A growth curve for the weights of mice

Contents

1. Introduction

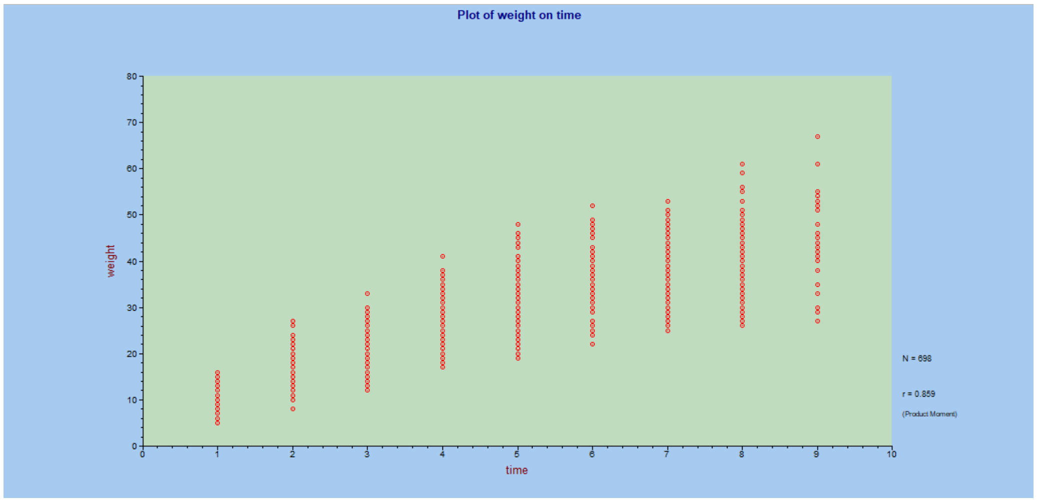
The data for this example contain repeated measurements on 82 striped mice and were obtained from the Department of Zoology at the University of Pretoria, South Africa (Du Toit, 1979). A number of male and female mice were released in an outdoor enclosure with nest boxes and sufficient food and water. They were allowed to multiply freely. Occurrence of birth was recorded daily and newborn mice were weighed weekly, from the end of the second week after birth until physical maturity was reached. The data set consists of the weights of 42 male and 40 female striped mice, measured at nine occasions in the case of males, and eight occasions in the case of females. Of interest here is to find a model that best describes the weight gain of the mice over time, while taking the gender into account. The data are in the file **mouse2.lsf**. The image below shows the data for the first two (male) mice:

1	1.00	1.00	15.00	1.00	1.00	1.00	0.00
2	1.00	2.00	17.00	1.00	2.00	4.00	0.00
3	1.00	3.00	23.00	1.00	3.00	9.00	0.00
4	1.00	4.00	24.00	1.00	4.00	16.00	0.00
5	1.00	5.00	26.00	1.00	5.00	25.00	0.00
6	1.00	6.00	31.00	1.00	6.00	36.00	0.00
7	1.00	7.00	37.00	1.00	7.00	49.00	0.00
8	1.00	8.00	42.00	1.00	8.00	64.00	0.00
9	1.00	9.00	46.00	1.00	9.00	81.00	0.00
10	2.00	1.00	11.00	1.00	1.00	1.00	0.00
11	2.00	2.00	14.00	1.00	2.00	4.00	0.00
12	2.00	3.00	20.00	1.00	3.00	9.00	0.00
13	2.00	4.00	24.00	1.00	4.00	16.00	0.00
14	2.00	5.00	29.00	1.00	5.00	25.00	0.00
15	2.00	6.00	35.00	1.00	6.00	36.00	0.00
16	2.00	7.00	36.00	1.00	7.00	49.00	0.00
17	2.00	8.00	41.00	1.00	8.00	64.00	0.00
18	2.00	9.00	43.00	1.00	9.00	81.00	0.00

The data set contains the following variables:

- ID2: Mouse identifier
- ID1: Measurement occasion identification
- Weight: weight in grams
- Intcept: A column of 1's that can be used to represent an intercept term
- Time: Time of measurement in
- Timesq: Time * Time
- Gender: the gender of mouse: 0 = male, 1 = female
- lwgt: the natural log of weight.

A scatter plot of the observed weights over the period of measurement is shown below. The variation in observed weight increases over the measurement period.



In a previous multilevel example, we considered models with quadratic terms and covariates. In this example, we will fit two nonlinear models to these data.

