Latent Variable Scores and Their Uses

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1 Latent Variable Scores

In LISREL 8.30 there is a new statistical feature called *Latent Variable Scores* described in Jöreskog, *et al.* (2000, pp. 168–171). This note explains what these scores are and how they are computed. I also discuss some of their possible uses.

Methods for estimating factor scores in exploratory factor analysis are given in Lawley & Maxwell (1971, Chapter 8). Here I am concerned with the estimation of scores of the latent variables in any single-group LISREL model estimated with any estimation method. In the first part of the note I show how such scores can be obtained such that these scores satisfy the same relationships as the latent variables themselves. In the second part of the paper I give an example of how such latent variable scores can be used to estimate a latent nonlinear relationship.

In its most general form the LISREL model is defined as follows. Consider random vectors $\boldsymbol{\eta}' = (\eta_1, \eta_2, \ldots, \eta_m)$ and $\boldsymbol{\xi}' = (\xi_1, \xi_2, \ldots, \xi_n)$ of latent dependent and independent variables, respectively, and the following system of linear structural relations

$$\boldsymbol{\eta} = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta} \ , \tag{1}$$

where $\boldsymbol{\alpha}$ is a vector of intercept terms, $\mathbf{B}$ and $\boldsymbol{\Gamma}$ are coefficient matrices and $\boldsymbol{\zeta}' = (\zeta_1, \zeta_2, \ldots, \zeta_m)$ is a random vector of residuals (errors in equations, random disturbance terms). The elements of $\mathbf{B}$ represent direct effects of η-variables on other η-variables and the elements of $\boldsymbol{\Gamma}$ represent direct effects of ξ-variables on η-variables. It is assumed that $\boldsymbol{\zeta}$ is uncorrelated with $\boldsymbol{\xi}$ and that $\mathbf{I}$ - $\mathbf{B}$ is non-singular.

Vectors $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ are not observed, but instead vectors $\mathbf{y}' = (y_1, y_2, \ldots, y_p)$ and $\mathbf{x}' = (x_1, x_2, \ldots, x_q)$ are observed, such that

$$\mathbf{y} = \boldsymbol{\tau}_y + \boldsymbol{\Lambda}_y\boldsymbol{\eta} + \boldsymbol{\epsilon} \ , \tag{2}$$

and

$$\mathbf{x} = \boldsymbol{\tau}_x + \boldsymbol{\Lambda}_x\boldsymbol{\xi} + \boldsymbol{\delta} \ , \tag{3}$$

where $\boldsymbol{\epsilon}$ and $\boldsymbol{\delta}$ are vectors of error terms (errors of measurement or measure-specific components) assumed to be uncorrelated with $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$, respectively. It is assumed that all error variables have mean zero.

Let $\boldsymbol{\kappa}$ be the mean vector of $\boldsymbol{\xi}$, $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ the covariance matrices of $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$, $\boldsymbol{\Theta}_\epsilon$ and $\boldsymbol{\Theta}_\delta$ the covariance matrices of $\boldsymbol{\epsilon}$ and $\boldsymbol{\delta}$, and $\boldsymbol{\Theta}_{\delta\epsilon}$ the covariance matrix between $\boldsymbol{\delta}$ and $\boldsymbol{\epsilon}$. From (1) and the assumptions made it follows that the mean vector $\boldsymbol{\kappa}^*$ and covariance matrix $\boldsymbol{\Phi}^*$ of $\boldsymbol{\xi}^* = (\boldsymbol{\eta}', \boldsymbol{\xi}')'$ are

$$\boldsymbol{\kappa}^* = \begin{pmatrix} (\mathbf{I} - \mathbf{B})^{-1}(\boldsymbol{\alpha} + \boldsymbol{\Gamma}\boldsymbol{\kappa}) \\ \boldsymbol{\kappa} \end{pmatrix} , \tag{4}$$

$$\boldsymbol{\Phi}^* = \begin{pmatrix} \mathbf{A}(\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Gamma}' + \boldsymbol{\Psi})\mathbf{A}' & \mathbf{A}\boldsymbol{\Gamma}\boldsymbol{\Phi} \\ \boldsymbol{\Phi}\boldsymbol{\Gamma}'\mathbf{A}' & \boldsymbol{\Phi} \end{pmatrix} , \tag{5}$$

where $\mathbf{A} = (\mathbf{I} - \mathbf{B})^{-1}$.

