



Confirmatory factor analysis for multiple groups using school data

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1. Introduction

In this example (Holzinger & Swineford (1939)) nine variables were selected to measure three latent factors: Space, Verbal and Memory. The groups consist of eighth-grade children from two schools in Chicago: the Pasteur and the Grant-White schools. The children from each school were divided into two groups according to whether they scored above or below the median on a speeded addition test. Thus the groups are:

1. Pasteur Low ($N_1 = 77$)
2. Pasteur High ($N_2 = 79$)
3. Grant-White Low ($N_3 = 74$)
4. Grant-White High ($N_4 = 71$)

The variables and their correlations are given in the table below.

Table: Nine psychological variables: correlations

Pasteur	Group 1 above diagonal; Group 2 below diagonal								
Test	1	2	3	4	5	6	7	8	9
Visual Perception	-	.32	.48	.28	.26	.40	.42	.12	.23
Cubes	.24	-	.33	.01	.01	.26	.32	.05	-.04
Paper Form Board	.23	.22	-	.06	.01	.10	.22	.03	.01
General Information	.43	.05	.23	-	.75	.60	.15	-.08	-.05
Sentence Completion	.35	.23	.18	.68	-	.63	.07	.06	.10
Word Classification	.36	.10	.11	.59	.66	-	.36	.19	.24
Figure Recognition	.22	.01	-.07	.09	.11	.12	-	.29	.19
Object-Number	-.02	-.01	-.13	.05	.08	.03	.19	-	.38
Number-Figure	.09	-.14	-.06	.16	.02	.12	.15	.29	-

Grant-White	Group 3 above diagonal; Group 4 below diagonal								
Test	1	2	3	4	5	6	7	8	9
Visual Perception	-	.34	.41	.38	.40	.42	.35	.16	.35
Cubes	.32	-	.21	.32	.16	.13	.27	.01	.27
Paper Form Board	.34	.18	-	.31	.24	.35	.30	.09	.09
General Information	.41	.24	.31	-	.69	.55	.17	.31	.34
Sentence Completion	.22	.16	.29	.62	-	.65	.20	.30	.27
Word Classification	.27	.20	.32	.57	.61	-	.31	.34	.27
Figure Recognition	.48	.31	.32	.18	.20	.29	-	.31	.38
Object-Number	.20	.01	.15	.06	.19	.15	.36	-	.38
Number-Figure	.42	-.14	.40	.11	.07	.18	.35	.44	-

The first few lines of the data sets **Pasteur.lsf** and **grantwhite.lsf** are shown below.

