

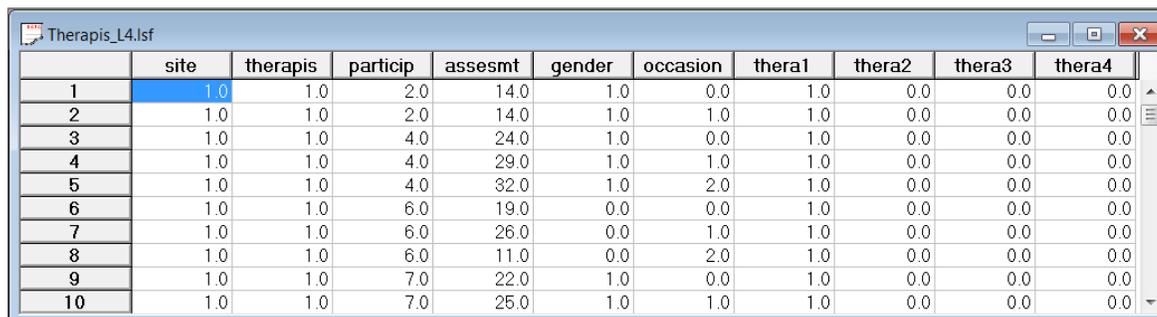
## Four-level model for assessment data

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### 1. The data

The data set used here consists of four levels of nesting. Information is available on repeated measurements made on 1,192 participants at three occasions. In the case of some of the participants, measurements were made on only one or two occasions. Data for the first 10 participants on most of the variables used in this section are shown below in the form of a LISREL spreadsheet file, named **therapist\_L4.Isf** in the **Multilevel Examples** folder.



|    | site | therapis | particip | assesmt | gender | occasion | thera1 | thera2 | thera3 | thera4 |
|----|------|----------|----------|---------|--------|----------|--------|--------|--------|--------|
| 1  | 1.0  | 1.0      | 2.0      | 14.0    | 1.0    | 0.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 2  | 1.0  | 1.0      | 2.0      | 14.0    | 1.0    | 1.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 3  | 1.0  | 1.0      | 4.0      | 24.0    | 1.0    | 0.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 4  | 1.0  | 1.0      | 4.0      | 29.0    | 1.0    | 1.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 5  | 1.0  | 1.0      | 4.0      | 32.0    | 1.0    | 2.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 6  | 1.0  | 1.0      | 6.0      | 19.0    | 0.0    | 0.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 7  | 1.0  | 1.0      | 6.0      | 26.0    | 0.0    | 1.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 8  | 1.0  | 1.0      | 6.0      | 11.0    | 0.0    | 2.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 9  | 1.0  | 1.0      | 7.0      | 22.0    | 1.0    | 0.0      | 1.0    | 0.0    | 0.0    | 0.0    |
| 10 | 1.0  | 1.0      | 7.0      | 25.0    | 1.0    | 1.0      | 1.0    | 0.0    | 0.0    | 0.0    |

The variables of interest are:

- site is the level-4 identification variable (49 units in total).
- therapis is the level-3 identification variable (187 units in total).
- particip is the level-2 identification variable (1192 units in total).
- assesmt is a score assigned by a therapist to a particular participant on occasion 0, 1 or 2.
- gender is a gender indicator, with a value of 0 indicating a male participant and 1 a female participant.
- occasion is a predictor variable coded 0, 1 and 2.
- thera1 - thera4 are dummy coded variables indicating four types of therapy.

## 2. The model

In this example, the identification variable *site* is used as the level-4 identifier, *therapis* is used as the level-3 identifier, and *particip* serves to identify level-2 units.