In the most common type of LISREL model, namely confirmatory factor analysis, only equation (3) is involved and $\boldsymbol{\kappa} = 0$ and $\boldsymbol{\tau}_x$ is the mean vector of $\mathbf{x}$.

The objective of this note is to construct individual scores on $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ for every individual in the sample such that their sample mean vector and covariance matrix equal $\boldsymbol{\kappa}^*$ and $\boldsymbol{\Phi}^*$, respectively. If this is achieved, one can estimate (1) from the latent variable scores and obtain the same parameter estimates as with LISREL. The next section gives an example of how these latent variable scores can be used to estimate a nonlinear relationship among latent variables.

The model may involve all these parameter matrices: $\boldsymbol{\kappa}$, $\boldsymbol{\alpha}$, $\boldsymbol{\tau}_y$, $\boldsymbol{\tau}_x$, $\boldsymbol{\Lambda}_y$, $\boldsymbol{\Lambda}_x$, $\mathbf{B}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Phi}$, $\boldsymbol{\Psi}$, $\boldsymbol{\Theta}_\epsilon$, $\boldsymbol{\Theta}_\delta$, and $\boldsymbol{\Theta}_{\delta\epsilon}$, the elements of which are of three kinds:

- *fixed parameters* that have been assigned specified values,
- *constrained parameters* that are unknown but linear or non-linear functions of one or more other parameters, and
- *free parameters* that are unknown and not constrained.

In the following, I assume that all parameter matrices are known. In practice they would be equal to their parameter estimates after the model has been fitted.

Equations (2) and (3) may be combined into one single equation:

$$\begin{pmatrix} \mathbf{y} \\ \mathbf{x} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\tau}_y \\ \boldsymbol{\tau}_x \end{pmatrix} + \begin{pmatrix} \boldsymbol{\Lambda}_y & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_x \end{pmatrix} \begin{pmatrix} \boldsymbol{\eta} \\ \boldsymbol{\xi} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon} \\ \boldsymbol{\delta} \end{pmatrix} , \tag{6}$$

which I write as

$$\mathbf{x}^* = \boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\xi}^* + \boldsymbol{\delta}^* \ . \tag{7}$$

Since I am running out of symbols I will reuse old symbols with new meanings. Let $\boldsymbol{\xi} = \boldsymbol{\xi}^* - \boldsymbol{\kappa}^*$, $\mathbf{x} = \mathbf{x}^* - \boldsymbol{\tau} - \boldsymbol{\Lambda}\boldsymbol{\kappa}^*$, and $\boldsymbol{\delta} = \boldsymbol{\delta}^*$. Then (7) becomes

$$\mathbf{x} = \boldsymbol{\Lambda}\boldsymbol{\xi} + \boldsymbol{\delta} \ . \tag{8}$$

Estimating $\boldsymbol{\xi}^*$ is equivalent to estimating $\boldsymbol{\xi}$.

The sample observations on $\mathbf{x}$ are denoted $\mathbf{x}_1, \ldots, \mathbf{x}_N$, where N is the sample size. Following Anderson & Rubin (1956, eq. 9.5). I estimate $\boldsymbol{\xi}$ for every individual in the sample by minimizing

$$\sum_{a=1}^{N} (\mathbf{x}_a - \boldsymbol{\Lambda}\boldsymbol{\xi}_a)' \boldsymbol{\Theta}^{-1} (\mathbf{x}_a - \boldsymbol{\Lambda}\boldsymbol{\xi}_a) \ , \tag{9}$$

subject to the constraint

$$(1/N) \sum_{a=1}^{N} \boldsymbol{\xi}_a \boldsymbol{\xi}'_a = \boldsymbol{\Phi} \ . \tag{10}$$

Here

$$\boldsymbol{\Theta} = \begin{pmatrix} \boldsymbol{\Theta}_\epsilon & \boldsymbol{\Theta}_{\delta\epsilon}{}' \\ \boldsymbol{\Theta}_{\delta\epsilon} & \boldsymbol{\Theta}_\delta \end{pmatrix} \ , \tag{11}$$

is the covariance matrix $\boldsymbol{\delta}$. It is assumed that $\boldsymbol{\Theta}$ and $\boldsymbol{\Phi}$ are positive definite and that $\boldsymbol{\Lambda}$ has full column rank.

