Second-order factor analysis of nine psychological variables

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1. Introduction

The classic Holzinger and Swineford (1939) dataset consists of mental ability test scores of seventh- and eighth-grade Chicago children from two different schools (Pasteur and Grant-White). A smaller subset with 9 of the original 26 test variables is used here.

The variables are:

- VIS PERC: Visual perception
- CUBES: Cubes
- LOZENGES: Lozenges
- PAR COMP: Paragraph completion
- SEN COMP: Sentence completion
- WORDMEAN: Word meaning
- ADDITION: Addition
- COUNTDOT: Counting dots
- S-C CAPS: Straight-curved capitals

Additional information on the students include

- SCHOOL: 0 for Pasteur school, 1 for Grant-White school
- GENDER: coded 0 for boys and 1 for girls
- AGEYEAR: Age in years
- BIRTHMON: Month of birth

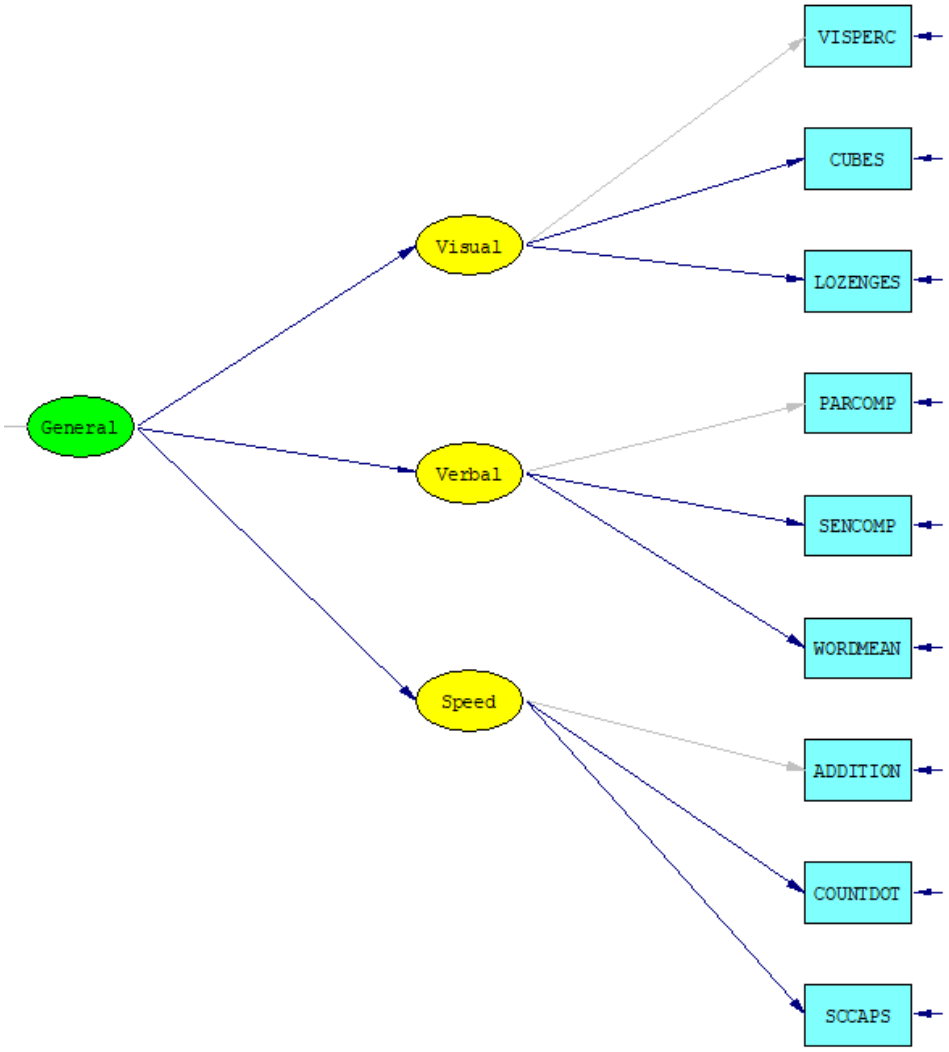
These data are available in the file **npv.lsf**. The first few lines of this file are shown below.