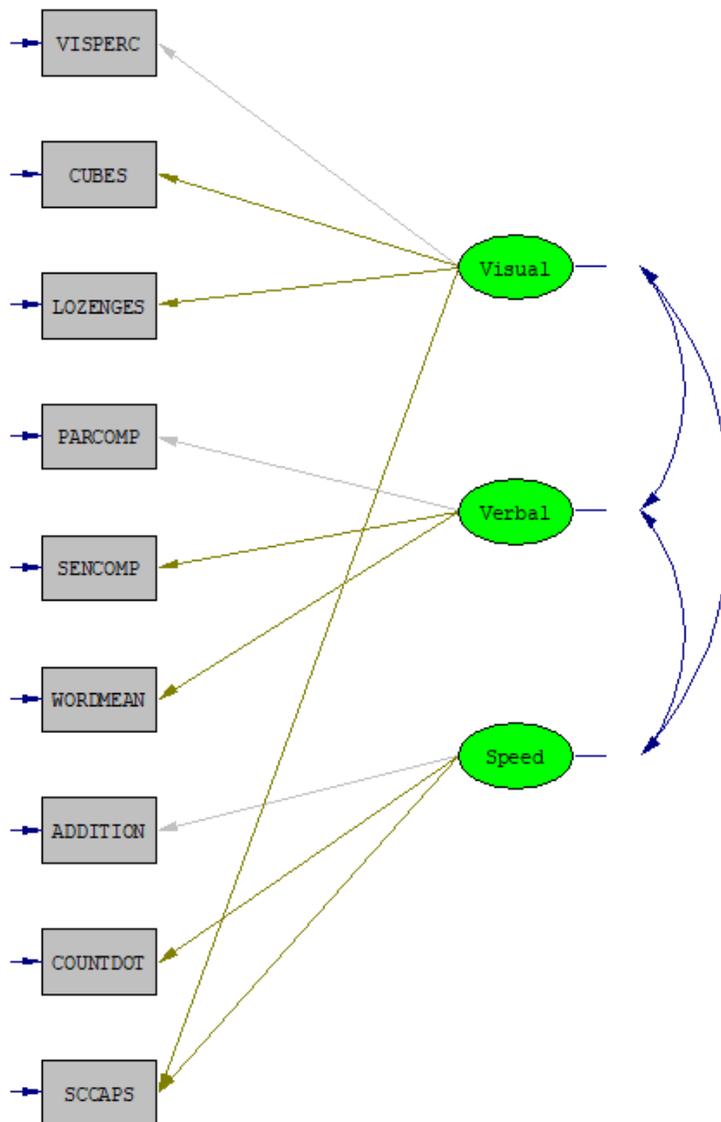
	VISPERC	CUBES	LOZENGES	PARCOMP	SENCOMP	WORDMEAN	ADDITION	COUNTDOT	SCCAPS
1	20.00	31.00	3.00	7.00	23.00	9.00	78.00	115.00	229.00
2	32.00	21.00	17.00	5.00	12.00	9.00	87.00	125.00	285.00
3	27.00	21.00	15.00	3.00	7.00	3.00	75.00	78.00	159.00
4	32.00	31.00	24.00	8.00	18.00	17.00	69.00	106.00	175.00
5	29.00	19.00	7.00	8.00	16.00	18.00	85.00	126.00	213.00
6	32.00	20.00	18.00	3.00	12.00	6.00	100.00	133.00	270.00
7	17.00	24.00	8.00	10.00	24.00	20.00	108.00	124.00	175.00
8	34.00	25.00	15.00	11.00	17.00	9.00	78.00	103.00	132.00
9	27.00	23.00	12.00	8.00	23.00	19.00	104.00	93.00	265.00
10	21.00	21.00	6.00	8.00	20.00	18.00	95.00	91.00	157.00

	VISPERC	CUBES	LOZENGES	PARCOMP	SENCOMP	WORDMEAN	ADDITION	COUNTDOT	SCCAPS
1	23.00	19.00	4.00	10.00	17.00	10.00	69.00	82.00	156.00
2	33.00	22.00	17.00	8.00	17.00	10.00	65.00	98.00	195.00
3	34.00	24.00	22.00	11.00	19.00	19.00	50.00	86.00	228.00
4	29.00	23.00	9.00	9.00	19.00	11.00	114.00	103.00	144.00
5	16.00	25.00	10.00	8.00	25.00	24.00	112.00	122.00	160.00
6	30.00	25.00	20.00	10.00	23.00	18.00	94.00	113.00	201.00
7	36.00	33.00	36.00	17.00	25.00	41.00	129.00	139.00	333.00
8	28.00	25.00	9.00	10.00	18.00	11.00	96.00	95.00	174.00
9	30.00	25.00	11.00	11.00	21.00	8.00	103.00	114.00	197.00
10	20.00	25.00	6.00	9.00	21.00	16.00	89.00	101.00	178.00

In this example, we are going to take a closer look at factorial invariance using these data.

2. Testing hypotheses of invariance

The model we are interested in fitting to these data is shown below. Note that the variable SCCAPS is allowed to load on both factors Visual and Speed.



The question we would like to investigate is the extent to which this model holds in both schools. Initial separate analyses for each school produced a χ^2 of 53.253 for the Pasteur school and 28.293 for the Grant-White school. Both of these χ^2 had 23 degrees of freedom, leading to the conclusion that the model does not fit equally well in both schools. We can use the sum of these χ^2 and their degrees of freedom (81.546, 46 degrees of freedom) as a baseline for testing, as these two groups are independent.