Hence, I minimize

$$F = \sum_{a=1}^{N} (\mathbf{x}_a - \boldsymbol{\Lambda}\boldsymbol{\xi}_a)' \boldsymbol{\Theta}^{-1} (\mathbf{x}_a - \boldsymbol{\Lambda}\boldsymbol{\xi}_a) + tr[\mathbf{X}(\sum_{a=1}^{N} \boldsymbol{\xi}_a \boldsymbol{\xi}'_a - N\boldsymbol{\Phi})] \ , \tag{12}$$

where $\mathbf{X}$ is a symmetric matrix of Lagrange multipliers.

The solution is

$$\hat{\boldsymbol{\xi}}_a = (\boldsymbol{\Lambda}'\boldsymbol{\Theta}^{-1}\boldsymbol{\Lambda} + \mathbf{X})^{-1} \boldsymbol{\Lambda}'\boldsymbol{\Theta}^{-1}\mathbf{x}_a \ , \tag{13}$$

where $\mathbf{X}$ must satisfy

$$(\boldsymbol{\Lambda}'\boldsymbol{\Theta}^{-1}\boldsymbol{\Lambda} + \mathbf{X})\boldsymbol{\Phi}(\boldsymbol{\Lambda}'\boldsymbol{\Theta}^{-1}\boldsymbol{\Lambda} + \mathbf{X}) = \boldsymbol{\Lambda}'\boldsymbol{\Theta}^{-1}\mathbf{A}\boldsymbol{\Theta}^{-1}\boldsymbol{\Lambda} \ , \tag{14}$$

and

$$\mathbf{A} = \sum_{a=1}^{N} \mathbf{x}_a \mathbf{x}'_a \ , \tag{15}$$

see Anderson & Rubin (1956, eqs. 9.6 and 9.8). Let $\mathbf{Y} = \mathbf{\Lambda}'\mathbf{\Theta}^{-1}\mathbf{\Lambda} + \mathbf{X}$ and $\mathbf{B} = \mathbf{\Lambda}'\mathbf{\Theta}^{-1}\mathbf{A}\mathbf{\Theta}^{-1}\mathbf{\Lambda}$. Solving for $\mathbf{X}$ is equivalent to solving for $\mathbf{Y}$, where $\mathbf{Y}$ satisfies

$$\mathbf{Y}\mathbf{\Phi}\mathbf{Y} = \mathbf{B} \ . \tag{16}$$

Let $\mathbf{\Phi} = \mathbf{U}\mathbf{D}\mathbf{U}'$ be the singular value decomposition of $\mathbf{\Phi}$, where $\mathbf{D}$ is a diagonal matrix of eigenvalues of $\mathbf{\Phi}$ and the columns of $\mathbf{U}$ are the corresponding eigenvectors. Then (16) is

$$\mathbf{Y}\mathbf{U}\mathbf{D}\mathbf{U}'\mathbf{Y} = \mathbf{B} \ . \tag{17}$$

Premultiplying by $\mathbf{D}^{\frac{1}{2}}\mathbf{U}'$ and postmultiplying by $\mathbf{U}\mathbf{D}^{\frac{1}{2}}$ gives

$$\mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{Y}\mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{Y}\mathbf{U}\mathbf{D}^{\frac{1}{2}} = \mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{B}\mathbf{U}\mathbf{D}^{\frac{1}{2}} \ . \tag{18}$$

