



Poisson log model with an offset variable

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1. The model

The previous analysis assumed that the counts were all observed for the same number of days. However, this was not the case since the number of treatment days in the period did vary to some degree. Most of the counts were based on the full seven days in the week; however, some observations were made only for 1 day in the given week. To take this into account, we need to specify a so-called OFFSET variable. The offset variable indicates the amount of time that each count is based on. If OFFSET = no is specified, as was the case in the previous example, it is assumed that all counts are based on the same amount of time.

The offset variable is introduced into the Poisson model in the following way:

$$\log\left(\hat{\lambda}_{ij}\right) = \log(\text{offset variable}) + [\mathbf{x}'_{ij}\mathbf{b}_i]$$

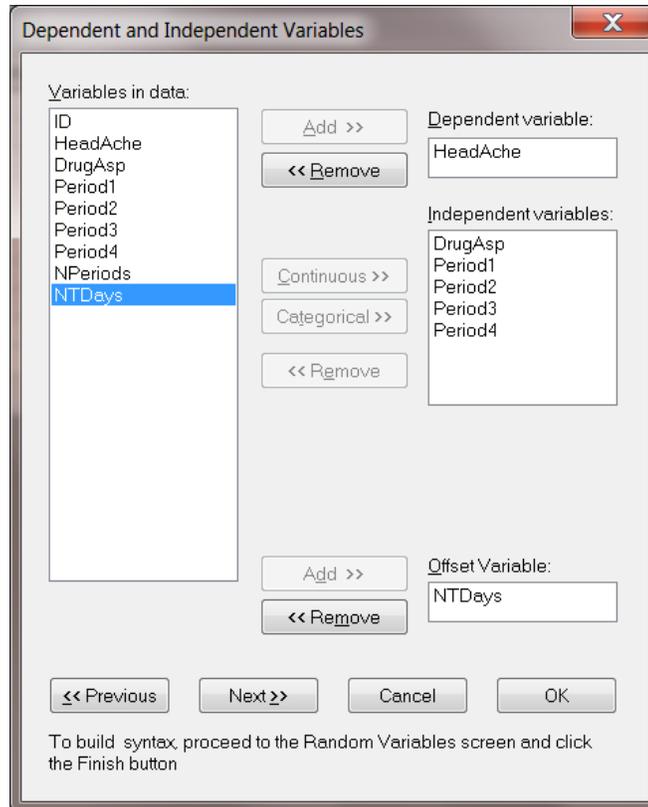
where \mathbf{x}_{ij} represent the values of the covariates corresponding to level-1 unit j nested within level-2 unit i and \mathbf{b}_i denotes the coefficient vector containing both fixed and random effects.

In the current situation, the variable NTDays is the appropriate choice as the OFFSET variable. The model to be fitted to the data now changes to:

$$\begin{aligned} \log(\text{Headache}_{ij}) = & \log(\text{NTDays}) + (\beta_0 + \beta_1 \times \text{Period1}_{ij} + \beta_2 \times \text{Period2}_{ij} \\ & + \beta_3 \times \text{Period3}_{ij} + \beta_4 \times \text{Period4}_{ij} + \beta_5 \times \text{DrugAsp}_{ij} + v_{i0}). \end{aligned}$$

2. Setting up the analysis

To create the model specifications for this model, ensure that **aspart.lsf** is displayed as the active window. Select the variable **NTDays** from the **Variables in data:** list as the **Offset Variable** by proceeding to the **Dependent and Independent Variables** dialog.



Click the **Next** button to proceed to the **Random Variables** dialog and then click **Finish** to create the syntax file (saved as **Aspart2.prl**) shown below.

```
MGLimOptions Converge=0.0001 MaxIter=100 MissingCode=-999999
IterDetails=No Method=Quad NQUADPTS=20 Output = Residuals;
Title=2-level Poisson log random intercept model, ASPART data;
SY=aspart.lsf;
ID2=ID;
Distribution=POI;
Link=LOG;
Intercept=Yes;
Scale=None;
DepVar=HeadAche;
CoVars=DrugAsp Period1 Period2 Period3 Period4;
Offset=NTDays;
RANDOM2=intcept;
```