2. Logistic model

A commonly used model in the case of growth models is the logistic model. The logistic model fitted here can be defined as

$$y = b_1 / [1 + s * \exp(b_2 - b_3 * Time)] + e$$

$$b_1 = \beta_1 + \gamma_1 * Gender + u_1$$
$$b_2 = \beta_2 + \gamma_2 * Gender + u_2$$
$$b_3 = \beta_3 + \gamma_3 * Gender + u_3$$

The syntax for fitting this model is shown below.

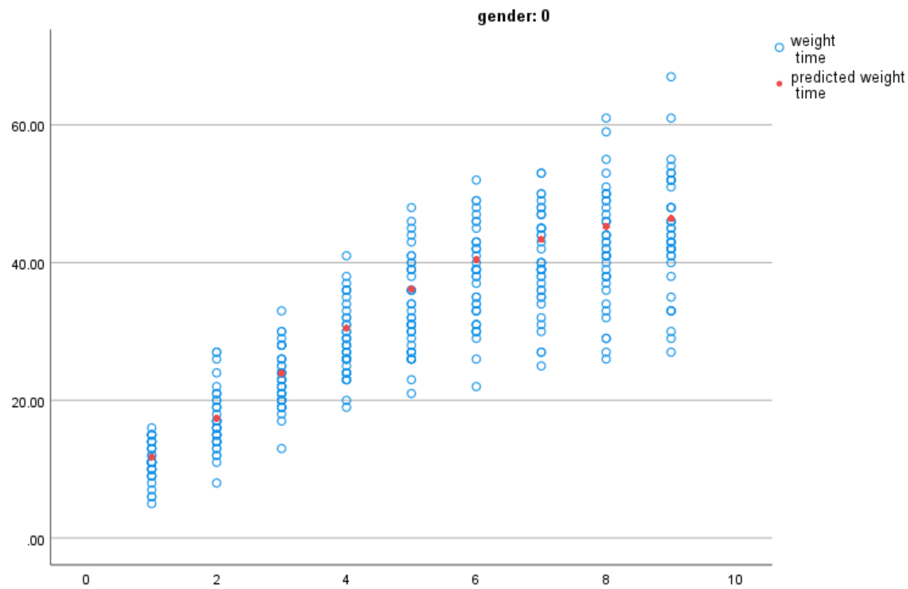
mouse1.prl

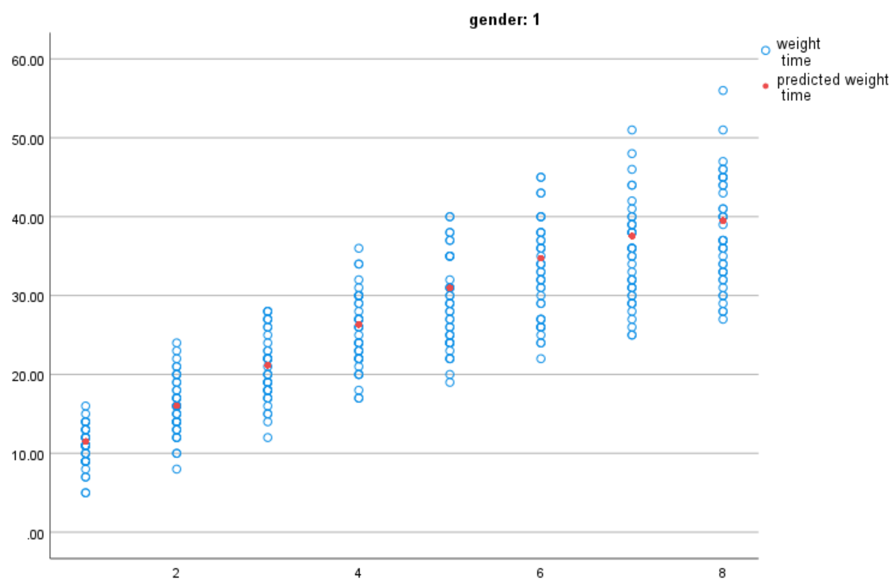
```
!-----------------------------------------------------------
! Repeated weight  measurements on 82 striped mice
! 42 males and 40 females. Gender=1 males , Gender=-1 females
!
! The selected model is the Logistic function
!
!    Level 1 model:
!       weight= b1/[1+s*exp(b2-b3*time)]+ e
!       note: s=  1
!
!    Level 2 model:
!       b1= beta1+ gamma1*gender+ u1
!       b2= beta2+ gamma2*gender+ u2
!       b3= beta3+ gamma3*gender+ u3
!-----------------------------------------------------------
OPTIONS METHOD = ML CONVERGE = 0.0000010 MAXITER =30 QUADPTS =50;
TITLE = Male and female mice weight measurements;
SY=mouse2.LSF;
ID1 = id1;
ID2 = id2;
RESPONSE = weight;
FIXED = time;
MODEL = Logistic ;
COVARIATES b1 = gender
           b2 = gender
           b3 = gender;
```