Let $\mathbf{Z} = \mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{Y}\mathbf{U}\mathbf{D}^{\frac{1}{2}}$ and $\mathbf{C} = \mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{B}\mathbf{U}\mathbf{D}^{\frac{1}{2}}$. Then both $\mathbf{Z}$ and $\mathbf{C}$ are symmetric matrices and (18) becomes

$$\mathbf{Z}^2 = \mathbf{C} \ . \tag{19}$$

Hence, $\mathbf{Z}$ is the symmetric square root of $\mathbf{C}$. Let $\mathbf{C} = \mathbf{V}\mathbf{L}\mathbf{V}'$ be the singular value decomposition of $\mathbf{C}$, where $\mathbf{L}$ is a diagonal matrix of eigenvalues of $\mathbf{C}$ and the columns of $\mathbf{V}$ are the corresponding eigenvectors. Then

$$\mathbf{Z} = \mathbf{V}\mathbf{L}^{\frac{1}{2}}\mathbf{V}' \ . \tag{20}$$

The final solution can then be expressed as

$$\hat{\boldsymbol{\xi}}_a = \mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{Z}^{-1}\mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{\Lambda}'\mathbf{\Theta}^{-1}\mathbf{x}_a \ , \tag{21}$$

or

$$\hat{\boldsymbol{\xi}}_a = \mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{V}\mathbf{L}^{-\frac{1}{2}}\mathbf{V}'\mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{\Lambda}'\mathbf{\Theta}^{-1}\mathbf{x}_a \ . \tag{22}$$

In terms of $\mathbf{x}^*$ and $\boldsymbol{\xi}^*$ in (7), the solution is

$$\hat{\boldsymbol{\xi}}^*_a = \boldsymbol{\kappa}^* + \mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{Z}^{-1}\mathbf{D}^{\frac{1}{2}}\mathbf{U}'\mathbf{\Lambda}'\mathbf{\Theta}^{-1}(\mathbf{x}^*_a - \boldsymbol{\tau} - \mathbf{\Lambda}\boldsymbol{\kappa}^*) \ . \tag{23}$$

It can be verified that

$$E(\hat{\boldsymbol{\xi}}^*_a) = \boldsymbol{\kappa}^* \ , \tag{24}$$

$$E(\hat{\boldsymbol{\xi}}^*_a\hat{\boldsymbol{\xi}}^{*\prime}_a) = \mathbf{\Phi}^* \ , \tag{25}$$

so that $\hat{\boldsymbol{\xi}}^*_a$ is unbiased and has the correct covariance matrix. Furthermore, if the fit of the mean vector is perfect, which is always the case when $\boldsymbol{\tau}$ is unconstrained (in other words, almost always), then these two properties also hold in the sample:

$$(1/N)\sum_{a=1}^{N}\hat{\boldsymbol{\xi}}^*_a = \boldsymbol{\kappa}^* \ , \tag{26}$$

$$(1/N)\sum_{a=1}^{N}\hat{\boldsymbol{\xi}}^*_a\hat{\boldsymbol{\xi}}^{*\prime}_a = \boldsymbol{\Phi}^* \ . \tag{27}$$

This means that one can use the estimated latent variable scores on $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ and reestimate (1) with $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ as observed variables. One will then get the same parameter estimates of $\mathbf{B}$ and $\boldsymbol{\Gamma}$. However, one will not necessarily get the same standard errors. But why would anyone want to do that? Possible uses of latent variable scores are

- Select subgroups of individuals on the basis of the latent variable scores
- Rank individuals on the basis of the scores of one latent variable
- Correlate latent variable scores with external variables
- Estimate the error variables $\boldsymbol{\zeta}$ and study their distribution
- Estimate nonlinear relationships among latent variables

In the LISREL 8.30 implementation of latent variable scores, the scores on $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ are automatically appended to the PSF file containing the data on $\mathbf{y}$ and $\mathbf{x}$, see Jöreskog, *et al.* (2000, pp. 168–171). This facilitates easy use of the latent variable scores for most purposes. For example, using Interactive LISREL one can

- plot the univariate and bivariate distributions of the latent variable scores and the observed variables
- correlate the latent variable scores with the observed variables
- estimate the regression of the observed variables on the latent variable scores

2 Estimating Latent Nonlinear Equations

To illustrate how latent variable scores may be used to estimate a nonlinear relationship between latent variables, I use an example previously analyzed by Baumgartner & Bagozzi (1995), Jöreskog & Yang (1996), and Yang Jonsson (1997, 1998). Another example based on generated data for the Kenny & Judd (1984) model is given in Jöreskog, *et al.* (2000, pp. 171–175).

