Exponential Logistic Curve

Contents

1. Introduction

In this example we consider the fitting of a combination curve, with one component exponential and one logistic, to simulated longitudinal data. Values were simulated for 17 time points and 100 cases. Data are given in **exp_logist.lsf** and the data for the first two cases are shown below.

exp_logist.lsf

	id2	id1	y	time
1	1.00	1.00	41.88	2.00
2	1.00	2.00	39.90	3.00
3	1.00	3.00	43.42	4.00
4	1.00	4.00	42.48	5.00
5	1.00	5.00	44.05	6.00
6	1.00	6.00	43.04	7.00
7	1.00	7.00	46.26	8.00
8	1.00	8.00	51.08	9.00
9	1.00	9.00	58.69	10.00
10	1.00	10.00	64.90	11.00
11	1.00	11.00	69.81	12.00
12	1.00	12.00	71.93	13.00
13	1.00	13.00	72.39	14.00
14	1.00	14.00	72.43	15.00
15	1.00	15.00	75.19	16.00
16	1.00	16.00	72.06	17.00
17	1.00	17.00	76.46	18.00
18	2.00	1.00	43.60	2.00
19	2.00	2.00	48.67	3.00
20	2.00	3.00	52.30	4.00
21	2.00	4.00	53.38	5.00
22	2.00	5.00	58.42	6.00
23	2.00	6.00	61.43	7.00
24	2.00	7.00	64.88	8.00
25	2.00	8.00	75.02	9.00
26	2.00	9.00	79.43	10.00
27	2.00	10.00	93.34	11.00
28	2.00	11.00	103.93	12.00
29	2.00	12.00	111.61	13.00
30	2.00	13.00	120.67	14.00
31	2.00	14.00	124.34	15.00
32	2.00	15.00	132.24	16.00
33	2.00	16.00	140.87	17.00
34	2.00	17.00	147.56	18.00

The model is defined as

$$y = b_1 * \exp(-b_2 * Time) + c_1 / [1 + s * \exp(c_2 - c_3 * Time] + e$$

with b_1 = 40.00, b_2 = -0.04, c_1 = 24.00, c_2 = 12.50, c_3 = 1.05, and where

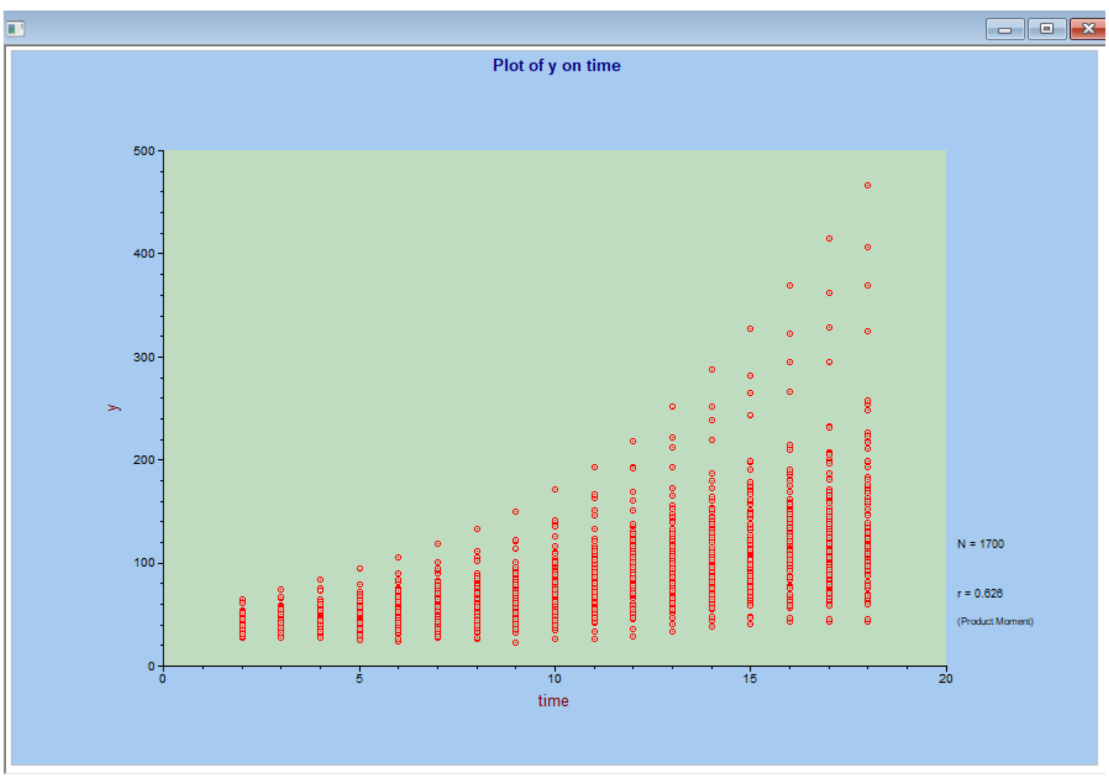
$$\begin{aligned} b_1 &= \beta_1 + u_1 \\ b_2 &= \beta_2 + u_2 \\ c_1 &= \beta_3 + u_3 \\ c_2 &= \beta_4 + u_4 \\ c_3 &= \beta_5 + u_5 \end{aligned}$$