mouse1.OUT

```
  ---------------------------------------------------------------------------
  Coefficients           Beta          Std.Err.      Z-value        P > |z|
  ---------------------------------------------------------------------------
               b1      48.03676       1.15271      41.67275       0.00000
               b2       1.68784       0.04417      38.21090       0.00000
               b3       0.56049       0.01750      32.02994       0.00000


  ---------------------------------------------------------------------------
  Covariate Names        Gamma         Std.Err.      Z-value        P > |z|
  ---------------------------------------------------------------------------
       gender          -4.93925       1.71157      -2.88581       0.00390
       gender          -0.19079       0.06350      -3.00452       0.00266
       gender          -0.07345       0.02600      -2.82451       0.00474


  ---------------------------------------------------------------------------
  Variance estimate      Level 1       Std.Err.      Z-value        P > |z|
  ---------------------------------------------------------------------------
   Sigma**2             2.73018       0.10424      26.19085       0.00000



  ---------------------------------------------------------------------------
  Covariances            Level 2       Std.Err.      Z-value        P > |z|
  ---------------------------------------------------------------------------
          u1,u1        99.88239      12.80908       7.79778       0.00000
          u2,u1         2.09382       0.38121       5.49261       0.00000
          u2,u2         0.13358       0.01821       7.33712       0.00000
          u3,u1        -0.60819       0.15473      -3.93071       0.00008
          u3,u2         0.00423       0.00529       0.80010       0.42365
          u3,u3         0.01935       0.00294       6.57953       0.00000
```

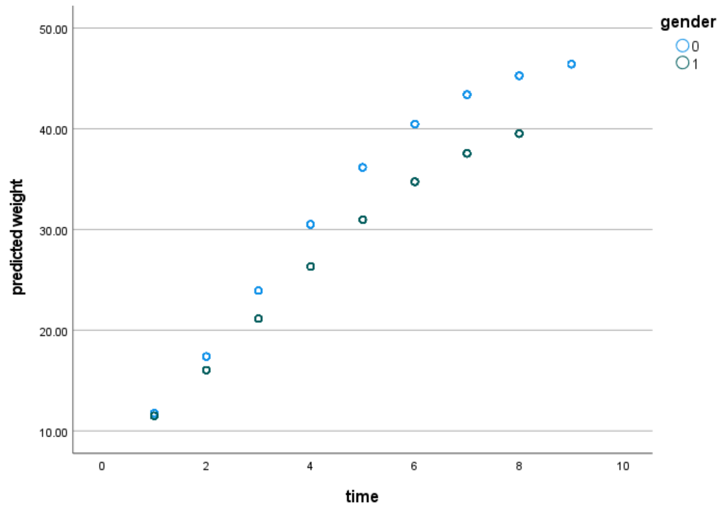
As we have a monotonic increase in function values over time, $s = 1$. Graphs for the two gender groups show that in general the logistic model describes the average growth curve well for both groups.





For the logistic model, the sum of squared residuals = 223266.

A scatter plot of expected weight over time for the two gender groups as shown below indicated higher expected weights for the male mice (Gender = 0).



3. Exponential model

Another model that may be used to described growth is the exponential model. In the exponential model, the independent variable (in this case time) becomes the exponent. The model we fit here is defined as

$$weight = b_1 \exp(-b_2 * time) + e$$

with associated level-2 model

$$b_1 = \beta_1 + \gamma_1 * gender + u_1$$
$$b_2 = \beta_2 + u_2$$

This model is given in **mouse2.prl** shown below. Note that we opted to use a covariate on the first component b_1 and not on the second component b_2.

mouse2.PRL

```
!----------------------------------------------------------
! Repeated weight  measurements on 82 striped mice
! 42 males and 40 females. Gender=1 males , Gender=-1 females
!----------------------------------------------------------
OPTIONS METHOD = ML CONVERGE = 0.0000010 MAXITER =30 QUADPTS =50;
TITLE = Male and female mice weight measurements;
SY=mouse2.LSF;
ID1 = id1;
ID2 = id2;
RESPONSE = weight;
FIXED = time;
MODEL = Exponential ;
COVARIATES b1 = gender;
```