A theory involving interactions among theoretical constructs is the theory of reasoned action or expectancy-value attitude theory in social psychology (Fishbein & Ajzen, 1975; Ajzen & Fishbein, 1980). See Bagozzi (1986) for further explanation of this theory and Bagozzi, Baumgartner, & Yi (1992) and references therein for its interpretation and application in consumer research. In essence, this theory states that beliefs about consequences of an act and evaluations of consequences of the same act interact in determining a person's overall attitude or intention to perform the act. This theory can be and has been applied to many different acts such as using public transportation rather than a private car to commute to work, or to donate or not to donate blood or an organ. There are various extensions of this theory but, for the present illustration, only the basic theory here referred will be used.

In Bagozzi, Baumgartner, & Yi (1992), the attitudinal act of interest was consumers' use of coupons for grocery shopping and the two consequences of coupon use were the beliefs that using coupons would lead to savings on the grocery bill and that using coupons would contribute to a feeling of being a good shopper.

Beliefs about consequences of using coupons were measured by the responses to the following two questions:

> *Please indicate how likely it is that each of the following consequences would occur to you if you were to use coupons for shopping in the supermarket in the upcoming week. (Ignore how important each consequence is to you at this time. Here we would like you to simply indicate the likely amount of each consequence occurring to you if you were to use coupons.)*

BE1 Much time and effort would be required to search for, gather, and organize coupons.

likely						unlikely
extremely	quite	slightly	neither	slightly	quite	extremely
1	2	3	4	5	6	7

BE2 Much time and effort would be required to plan the use of and actually redeem coupons in the supermarket.

likely						unlikely
extremely	quite	slightly	neither	slightly	quite	extremely
1	2	3	4	5	6	7

Evaluations of consequences of using coupons were measured by responses to the following two questions:

Please express your evaluations of each of the following consequences of using coupons for shopping in the supermarket in the upcoming week.

VL1 The time and effort required to search for, gather, and organize coupons makes me feel:

bad						good
extremely	quite	slightly	neither	slightly	quite	extremely
1	2	3	4	5	6	7

VL2 The time and effort required to plan the use of and actually redeem coupons in the supermarket makes me feel:

bad						good
extremely	quite	slightly	neither	slightly	quite	extremely
1	2	3	4	5	6	7

The overall attitude toward coupons was measured by the responses to the following three items.

My attitude toward coupons can be best summarized as

	favorable						unfavorable
AA1	extremely	quite	slightly	neither	slightly	quite	extremely
	1	2	3	4	5	6	7
	negative						positive
AA2	extremely	quite	slightly	neither	slightly	quite	extremely
	1	2	3	4	5	6	7
	good						bad
AA3	extremely	quite	slightly	neither	slightly	quite	extremely
	1	2	3	4	5	6	7

After data collection, items **AA1** and **AA3** were recoded in the opposite direction.

The data[1] comes from a survey of 253 female staff members at two large public universities in United States. The variables are ordinal but I treat them here as continuous. The term BEA data will be used as a working name for this dataset (BEA = Beliefs, Evaluations, Attitudes).

Writing

$$\eta = \mathbf{At} = \mathit{attitude} \ ,$$

$$\xi_1 = \mathbf{Be} = \mathit{belief} \ ,$$

$$\xi_2 = \mathbf{Vl} = \mathit{evaluation} \ ,$$

the expectancy-value model is

$$\mathbf{At} = \alpha + \gamma_1 \mathbf{Be} + \gamma_2 \mathbf{Vl} + \gamma_3 (\mathbf{Be} \times \mathbf{Vl}) + \zeta \ ,$$

or in LISREL notation

$$\eta = \alpha + \gamma_1 \xi_1 + \gamma_2 \xi_2 + \gamma_3 \xi_1 \xi_2 + \zeta \ . \tag{28}$$

[1]The author is grateful to Professor Richard P. Bagozzi for kindly providing this data to me.

