Non recursive model for income data

Contents

1. Introduction

In a previous example we showed how individual equations for a non-recursive system can be estimated separately using Two-Stage Least Squares (TSLS) and SIMPLIS syntax if a sufficient number of instrumental variables is available in the data. Here we illustrate how a non-recursive system can be estimated using LISREL syntax and maximum likelihood estimation. We again use the data from Gujarati (1995) to illustrate. Of interest here is the relationship between income and government spending.

The data given in **incomemoney.lsf** are selected macro-economic data for the USA for the period 1970 to 1991. This file can be found in the **MVABOOK Examples\Chapter 2** folder.

The variables are:

- y_1 = income
- y_2 = money supply
- x_1 = investment expenditure
- x_2 = government spending on goods and services
- x_3 = interest rate on 6-month Treasury bills in %.

All variables except for x_3 are measured in billions of dollars.

incomemoney.lsf

	Y1	Y2	X1	X2	X3
1	10.11	6.28	1.50	2.08	6.56
2	10.97	7.17	1.75	2.24	4.51
3	12.07	8.05	2.06	2.49	4.47
4	13.50	8.61	2.43	2.70	7.18
5	14.59	9.09	2.46	3.06	7.93
6	15.86	10.23	2.26	3.64	6.12
7	17.68	11.64	2.86	3.93	5.27
8	19.74	12.87	3.58	4.26	5.51
9	22.33	13.89	4.34	4.69	7.57
10	24.89	14.97	4.80	5.20	10.02
11	27.08	16.29	4.68	6.13	11.37
12	30.31	17.93	5.58	6.98	13.78
13	31.50	19.52	5.03	7.71	11.08
14	34.05	21.86	5.47	8.40	8.75
15	37.77	23.74	7.19	8.93	9.80
16	40.39	25.69	7.14	9.70	7.66
17	42.69	28.11	7.18	10.28	6.03
18	45.40	29.11	7.49	10.66	6.05
19	49.00	30.71	7.94	11.09	6.92
20	52.51	32.27	8.32	11.82	8.04
21	55.22	33.39	7.99	12.74	7.47
22	56.77	34.40	7.21	13.33	5.49

2. Using LISREL syntax

The model previously considered was

$$\begin{aligned} y_1 &= \alpha_1 + \beta_1 y_2 + \gamma_1 x_1 + \gamma_2 x_2 + u_1 \\ y_2 &= \alpha_2 + \beta_2 y_1 + u_2 \end{aligned} \qquad (1)$$

The model parameter matrices $\mathbf{B}$ and $\mathbf{\Gamma}$ are

$$\mathbf{B} = \begin{bmatrix} 0 & \beta_1 \\ \beta_1 & 0 \end{bmatrix}, \quad \mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & \gamma_2 \\ 0 & 0 \end{bmatrix}$$

Syntax is given in **incomemoney1b.lis**.

incomemoney1b.lis

```
!Income and Money Supply Model Estimated by ML
!Using LISREL Syntax
da ni=5
ra=incomemoney.lsf
mo ny=2 nx=2 be=fi al=fr ka=fr
fr be(1,2) be(2,1)
fi ga(2,1) ga(2,2)
ou
```

Output for this analysis is as follows:

```
LISREL Estimates (Maximum Likelihood)

         BETA

                  Y1          Y2
            --------    --------
       Y1       - -       -2.165
                         (1.847)
                          -1.173

       Y2      0.614        - -
             (0.009)
              67.794


         GAMMA

                  X1          X2
            --------    --------
       Y1      1.504       8.703
             (1.089)     (4.268)
               1.381       2.039

       Y2       - -         - -


         Covariance Matrix of Y and X

                  Y1          Y2          X1          X2
            --------    --------    --------    --------
       Y1    235.456
       Y2    144.314      88.836
       X1     34.575      21.244       5.407
       X2     56.749      34.868       8.324      13.758

         Mean Vector of Eta-Variables

                  Y1          Y2
            --------    --------
              30.200      18.901

         PHI

                  X1          X2
            --------    --------
       X1      5.407
             (1.710)
               3.162

       X2      8.324      13.758
             (2.680)     (4.351)
               3.106       3.162
```

```
 PSI
         Note: This matrix is diagonal.

                  Y1         Y2
            --------   --------
               4.815      0.385
             (4.930)    (0.122)
               0.977      3.158


         Squared Multiple Correlations for Structural Equations

                  Y1         Y2
            --------   --------
               0.996      0.999
```

Comparison of these results with the two other analyses of the same data shows that these results are different from those obtained using TSLS. TSLS works better than maximum likelihood in small samples, but ML has better large sample properties. While ML makes assumptions about multivariate normality, this may not hold for these data and TSLS, with less strong distributional assumptions, may be more appropriate.