A general four-level model for a response variable  $y$  depending on a set of  $r$  predictors  $x_1, x_2, \dots, x_r$  can be written in the form

$$y_{ijkl} = \mathbf{x}'_{(f)ijkl} \boldsymbol{\beta} + \mathbf{x}'_{(4)ijkl} \mathbf{v}_i + \mathbf{x}'_{(3)ijkl} \mathbf{v}_{ij} + \mathbf{x}'_{(2)ijkl} \mathbf{u}_{ijk} + \mathbf{x}'_{(1)ijkl} \mathbf{e}_{ijkl}$$

where  $i = 1, 2, \dots, N$  denotes the level-4 units,  $j = 1, 2, \dots, n_i$  the level-3 units,  $k = 1, 2, \dots, n_{ij}$  the level-3 units, and  $l = 1, 2, \dots, n_{ijk}$  the level-1 units. In this context,  $y_{ijkl}$  represents the response of individual  $l$ , nested within level-2 unit  $k$ , level-3 unit  $j$  and level-4 unit  $i$ . The model shown here consists of a fixed and a random part. The fixed part of the model is represented by the vector product  $\mathbf{x}'_{(f)ijkl} \boldsymbol{\beta}$ , where  $\mathbf{x}'_{(f)ijkl}$  is a typical row of the design matrix of the fixed part of the model with, as elements, a subset of the  $r$  predictors. The vector  $\boldsymbol{\beta}$  contains the fixed, but unknown parameters to be estimated. The vector products  $\mathbf{x}'_{(4)ijkl} \mathbf{v}_i$ ,  $\mathbf{x}'_{(3)ijkl} \mathbf{v}_{ij}$ ,  $\mathbf{x}'_{(2)ijkl} \mathbf{u}_{ijk}$ , and  $\mathbf{x}'_{(1)ijkl} \mathbf{e}_{ijkl}$  denote the random part of the model at levels 4, 3, 2, and 1 respectively. For example,  $\mathbf{x}'_{(3)ijkl}$  represents a typical row of the design matrix of the random part at level-3, and  $\mathbf{v}_{ij}$  the vector of random level-3 coefficients to be estimated. The products  $\mathbf{x}'_{(2)ijkl} \mathbf{u}_{ijk}$  and  $\mathbf{x}'_{(1)ijkl} \mathbf{e}_{ijkl}$  serve the same purpose at levels 2 and 1 respectively. It is assumed that  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_N$  are independently and identically distributed (i.i.d.) with mean vector  $\mathbf{0}$  and covariance matrix  $\Phi_{(4)}$  and that  $\mathbf{v}_{i1}, \mathbf{v}_{i2}, \dots, \mathbf{v}_{in_i}$  are independently and identically distributed (i.i.d.) with mean vector  $\mathbf{0}$  and covariance matrix  $\Phi_{(3)}$ . Similarly,  $\mathbf{u}_{ij1}, \mathbf{u}_{ij2}, \dots, \mathbf{u}_{ijn_{ij}}$  are assumed i.i.d., with mean vector  $\mathbf{0}$  and covariance matrix  $\Phi_{(2)}$ , and  $\mathbf{e}_{ijk1}, \mathbf{e}_{ijk2}, \dots, \mathbf{e}_{ijkn_{ijk}}$  are assumed i.i.d., with mean vector  $\mathbf{0}$  and covariance matrix  $\Phi_{(1)}$ .

Within this hierarchical framework, the model fitted to the data uses the participant's gender and type of therapy treatment to predict the assessment scores on three occasions.

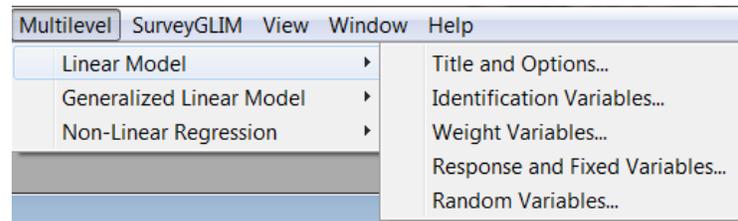
$$\begin{aligned} \text{assessment}_{ijkl} = & \beta_1 * \text{gender}_{ijkl} + \beta_2 * \text{occasion}_{ijkl} + \beta_3 * \text{thera1}_{ijkl} + \\ & \beta_4 * \text{thera2}_{ijkl} + \beta_5 * \text{thera3}_{ijkl} + \beta_6 * \text{thera4}_{ijkl} + \\ & + v_{i0} + v_{ij0} + u_{ijk0} + u_{ijk1} * \text{occasion}_{ijkl} + e_{ijkl} \end{aligned}$$

where  $\beta_1, \beta_2, \dots, \beta_6$  indicate the estimated coefficients associated with the fixed part of the model that contains the predictor variables gender, occasion, and the four indicator variables. The random part of the model is represented by  $v_{i0}$ ,  $v_{ij0}$ ,  $u_{ijk0}$ ,  $u_{ijk1}$  and  $e_{ijkl}$ , which denote the variation in average assessment across sites, therapists, participants and measurement occasions.