3. Discussion of results

Fixed and random effect results

ASPART2.OUT

Number of quadrature points = 20
 Number of free parameters = 7
 Number of iterations used = 3

-2lnL (deviance statistic) = 404.86839
 Akaike Information Criterion = 418.86839
 Schwarz Criterion = 438.49654

Estimated regression weights

Parameter	Estimate	Standard Error	z Value	P Value
intcept	-1.7127	0.2105	-8.1371	0.0000
DrugAsp	0.2797	0.1641	1.7044	0.0883
Period1	0.1001	0.2357	0.4247	0.6711
Period2	0.0879	0.2250	0.3909	0.6959
Period3	-0.2116	0.2567	-0.8242	0.4098
Period4	-0.0787	0.2545	-0.3092	0.7571

Event Rate Ratio and 95% Event Rate Confidence Intervals

Parameter	Estimate	Event Rate	Bounds	
			Lower	Upper
intcept	-1.7127	0.1804	0.1194	0.2725
DrugAsp	0.2797	1.3227	0.9589	1.8245
Period1	0.1001	1.1053	0.6963	1.7544
Period2	0.0879	1.0919	0.7026	1.6971
Period3	-0.2116	0.8093	0.4894	1.3385
Period4	-0.0787	0.9243	0.5613	1.5220

Estimated level 2 variances and covariances

Parameter	Estimate	Standard Error	z Value	P Value
intcept/intcept	0.4775	0.1924	2.4811	0.0131

Results for this model differ from those obtained for the model without offset variable discussed in the previous section. While the overall trend in predictor coefficient estimates is similar, the intercept is now estimated as -1.7127 , compared to 0.2572 previously. The variance in intercept over patients for this model is estimated as 0.4775 compared to 0.4290 previously.

4. Interpreting the results

Estimated outcomes for groups: unit-specific results

The expected number of headaches can be obtained in the following fashion. First, we substitute the estimated coefficients in the model formulation

$$\begin{aligned}
 \log\left(\widehat{Headache}_{ij}\right) &= \log\left(NTDays_{ij}\right) + (\beta_0 + \beta_1 \times \text{Period1}_{ij} + \beta_2 \times \text{Period2}_{ij} \\
 &\quad + \beta_3 \times \text{Period3}_{ij} + \beta_4 \times \text{Period4}_{ij} + \beta_5 \times \text{DrugAsp}_{ij}) \\
 &= \log\left(NTDays_{ij}\right) + (-1.7127 + 0.1001 \times \text{Period1}_{ij} + 0.0879 \times \text{Period2}_{ij} \\
 &\quad - 0.2116 \times \text{Period3}_{ij} - 0.0787 \times \text{Period4}_{ij} + 0.2797 \times \text{DrugAsp}_{ij}),
 \end{aligned}$$

or, after taking exponents on both sides, as

$$\begin{aligned} \widehat{Headache}_{ij} = & NTDays_{ij} \times \exp(-1.7127 + 0.1001 \times Period1_{ij} + 0.0879 \times Period2_{ij} \\ & - 0.2116 \times Period3_{ij} - 0.0787 \times Period4_{ij} + 0.2797 \times DrugAsp_{ij}). \end{aligned}$$

As most observations had a value of $NTDays = 7$, we start by considering typical patients with a full set of treatment days. We also assume that $DrugAsp = 1$, in other words, that aspartame rather than a placebo was administered.

During the first treatment period, we find that for such a patient

$$\begin{aligned} \widehat{Headache}_{ij} &= 7 \exp(-1.7127 + 0.1001 + 0.2797) \\ &= 7 \exp(-1.3329) \\ &= 1.8460. \end{aligned}$$

The expected numbers of headaches for a typical patient (again with $NTDays = 7$ and $DrugAsp = 1$) for the second, third, and fourth treatment periods are calculated as

$$\begin{aligned} \widehat{Headache}_{ij} &= 7 \exp(-1.7127 + 0.0879 + 0.2797) \\ &= 1.8236, \\ \widehat{Headache}_{ij} &= 7 \exp(-1.7127 - 0.2116 + 0.2797) \\ &= 1.3516, \end{aligned}$$

and

$$\begin{aligned} \widehat{Headache}_{ij} &= 7 \exp(-1.7127 - 0.0787 + 0.2797) \\ &= 1.5437 \end{aligned}$$

respectively.