npv.LSF

	VISPERC	CUBES	LOZENGES	PARCOMP	SENCOMP	WORDMEAN	ADDITION	COUNTDOT	SCCAPS
1	23.00	19.00	4.00	10.00	17.00	10.00	69.00	82.00	156.00
2	33.00	22.00	17.00	8.00	17.00	10.00	65.00	98.00	195.00
3	34.00	24.00	22.00	11.00	19.00	19.00	50.00	86.00	228.00
4	29.00	23.00	9.00	9.00	19.00	11.00	114.00	103.00	144.00
5	16.00	25.00	10.00	8.00	25.00	24.00	112.00	122.00	160.00
6	30.00	25.00	20.00	10.00	23.00	18.00	94.00	113.00	201.00
7	36.00	33.00	36.00	17.00	25.00	41.00	129.00	139.00	333.00
8	28.00	25.00	9.00	10.00	18.00	11.00	96.00	95.00	174.00
9	30.00	25.00	11.00	11.00	21.00	8.00	103.00	114.00	197.00
10	20.00	25.00	6.00	9.00	21.00	16.00	89.00	101.00	178.00
11	27.00	26.00	6.00	10.00	16.00	13.00	88.00	107.00	137.00
12	32.00	21.00	8.00	1.00	7.00	11.00	103.00	136.00	154.00
13	38.00	31.00	12.00	10.00	11.00	14.00	83.00	108.00	201.00
14	17.00	21.00	6.00	5.00	10.00	10.00	99.00	87.00	147.00
	34.00	28.00	24.00	14.00	22.00	26.00	49.00	84.00	171.00

2. Second-order factor analysis

When factors in an oblique factor solution depend on other factors, these factors are referred to as second-order factors. One of the earliest discussion of this topic was by Schmidt and Leiman (1957). For these data, we propose to fit the model shown in the conceptual path diagram below.



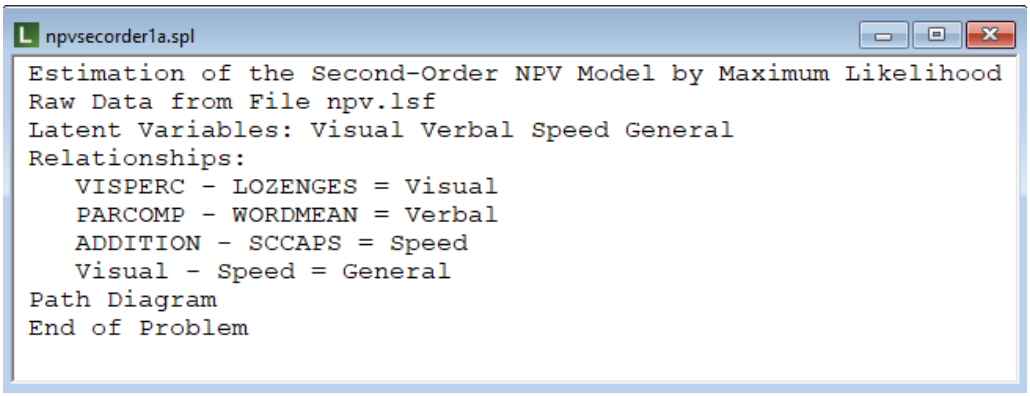
In this model, there are four latent variables: General, Visual, Verbal and Speed. There is a dependence structure on them so that Visual, Verbal and Speed depend on General. The latent variables Visual, Verbal and Speed are first-order factors and the latent variable General is a second-order factor.

In equation form, this model is given by two sets of equations:

$$\mathbf{y} = \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon}$$
$$\boldsymbol{\eta} = \boldsymbol{\Gamma} \boldsymbol{\xi} + \boldsymbol{\zeta}$$

In the figure above, $\mathbf{y}$ is supposed to satisfy a confirmatory factor model with three indicators of each of the first order factors. $\boldsymbol{\eta}$ is supposed to satisfy a confirmatory factor model with one factor.

The syntax to fit this model is shown below. The corresponding LISREL syntax is given in **npvsecorder1b.lis**.



While the solution obtained here is standardized in the sense that the latent variables are standardized, it is not a completely standardized solution for the observed variables that are analyzed in the original units of measurement. To obtain a completely standardized solution, the syntax file should be amended to include the line

```
Options SC
```

or

```
Analyze correlations.
```

The following second-order factor loadings are obtained:

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         Structural Equations


  Visual = 0.917*General, Errorvar.= 0.158 , R² = 0.842
Standerr  (0.176)                   (0.246)
Z-values   5.225                     0.644
P-values   0.000                     0.519

  Verbal = 0.589*General, Errorvar.= 0.653 , R² = 0.347
Standerr  (0.117)                   (0.145)
Z-values   5.037                     4.488
P-values   0.000                     0.000

   Speed = 0.570*General, Errorvar.= 0.675 , R² = 0.325
```

Standerr	(0.132)	(0.199)
Z-values	4.327	3.385
P-values	0.000	0.001

The goodness of fit statistics for this model are given as

Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)	24
Maximum Likelihood Ratio Chi-Square (C1)	51.542 (P = 0.0009)
Browne's (1984) ADF Chi-Square (C2_NT)	48.952 (P = 0.0019