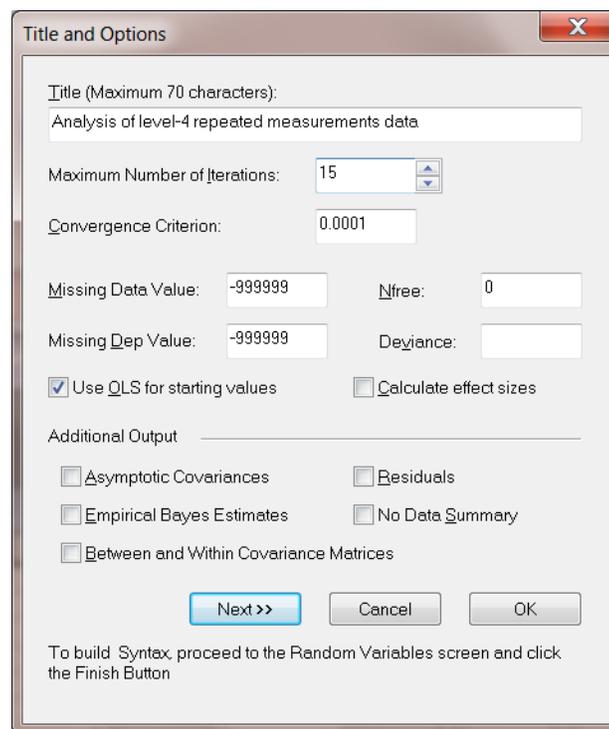
## 3. Setting up the analysis

The model is fitted to the data in **therapist\_L4.isf** by using the sequence of five dialog boxes accessed via the **Multilevel, Linear Model** option from the main menu bar in LISREL. Note that options such as **Multilevel** and **SurveyGLIM** are only available on the main menu bar when a \*.isf file is open.

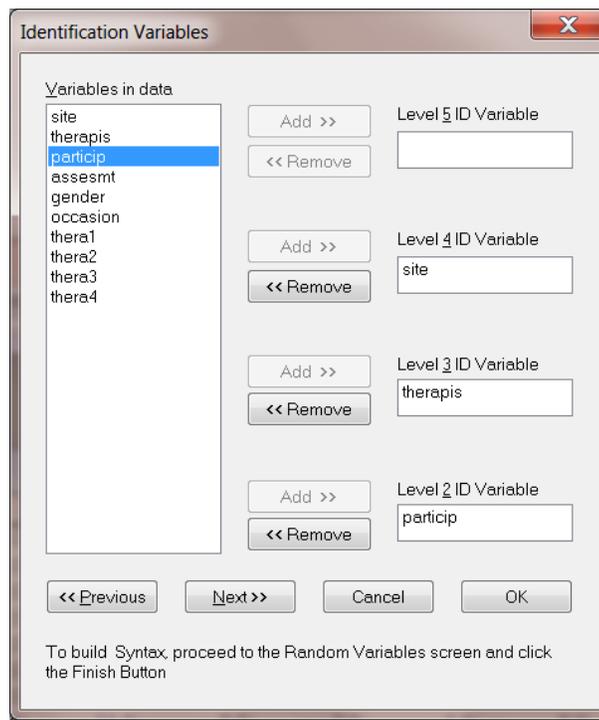
The first step is to open the LSF shown above. The next step is to describe the model to be fitted using the multilevel module in LISREL. From the main menu bar, select the **Multilevel** option. Here we limit our discussion to linear models, and thus the **Linear Model** option will be used throughout.