There are three indicators **AA1**, **AA2**, **AA3** of **At**, two indicators **BE1** and **BE2** of **Be**, and two indicators **VL1** and **VL2** of **Vl**. In LISREL notation these corresponds to $y_1, y_2, y_3, x_1, x_2, x_3, x_4$, respectively. The model is similar to the Kenny-Judd model (Kenny & Judd, 1984) and differs from it only in that there are three indicators of η rather than only one. A path diagram of the measurement model for **At**, **Be**, and **Vl** is shown in Figure 1.

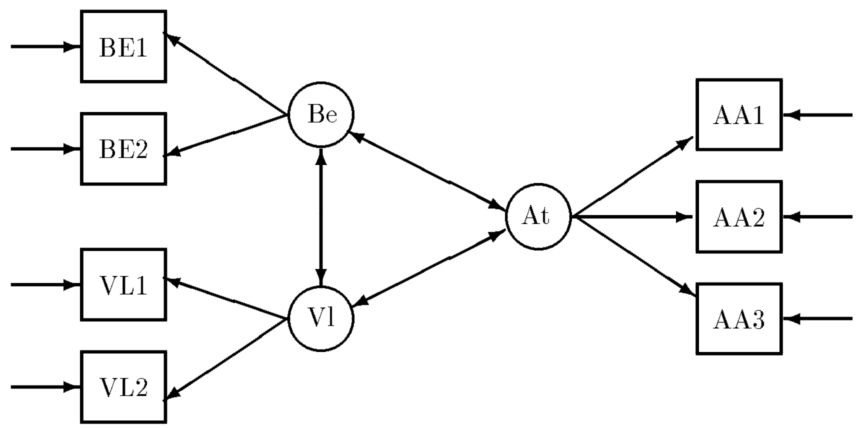

Figure 1: Path Diagram of Three-Factor Model for BEA Data

The raw data on the seven variables **AA1**, **AA2**, **AA3**, **BE1**, **BE2**, **VL1**, and **VL2** are in the file BEA.RAW. The PSF file corresponding to BEA.RAW is BEA.PSF. This can be obtained by running the following PRELIS command file (BEA1.PR2, say):

```
Computing PSF file from BEA.RAW
DA NI=7
LA
AA1 AA2 AA3 BE1 BE2 VL1 VL2
RA=BEA.RAW
CO ALL
OU RA=BEA.PSF
```

This run will also produce a DSF file called BEA1.DSF. To obtain the latent variable scores for **At**, **Be**, and **Vl** use the data system file BEA1.DSF and the following SIMPLIS command file:

```
Computing Latent Variable Scores
System File from File BEA1.DSF
Latent Variables At Be Vl
Relationships
AA1 = 1*At
AA2 - AA3 = At
BE1 = 1*Be
BE2 = Be
VL1 = 1*Vl
VL2 = Vl
PSFfile BEA.PSF
End of Problem
```

Verify that the PSF file BEA.PSF has been appended with the scores on **At**, **Be**, and **Vl**. One can now estimate the nonlinear equation directly using the following PRELIS command file:

```
Estimating the Nonlinear Equation
SY=BEA.PSF
NE BeV1=Be*Vl
RG At ON Be Vl BeVl
Output
```

The equation is estimated as

```
At =  - 0.0371 - 0.0644*Be + 1.036*Vl + 0.0614*BeVl + Error
        (0.0576) (0.0696)    (0.0878)   (0.0578)
        -0.644   -0.925       11.791     1.062
```

The interaction effect is not significant. This may be due to the small sample. But the variables are ordinal rather than continuous. So the validity of the t-values is questionable. Also, because there are very few observed variables there is very little construct validity of the three constructs. For a review of other approaches to estimating models with interaction effects, see Jöreskog (1998) and other chapters in Schumacker & Marcoulides (1998).

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