Double power curve

Contents

1. Introduction

In this example we consider the fitting of a double power curve to simulated longitudinal data. Values were simulated for 20 time points and 100 cases. Data are given in **power_power.lsf** and the data for the first case are shown below.

power_power.lsf

	id2	id1	y	time
1	1.00	1.00	12.37	1.00
2	1.00	2.00	14.19	2.00
3	1.00	3.00	15.11	3.00
4	1.00	4.00	19.10	4.00
5	1.00	5.00	22.00	5.00
6	1.00	6.00	22.34	6.00
7	1.00	7.00	24.96	7.00
8	1.00	8.00	26.72	8.00
9	1.00	9.00	31.14	9.00
10	1.00	10.00	33.70	10.00
11	1.00	11.00	35.24	11.00
12	1.00	12.00	38.31	12.00
13	1.00	13.00	38.71	13.00
14	1.00	14.00	40.19	14.00
15	1.00	15.00	44.81	15.00
16	1.00	16.00	46.18	16.00
17	1.00	17.00	46.16	17.00
18	1.00	18.00	49.70	18.00
19	1.00	19.00	52.59	19.00
	1.00	20.00	56.68	20.00

The model is defined as

$$y = b_1 * Time^{b_2} + c_1 * Time^{c_2} + e$$

with b_1 = 5.00, b_2 = 0.8, c_1 = 8.00, c_2 = -0.80, and where

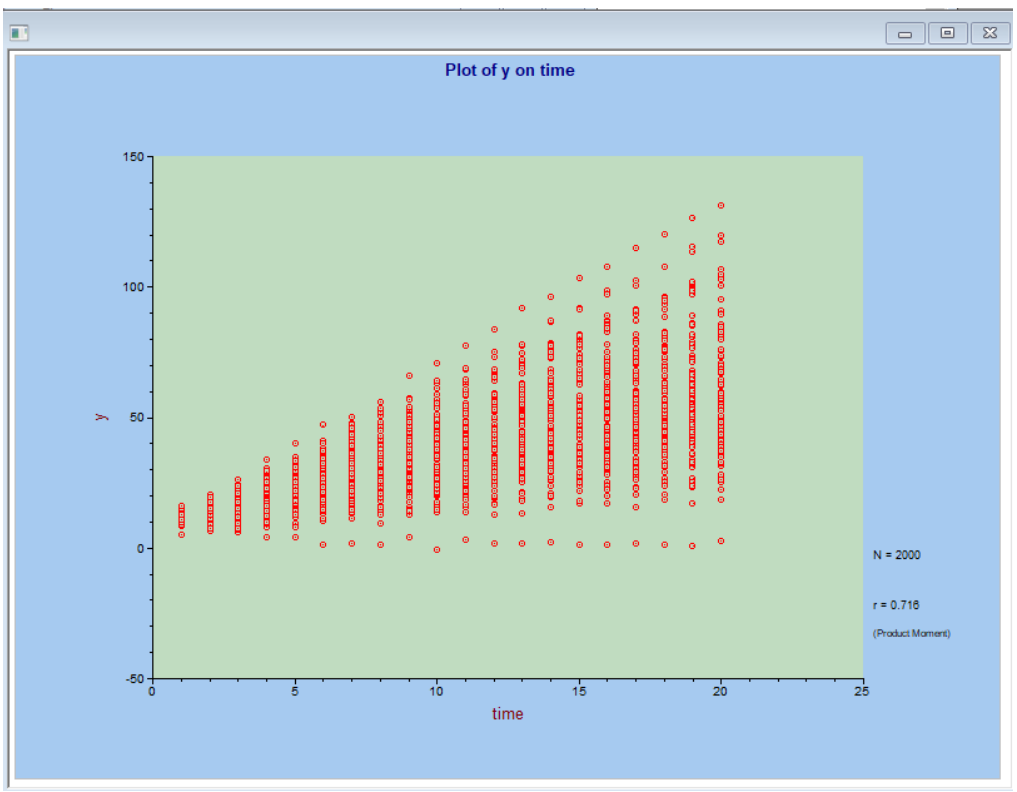
$$\begin{aligned} b_1 &= \beta_1 + u_1 \\ b_2 &= \beta_2 + u_2 \\ c_1 &= \beta_3 + u_3 \\ c_2 &= \beta_4 + u_4 \end{aligned}$$

The covariance matrix of the parameters b_1, b_2, c_1 and c_2 used for simulation was

$$\begin{bmatrix} 2.500 & & & \\ 0.050 & 0.0050 & & \\ -1.00 & -0.0300 & 1.6000 & \\ 0.005 & 0.0001 & 0.0250 & 0.003 \end{bmatrix}.$$

and a value of 1.5 was used for σ^2.