The first of the five options on the pop-up menu provide access to the **Title and Options** dialog box. Start by providing a title for the analysis in the **Title** field. In this example, we set the maximum number of iterations to 15 and the convergence criterion to 0.0001. Default settings for all other options associated with this dialog box are used. Click the **Next** button to go to the **Identification Variables** dialog box.

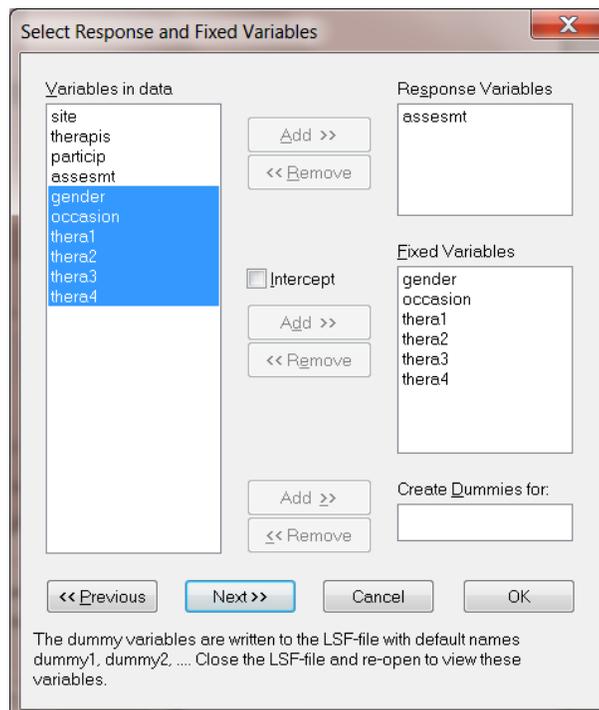


On the **Identification Variables** dialog box, enter the variables defining the hierarchical structure as ID variables.



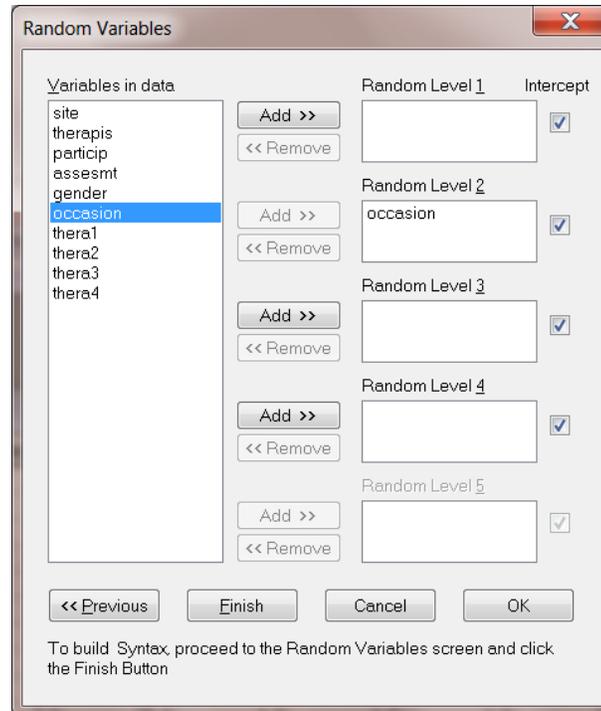
The next dialog box is used to provide information on weight variables, if any. In our case, weights are not available. Therefore, click the **Next** button to proceed to the **Select Response and Fixed Variables** dialog box.

The **Select Response and Fixed Variables** dialog box is used to identify the outcome variable and predictor variables, if any. Select and add the outcome variable **assesmt** to the **Response Variables** field. Next, select the variables starting from **gender** to **thera4** by dragging the mouse over them and click the **Add** button next to the **Fixed Variables** field to include these variables as predictors in the model. Note that the **Intercept** box should be unchecked. This completes the specification of the response and fixed variables. Click the **Next** button to proceed to the **Random Variables** dialog box.



The **Random Variables** dialog box shown below displays the default settings associated with this dialog box. In the current model, only intercept coefficients are allowed to vary randomly at the various levels of the hierarchy, except at

level-2 where we add the predictor occasion as a random effect. Once this is done, click the **Finish** button to generate the syntax for the model.