The covariance matrix of the parameters b_1, b_2, c_1, c_2 and c_3 used for simulation was

$$\begin{bmatrix} 35.000 & & & & \\ -0.070 & 0.001 & & & \\ -14.000 & 0.025 & 15.000 & & \\ 3.053 & -0.035 & -4.500 & 7.000 & \\ 0.160 & -0.003 & -0.275 & 0.400 & 0.030 \end{bmatrix}$$

and a value of 1.0 was used for σ^2. As in this case it is assumed that the function values increase with a monotonic increase in time, s = 1.

A scatterplot of the simulated y measurements over time is shown below. The relationship between y and time is nonlinear.



2. Exponential logistic curve

We now fit a two-component curve to the data in order to examine how the estimated and simulated coefficients compare. The syntax file for this model is shown in the syntax file **expon_logist.prl**. The variable id2 is used as level-2 identifier (ID2).

expon_logist.prl

```
OPTIONS METHOD = MAP CONVERGE = 0.000001 MAXITER =30 QUADPTS =15;
TITLE = Exponential Logistic combination curve fitted to simulated data ;
SY=exp_logist.LSF;
ID1 = id1;
ID2 = id2;
RESPONSE = y;
FIXED = time;
MODEL = exponential + logistic;
```

The MAP solution is as follows.

```
 --------------------------------------
     Coefficients           Beta
     --------------------------------------
                b1        40.18346
                b2        -0.04801
                c1        23.67227
                c2        12.58110
                c3         1.05500

     --------------------------------------
     Variance estimate     Level 1
     --------------------------------------
      Sigma**2             1.04730

     --------------------------------------
     Covariances           Level 2
     --------------------------------------
             u1,u1        28.20963
             u2,u1        -0.03935
             u2,u2         0.00088
             u3,u1        -8.02956
             u3,u2         0.01843
             u3,u3        15.97696
             u4,u1         1.33992
             u4,u2        -0.03046
             u4,u3        -6.14844
             u4,u4         5.09584
             u5,u1         0.02201
             u5,u2        -0.00306
             u5,u3        -0.38681
             u5,u4         0.33484
             u5,u5         0.02970

  Note: MAP estimates of individual coefficients written to file THETAI.MAP
```

The estimated beta coefficients are very close to the values used in simulation. While the estimates of the variance-covariance components are similar in size to the values used in simulation, the variance components are closer to the simulated values than the covariance estimates are.