The output for this model is as given below. All estimated coefficients except for $\mathrm{cov}(u_1, u_2)$ are statistically significant.

mouse2.OUT

Coefficients	Beta	Std.Err.	Z-value	P > |z|
b1	15.32344	0.31571	48.53661	0.00000
b2	-0.13137	0.00229	-57.34481	0.00000

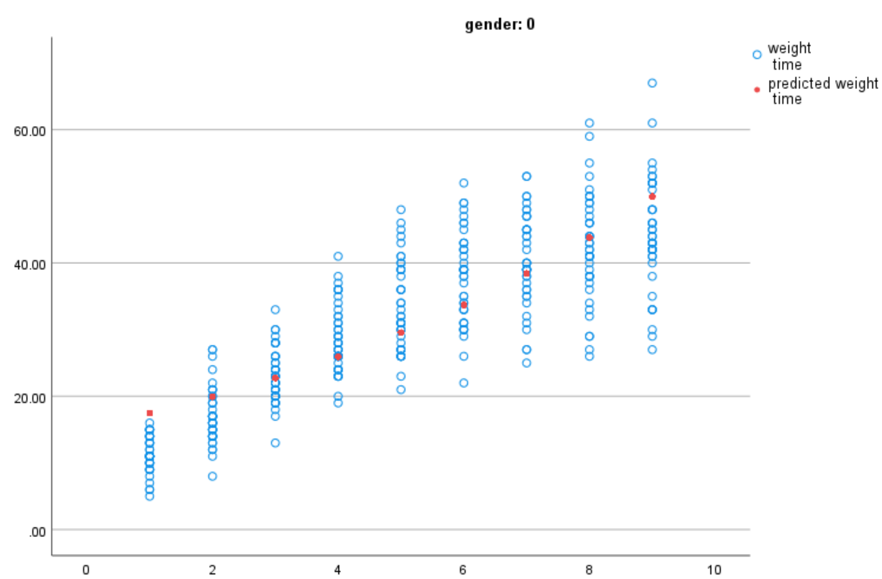
Covariate Names	Gamma	Std.Err.	Z-value	P > |z|
gender	-1.43060	0.36850	-3.88217	0.00010

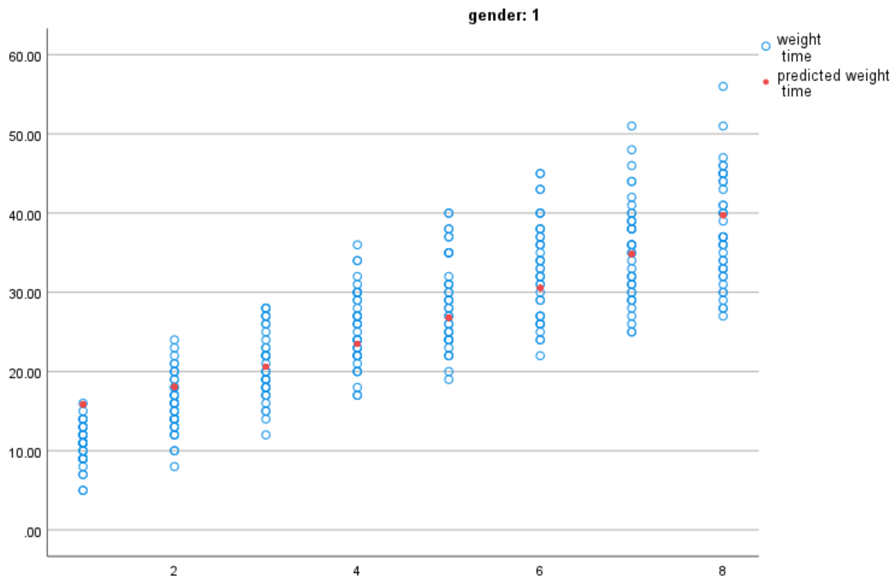
Variance estimate	Level 1	Std.Err.	Z-value	P > |z|
Sigma**2	18.07971	0.69473	26.02399	0.00000

Covariances	Level 2	Std.Err.	Z-value	P > |z|
u1,u1	6.34672	1.15943	5.47399	0.00000
u2,u1	0.02419	0.00896	2.70112	0.00691
u2,u2	0.00039	0.00009	4.22087	0.00002

Note: ML estimates of individual coefficients written to file THETAI.EST

Looking at the observed and predicted weights for the two gender groups, it is obvious that although the exponential function describes the general shape of the growth well towards the end of the time period, it overestimates at the beginning of the period for both gender groups.