The syntax shown below corresponds to the information entered via the dialog boxes above. Run the model by clicking the **Run Prelis** icon on the main menu bar.

## 4. Discussion of results

```
Therapis_L4.PRL
OPTIONS OLS=YES CONVERGE=0.000100 MAXITER=15 OUTPUT=STANDARD ;
TITLE=Analysis of level-4 repeated measurements data;
SY='C:\LISREL9 Examples\MLEVELEX\Therapis_L4.lsf';
ID4=site;
ID3=therapis;
ID2=particip;
RESPONSE=assesmt;
FIXED=gender occasion thera1 thera2 thera3 thera4;
RANDOM1=intcept;
RANDOM2=intcept occasion;
RANDOM3=intcept;
RANDOM4=intcept;
```

Only selected parts of the output are shown. The output describing the estimated **fixed effects** after convergence is shown next. The estimates are shown in the column with heading BETA-HAT, and correspond to the coefficients  $\beta_0, \beta_2, \dots, \beta_6$  in the model specification. From the z-values and associated exceedance probabilities, we see that except for the coefficient associated with gender, the remaining coefficients were all highly significant.

Therapis\_L4.OUT

ITERATION NUMBER 9

+-----+  
| FIXED PART OF MODEL |  
+-----+

| COEFFICIENTS | BETA-HAT | STD. ERR. | Z-VALUE  | PR >  Z |
|--------------|----------|-----------|----------|---------|
| gender       | -0.68445 | 0.31043   | -2.20484 | 0.02747 |
| occasion     | 2.52207  | 0.08571   | 29.42622 | 0.00000 |
| thera1       | 18.46583 | 0.52985   | 34.85093 | 0.00000 |
| thera2       | 22.81821 | 0.39635   | 57.57015 | 0.00000 |
| thera3       | 27.19290 | 0.39524   | 68.80062 | 0.00000 |
| thera4       | 30.63948 | 0.45528   | 67.29788 | 0.00000 |

+-----+  
| -2 LOG-LIKELIHOOD |  
+-----+

DEVIANCE= -2\*LOG(LIKELIHOOD) = 19795.3841910891  
NUMBER OF FREE PARAMETERS = 12

Therapis\_L4.OUT

+-----+  
| RANDOM PART OF MODEL |  
+-----+

| LEVEL 4           | TAU-HAT  | STD. ERR. | Z-VALUE  | PR >  Z |
|-------------------|----------|-----------|----------|---------|
| intcept /intcept  | 2.25830  | 0.74765   | 3.02055  | 0.00252 |
| LEVEL 3           | TAU-HAT  | STD. ERR. | Z-VALUE  | PR >  Z |
| intcept /intcept  | 0.36906  | 0.53853   | 0.68532  | 0.49314 |
| LEVEL 2           | TAU-HAT  | STD. ERR. | Z-VALUE  | PR >  Z |
| intcept /intcept  | 22.09563 | 1.57712   | 14.01007 | 0.00000 |
| occasion/intcept  | -0.50019 | 0.64561   | -0.77476 | 0.43848 |
| occasion/occasion | 0.21014  | 0.47079   | 0.44636  | 0.65534 |
| LEVEL 1           | TAU-HAT  | STD. ERR. | Z-VALUE  | PR >  Z |
| intcept /intcept  | 14.27640 | 0.63112   | 22.62080 | 0.00000 |

A study of the random part of the model shows that all the intercept effects are highly significant, except for the level-3 (therapists) intercept. From this, we conclude that intercept estimates vary significantly over sites, but not over therapists. Finally, we note that there is no significant occasion effect at level-2. This fact can be further substantiated by fitting an intercepts only model to the data. In this case, we obtain the following results:

DEVIANCE= -2\*LOG(LIKELIHOOD)= 19795.9780720184  
NUMBER OF FREE PARAMETERS= 10

Since the difference between the deviance statistics equals 0.58, this leads to the conclusion (at  $12 - 10 = 2$  degrees of freedom) that the level-2 random occasion effect is not significant.