For a typical patient with only 5 treatment days, the expected numbers of headaches in each of the four treatment periods are 1.3186, 1.3026, 0.9654, and 1.1027 respectively.

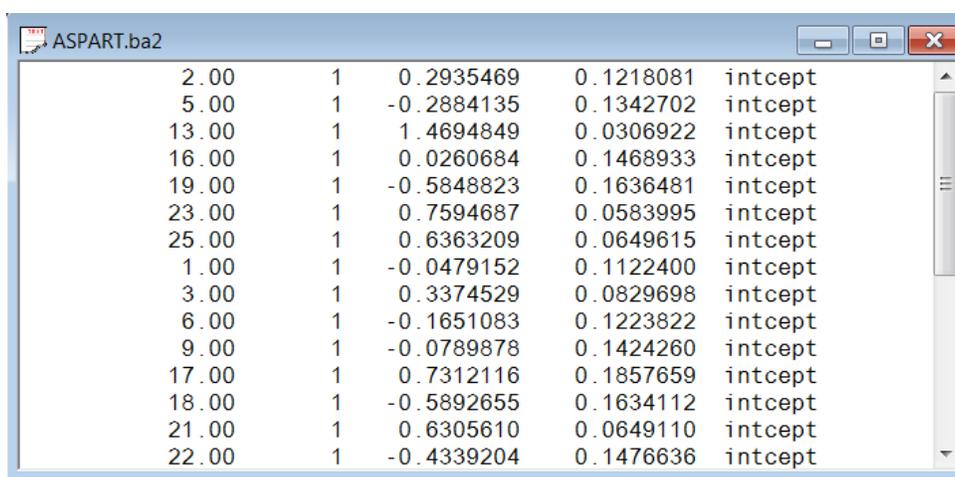
When the expected numbers of headaches for a typical patient receiving aspartame under the Poisson model without offset variable (see previous section) and the Poisson model with offset variable are compared, we clearly see the impact of the inclusion of the offset variable on the estimated coefficients. These results are shown below.

Comparison of results for Poisson models

Period	Without offset variable	With offset variable (NTDays = 7)	With offset variable (NTDays = 5)
1	1.7385	1.846	1.3186
2	1.6600	1.8236	1.3026
3	1.2784	1.3516	0.9654
4	1.3677	1.5437	1.1027

Level 2 Bayes results

As requested during the model specification stage, the empirical Bayes estimates of the random effects are written to the file **aspart2.ba2**. The first few lines of this file are shown below.



```
ASPART.ba2
2.00 1 0.2935469 0.1218081 intcept
5.00 1 -0.2884135 0.1342702 intcept
13.00 1 1.4694849 0.0306922 intcept
16.00 1 0.0260684 0.1468933 intcept
19.00 1 -0.5848823 0.1636481 intcept
23.00 1 0.7594687 0.0583995 intcept
25.00 1 0.6363209 0.0649615 intcept
1.00 1 -0.0479152 0.1122400 intcept
3.00 1 0.3374529 0.0829698 intcept
6.00 1 -0.1651083 0.1223822 intcept
9.00 1 -0.0789878 0.1424260 intcept
17.00 1 0.7312116 0.1857659 intcept
18.00 1 -0.5892655 0.1634112 intcept
21.00 1 0.6305610 0.0649110 intcept
22.00 1 -0.4339204 0.1476636 intcept
```

The file **aspart.ba2** contains five pieces of information per individual:

- the individual's ID,
- the number of the random effect (only intercept in this case),
- the empirical Bayes estimate for that individual (which is the mean of the posterior distribution),
- the associated posterior variance, and
- the name of the relevant random coefficient.

Since they are estimates of b_{i0} for each individual, the empirical Bayes estimates are expressed on the standard normal scale. Inspection of these estimates indicates that subject 13 has a very high score. This person's estimate of 1.469 (with variance .031) suggests a very high level of headaches. This agrees well with the raw data, which reveals that this person recorded 7 headaches on four occasions and 6 on the only other occasion.