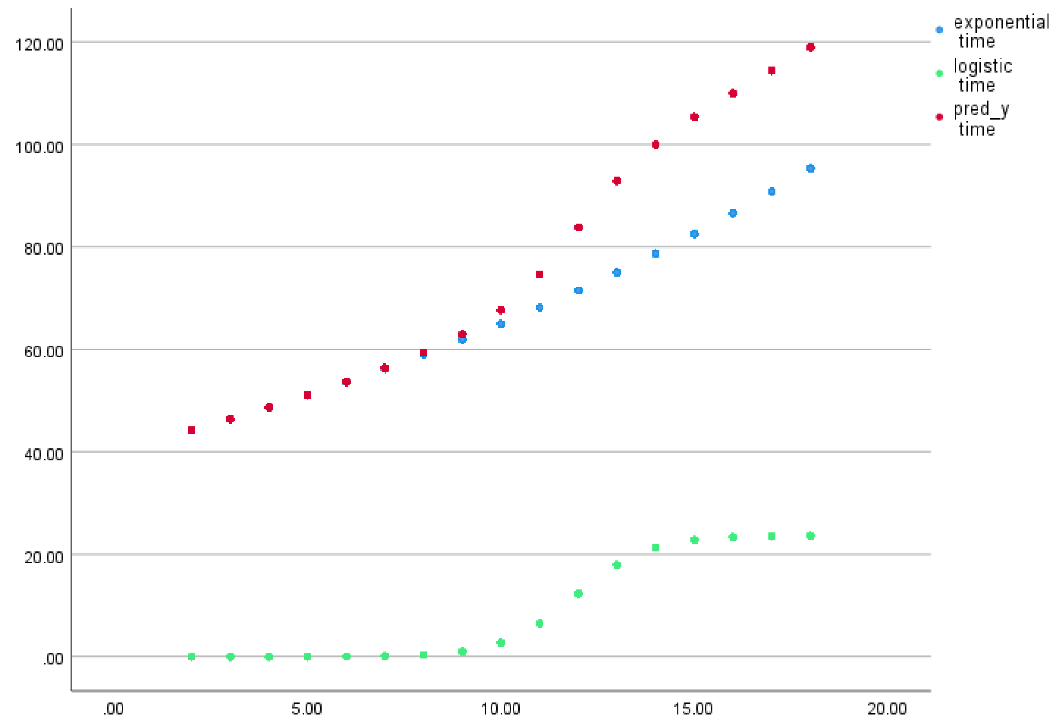
The average expected y can be calculated as

$$\hat{y} = 40.18346 * \exp(0.04801 * Time) + 23.67227 / [1 + \exp(12.58110 - 1.05500 * Time]$$

In the table below, the average predicted contribution of the two components to the average expected outcome (pred_y) are reported. The increased contribution to the average expected outcome of the logistic is greater towards the end of the time period.

exponential	logistic	pred_y	var
44.23	.00	44.23	
46.41	.00	46.41	
48.69	.01	48.70	
51.09	.02	51.10	
53.60	.05	53.64	
56.23	.13	56.36	
59.00	.37	59.37	
61.90	1.03	62.94	
64.95	2.75	67.69	
68.14	6.48	74.62	
71.49	12.30	83.79	
75.01	17.91	92.92	
78.70	21.29	99.98	
82.57	22.78	105.35	
86.63	23.36	109.98	

A plot of the components and the combined average expected outcome is given below.



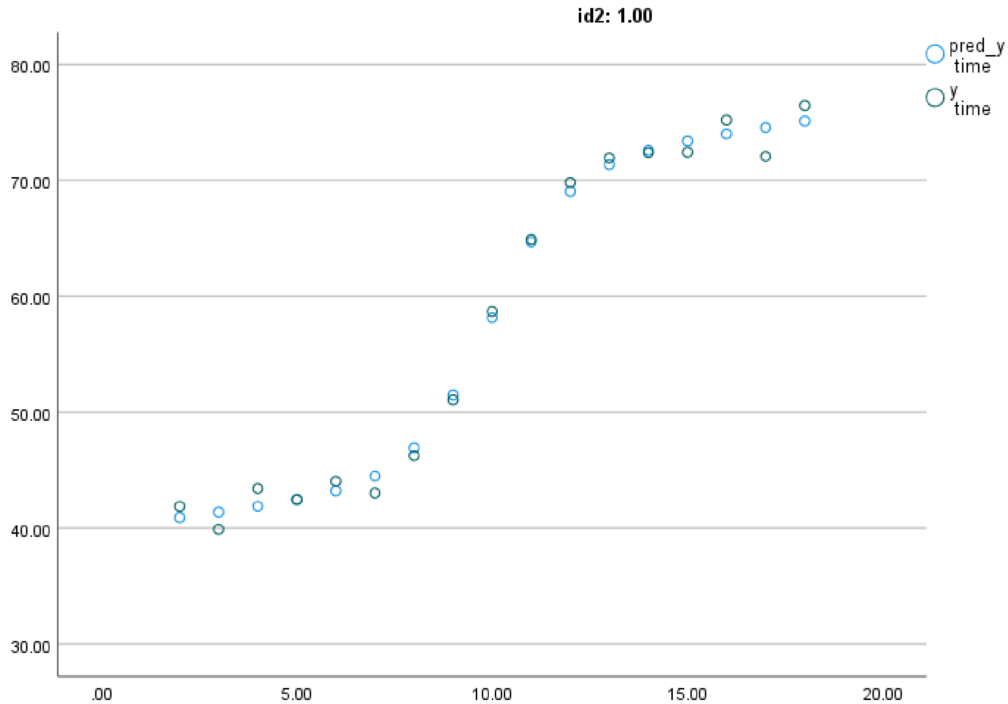
The program also writes the estimated parameters for each level-2 unit to an external text file named **thetai.map**. The contents of this file for the first few cases are as follows:

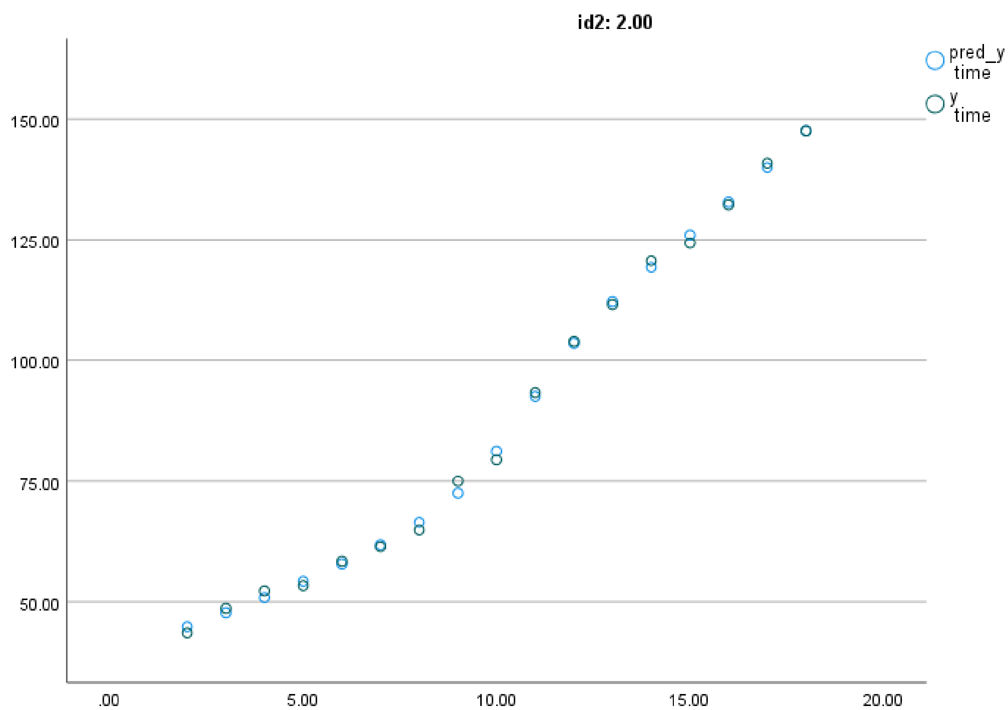
40.032	-0.10916E-01	26.389	10.011	1.0058
39.511	-0.63311E-01	24.052	12.324	1.1396
38.938	-0.46961E-01	21.338	12.908	1.1316
38.991	-0.28030E-01	26.781	9.7760	0.94645
44.719	-0.58698E-01	18.169	15.305	1.1278
46.408	-0.17760E-01	23.295	11.892	0.96081
31.076	-0.52641E-01	24.837	13.027	1.1415
42.484	-0.69559E-02	20.291	10.428	0.96747
40.912	-0.51075E-01	27.632	10.642	0.96548
31.499	-0.43648E-01	26.952	10.800	0.98206
41.805	-0.53991E-01	18.900	12.201	1.1869
31.512	0.30296E-01	24.950	10.934	0.84307
45.359	-0.32705E-01	18.520	14.042	1.0584
27.274	-0.35610E-01	28.024	12.630	1.0096
37.564	-0.85164E-01	23.223	14.527	1.1814

Using these results, the predicted outcome of, for example, the first case can be expressed as:

$$\hat{y} = 40.032 * \exp(0.01092 * Time) + 26.389 / [1 + \exp(10.011 - 1.0058 * Time]$$

When the observed and predicted heights are plotted, we see that the fitted curve describes the data well, as illustrated by the plots for the first 2 cases shown below.


id2: 1.00
pred_y
time
y
time
80.00
70.00
60.00
50.00
40.00
30.00
.00
5.00
10.00
15.00
20.00


id2: 2.00
pred_y
time
y
time
150.00
125.00
100.00
75.00
50.00
.00
5.00
10.00
15.00
20.00