A scatterplot of the simulated y measurements over time is shown below. The relationship between y and time is nonlinear.



2. Double power curve

We now fit a double power curve to the data in order to examine how the estimated and simulated coefficients compare. The syntax file for this model is shown in the syntax file **double_power.prl**. The variable Case is used as level-2 identifier (ID2).

double_power.prl

```
OPTIONS METHOD = ML CONVERGE = 0.0001 MAXITER =100 QUADPTS = 20;
TITLE = Double Power Curve fitted to simulated data ;
SY=power_power.LSF;
ID1 = id1;
ID2 = id2;
RESPONSE = y;
FIXED = time;
MODEL = power + power;
```

The ML solution is as follows.

double_power.OUT

```
 ----------------------------------------------------------------------------
  Coefficients             Beta          Std.Err.       Z-value         P > |z|
 ----------------------------------------------------------------------------
             b1          7.76778         0.14264       54.45794         0.00000
             b2         -0.82948         0.03577      -23.19192         0.00000
             c1          5.29427         0.12705       41.67172         0.00000
             c2          0.80755         0.00621      129.94141         0.00000


 ----------------------------------------------------------------------------
  Variance estimate        Level 1       Std.Err.       Z-value         P > |z|
 ----------------------------------------------------------------------------
   Sigma**2              1.42257         0.03221       44.17109         0.00000



 ----------------------------------------------------------------------------
  Covariances              Level 2       Std.Err.       Z-value         P > |z|
 ----------------------------------------------------------------------------
          u1,u1          2.33681         0.40436        5.77905         0.00000
          u2,u1         -0.03618         0.06700       -0.54007         0.58915
          u2,u2          0.01382         0.01699        0.81361         0.41587
          u3,u1         -1.14362         0.27850       -4.10638         0.00004
          u3,u2          0.01755         0.06066        0.28932         0.77234
          u3,u3          2.31187         0.31275        7.39198         0.00000
          u4,u1         -0.01929         0.01199       -1.60924         0.10756
          u4,u2          0.00058         0.00297        0.19426         0.84597
          u4,u3          0.04518         0.01009        4.48009         0.00001
          u4,u4          0.00422         0.00070        6.01631         0.00000

 Note: ML estimates of individual coefficients written to file THETAI.EST
```

It is interesting to see that the fitted curves are in reverse order of that simulated. Recall that the simulation parameter b_1 was 5.00, and c_1 was 8.00. The results above show b_1 estimated at approximately 8 and c_1 at 5. The value of sigma**2 is close to the value of 1.50 used in simulation.

The program also writes the estimated parameters for each level-2 unit to an external text file named **thetai.est**. The contents of this file for the first few cases are as follows:

thetai.est - Notepad

File Edit Format View Help

7.71367	-0.778618	5.23605	0.778357
10.0553	-0.741612	3.49112	0.662389
9.77306	-0.846204	4.67134	0.831417
7.92742	-0.747801	5.83015	0.850468
7.50234	-0.822205	5.95799	0.826503
7.81387	-0.837962	5.26809	0.886455
8.99532	-0.843137	5.81296	0.796649
7.17599	-0.825284	6.17404	0.793974
8.76194	-0.887671	5.29442	0.796043
7.89858	-0.849293	4.61045	0.854311
9.40786	-0.847895	4.01143	0.754379
8.43868	-0.833617	2.98976	0.776149
7.28652	-0.840807	6.79307	0.849256
5.30958	-0.796160	6.70639	0.912619
9.12587	-0.830954	4.18646	0.764005
8.29849	-0.761892	3.82594	0.713415
8.60005	-0.788437	4.70045	0.756298
8.01122	-0.859417	4.37720	0.838090
6.58409	-0.844638	7.50560	0.879024
7.07699	-0.809021	5.53517	0.861367
10.0188	-0.746039	0.468412E-01	0.701870
6.66040	-0.786863	7.29729	0.932037
4.83507	-0.795190	7.88137	0.862423