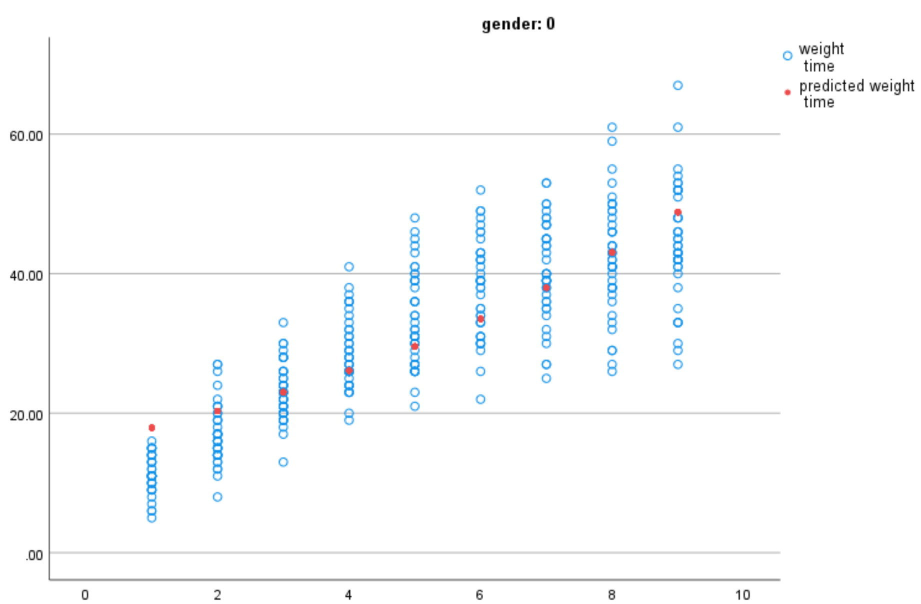
The squared sum of residuals for this model is 13912, compared to the 2232665 obtained for the logistic model.

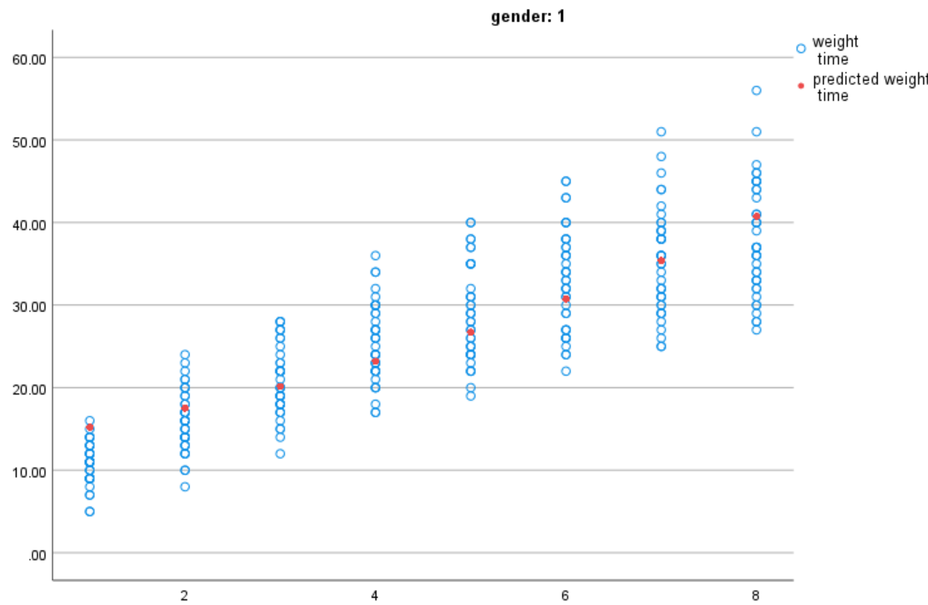
We conclude that the logistic model describes the data better than the exponential model.

4. Adding another covariate to the exponential model

The question may arise whether we should have included a covariate for both components in the exponential model described in the previous section. Perhaps this would have improved the fit? The short answer to this is no, but not by much. For this model, the squared sum of residuals is 14149. It is thus slightly higher than for the smaller exponential model fitted in the previous section and a lot smaller than that of the logistic model first considered. We conclude that the model with only one covariate described in Section 3 seems to describe the data the best of the three models considered here.

When we look at the scatterplot of predicted versus observed weight under the amended model, we note that there is very little change in the differences between the predicted and observed weights by gender.




gender: 1
weight
time
predicted weight
time
60.00
50.00
40.00
30.00
20.00
10.00
.00
2
4
6
8