To formally test the hypothesis

$$H_A : \Lambda_1 = \Lambda_2$$

we use the syntax file **twoschool1b.lis** shown below. The use of the keyword **LX = IN** on the **MO** command specifies that Λ_x should be invariant.

```

twoschools1b.lis
Group: Pasteur
DA NG=2 NI=9
RA=PASTEUR.LSF
MO NX=9 NK=3 PH=FR TD=DI
LK
Visual Verbal Speed
FR LX(2,1) LX(3,1) LX(5,2) LX(6,2) LX(8,3) LX(9,3),LX(9,1)
VA 1 LX(1,1) LX(4,2) LX(7,3)
OU

Group: Grant White
DA
RA=GRANTWHITE.LSF
MO LX=IN
PD
OU

```

For this model, we obtain the following goodness-of-fit statistics.

Global Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	53
Maximum Likelihood Ratio Chi-Square (C1)	88.973 (P = 0.0014)
Browne's (1984) ADF Chi-Square (C2_NT)	104.596 (P = 0.0000)

The difference between this χ^2 and that of the baseline model is 7.429 with 7 degrees of freedom. Note that the degrees of freedom is equal to the number of factor loadings in Λ_x that are equal across the two groups. The hypothesis

$$H_A : \Lambda_1 = \Lambda_2$$

cannot be rejected.

The question that now arises is whether the intercepts in the measurement equations are the same in both groups. To evaluate this, we now test the hypothesis

$$H_{\Lambda\tau_x} : \Lambda_1 = \Lambda_2 \quad \tau_{x1} = \tau_{x2}$$

Since the means of the latent variables are 0 this is the same as saying that the means of the observed variables are equal across groups. This hypothesis is specified in the syntax file shown below. Note the addition of the keyword TX = IN on the second MO line. On the first MO line, we add TX = FR.

```

twoschools2b.lis
Group: Pasteur
DA NG=2 NI=9
RA=PASTEUR.LSF
MO NX=9 NK=3 PH=FR TD=DI TX=FR
LK
Visual Verbal Speed
FR LX(2,1) LX(3,1) LX(5,2) LX(6,2) LX(8,3) LX(9,3),LX(9,1)
VA 1 LX(1,1) LX(4,2) LX(7,3)
OU

Group: Grant White
DA
RA=GRANTWHITE.LSF
MO LX=IN TX=IN
PD
OU

```

For the model we obtain the following

Global Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	62
Maximum Likelihood Ratio Chi-Square (C1)	169.452 (P = 0.0000)
Browne's (1984) ADF Chi-Square (C2_NT)	183.539 (P = 0.0000)

When this χ^2 is compared with the baseline, it is clear that the hypothesis that $\tau_{x1} = \tau_{x2}$ must be rejected.

Finally, we check whether the difference in means of the observed variables can be explained by difference in means of the latent variables. This is done by adding KA = FR to the first MO line and TX = IN to the second MO line (see **twoschools3b.lis**).

For this model, we obtain

Global Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	59
Maximum Likelihood Ratio Chi-Square (C1)	130.135 (P = 0.0000)
Browne's (1984) ADF Chi-Square (C2_NT)	146.202 (P = 0.0000)

When these results are compared with that of the previous analysis, we see an improvement in fit with the addition of 3 parameters. This leads us to reject the hypothesis.

However, this cannot necessarily be considered a good fit. It may be more reasonable to use the RMSEA to measure fit in this case.

For this model the RMSEA is

Root Mean Square Error of Approximation (RMSEA)	0.108
90 Percent Confidence Interval for RMSEA	(0.0884 ; 0.127)
P-Value for Test of Close Fit (RMSEA < 0.05)	0.000

A possible improvement on this model is given in the additional syntax file **twoschools4b.lis**.