Open** option on the **File** menu to open the SIMPLIS syntax file **IFLH1.SPL** in a text editor window. Delete lines 11-18. Use the **Save As** option on the **File** menu to load the **Save As** dialog box. Enter the name **IFLH0.SPL** in the **File name** string field. Click on the **Save** button to produce the following text editor window.

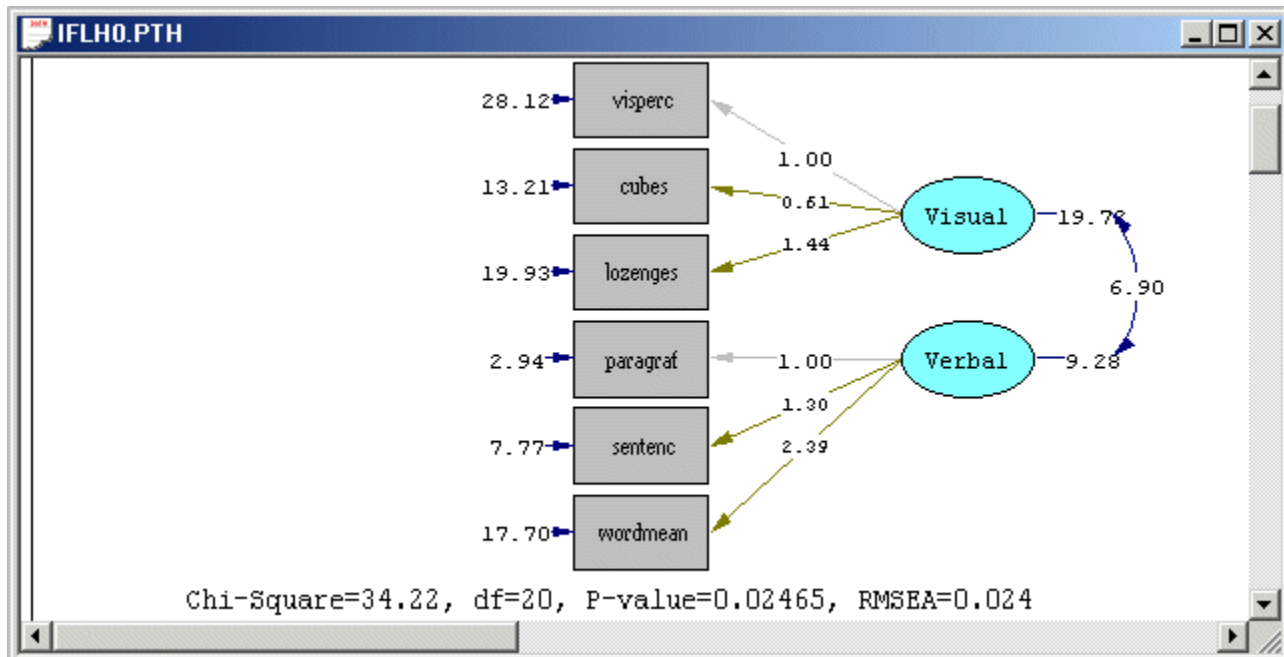
```

IFLH0.SPL
Group 1: Girls
Raw Data from File MGGIRLS.PSF
Latent Variables: Visual Verbal
visperc = 1*Visual
cubes lozenges = Visual
paragraf = 1*Verbal
sentenc wordmean = Verbal

Group 2: Boys
Raw Data from File MGBBOYS.PSF
Set the Variance of Verbal Free
Set the Variance of Visual Free
Set the Covariance of Visual Verbal Free
Set the Error Variance of visperc - wordmean Free
LISREL Output: ND=3 SC
Path Diagram
End of Problem

```

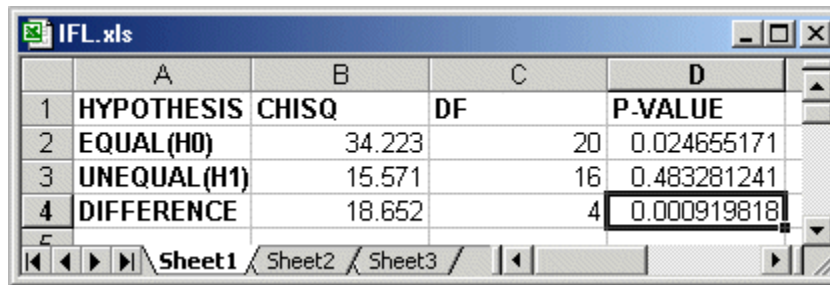
Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



Performing the Chi-square difference test

A Chi-square difference test is used to assess whether or not the factor loadings of the measurement model are invariant across the two gender groups. In other words, a Chi-square difference test is used to test the null and alternative hypotheses. The test statistic value for the Chi-square difference test is merely the difference between the goodness-of-fit Chi-square test statistic values of the multiple group measurement models under the null and the alternative hypotheses. The associated degrees of freedom are merely the difference between the degrees of freedom of the multiple group measurement models under the null and the alternative

hypotheses. The Chi-square difference test results for the measurement model for visual perception and verbal ability are summarized in the MS-Excel workbook **IFL.XLS**. The contents of this file are shown below.



	A	B	C	D
1	HYPOTHESIS	CHISQ	DF	P-VALUE
2	EQUAL (H0)	34.223	20	0.024655171
3	UNEQUAL (H1)	15.571	16	0.483281241
4	DIFFERENCE	18.652	4	0.000919818

The small P -value suggests that there is sufficient evidence that the null hypothesis should be rejected. In other words, there is sufficient evidence that the factor loadings for boys and girls are different.