Financial data also frequently tend to exhibit strong autocorrelation, in which case the assumption of independent observations become questionable. Multicollinearity also occurs frequently in data of this nature. Inspection of the covariance matrix for these data we find this to be the case here:

```
Income and Money Supply Model Estimated by ML

         Covariance Matrix

                  Y1         Y2         X1         X2
            --------   --------   --------   --------
       Y1    235.456
       Y2    144.314     88.836
       X1     34.531     21.264      5.407
       X2     56.757     34.865      8.324     13.758

 Total Variance = 343.457 Generalized Variance = 1.706

 Largest Eigenvalue = 342.789 Smallest Eigenvalue = 0.055

 Condition Number = 78.631

 WARNING: The Condition Number indicates severe multicollinearity.

          One or more variables may be redundant.
```

The condition number is the square root of the ratio of the largest and smallest eigenvalues. A large condition number, as is observed here, indicates the presence of multicollinearity in the data. The waning message is printed by LISREL if this ratio exceeds 30.

3. Direct, indirect and total effects

In previous analyses, we considered the model defined in (1). Also assume that we have only y_2 and y_1 having an effect on each other as shown below:

$$y_1 \underset{\beta_{12}}{\overset{\beta_{21}}{\rightleftarrows}} y_2$$

Note that x_1 has no effect on y_2. It does have an effect on y_1 though, and y_1 has an effect on y_2. The same argument can be made about x_2, which has no direct effect on y_1 but does have an effect on y_2. As a result, it has an effect on y_1.

Define a cycle for y_1 as one path to y_2 and one return to y_1. The effect of one cycle on y_1 is thus $\beta_{21}\beta_{12}$. It is clear that multiple cycles would lead to the total effect on y_1 being the sum of the infinite geometric series

$$\beta_{21}\beta_{12} + \left(\beta_{21}\beta_{12}\right)^2 + \left(\beta_{21}\beta_{12}\right)^3 + ...$$

which is $\beta_{21}\beta_{12} / \left(1 - \beta_{21}\beta_{12}\right)$ if $\beta_{21}\beta_{12} < 1$.

Similarly, the general effect of **y** on itself can then be written as

$$\mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + ... = (\mathbf{I} - \mathbf{B})^{-1} - \mathbf{I}$$

provided that the infinite series converges. The total effect of **x** on **y** can be written as

$$\left(\mathbf{I} + \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + ...\right)\boldsymbol{\Gamma} = (\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\Gamma}.$$

In order to ensure stability of this system, convergence of the series is needed. A necessary and sufficient condition is that all the eigenvalues of **B** are within the unit circle.

The table below summarizes the formulae for direct, indirect and total effects.

	$\mathbf{x} \rightarrow \mathbf{y}$	$\mathbf{y} \rightarrow \mathbf{y}$
Direct	$\boldsymbol{\Gamma}$	$\mathbf{B}$
Indirect	$(\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\Gamma} - \boldsymbol{\Gamma}$	$(\mathbf{I} - \mathbf{B})^{-1} - \mathbf{I} - \mathbf{B}$
Total	$(\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\Gamma}$	$(\mathbf{I} - \mathbf{B})^{-1} - \mathbf{I}$

To obtain estimates of the indirect and total effects and their standard errors in LISREL, the EF option should be added to the OU line. The following additional output is then available in the output file:

```
Total and Indirect Effects

         Total Effects of X on Y

                X1          X2
          --------    --------
Y1           0.645       3.734
           (0.330)     (0.209)
```

```
           1.956     17.873

 Y2        0.396      2.295
         (0.203)    (0.128)
           1.956     17.953


   Indirect Effects of X on Y

              X1         X2
        --------   --------
 Y1       -0.858     -4.969
         (0.825)    (4.052)
          -1.041     -1.226

 Y2        0.396      2.295
         (0.203)    (0.128)
           1.956     17.953


   Total Effects of Y on Y

              Y1         Y2
        --------   --------
 Y1       -0.571     -0.929
         (0.199)    (0.324)
          -2.863     -2.869

 Y2        0.264     -0.571
         (0.122)    (0.199)
           2.156     -2.863


 Largest Eigenvalue of B*B' (Stability Index) is   4.689

   Indirect Effects of Y on Y

              Y1         Y2
        --------   --------
 Y1       -0.571      1.236
         (0.199)    (1.437)
          -2.863      0.860

 Y2       -0.351     -0.571
         (0.123)    (0.199)
          -2.851     -2.863
```

Under ML estimation, it seems as if the system is diverging as the Stability Index is larger than 1. However, if the stability index is calculated from the TSLS estimates instead, it is found to be 0.377.