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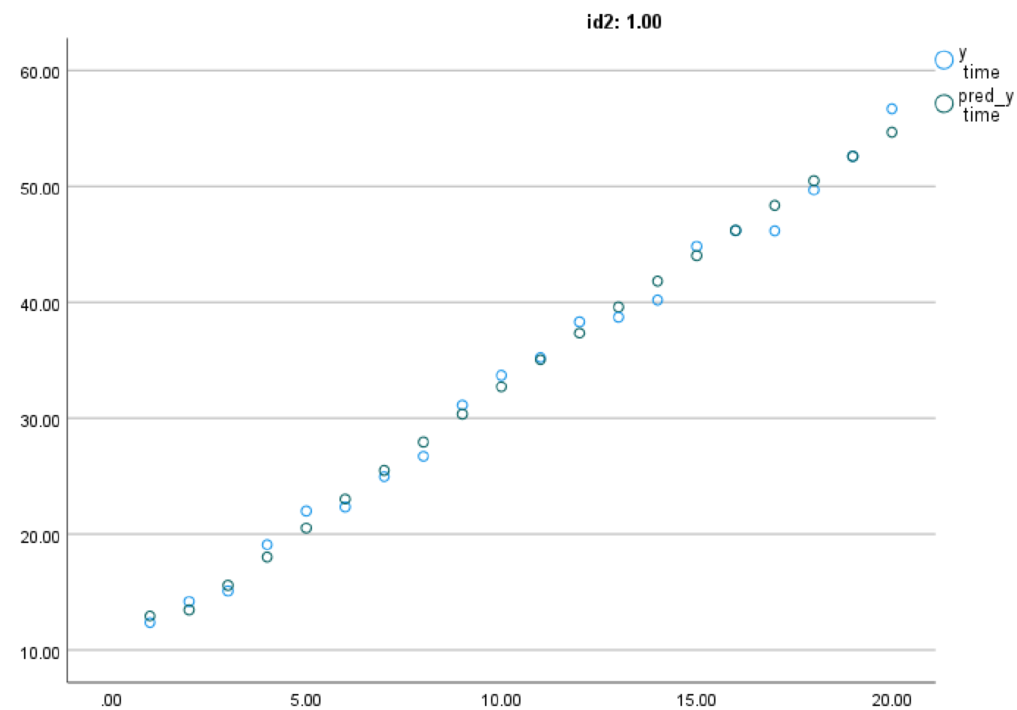
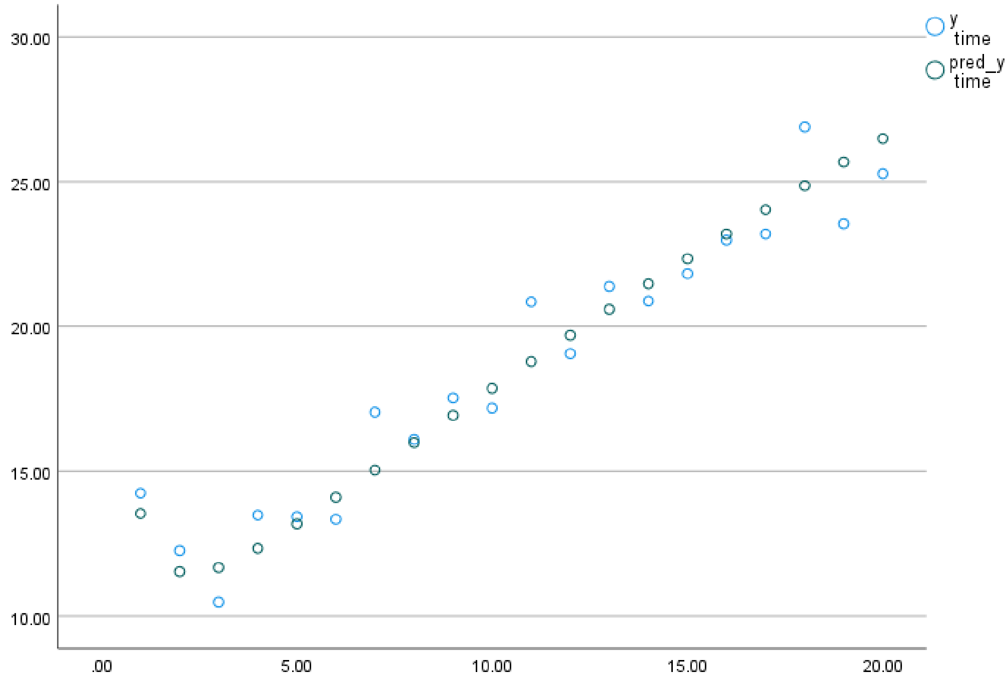
Using these results, the predicted outcome of, for example, the third case can be expressed as:

$$\hat{y} = 7.71367 * Time^{-0.778618} + 5.23605 * Time^{0.778357}$$

When the observed and predicted heights are plotted, we see that the fitted curve describes the data reasonably well, as illustrated by the plots for the first 3 cases shown below.



id2: 2.00



id2: 3.00