Gompertz curve

Contents

1. Introduction

In this example we consider the fitting of a Gompertz curve to simulated longitudinal data. Values were simulated for 8 time points and 100 cases. Data are given in **gompertz.lsf** and the data for the first two cases are shown below.

Gompertz.lsf

	id2	id1	y	time
1	1.00	1.00	248.33	0.00
2	1.00	2.00	166.64	1.00
3	1.00	3.00	126.63	2.00
4	1.00	4.00	100.81	3.00
5	1.00	5.00	88.16	4.00
6	1.00	6.00	79.12	5.00
7	1.00	7.00	69.65	6.00
8	1.00	8.00	65.43	7.00
9	2.00	1.00	194.79	0.00
10	2.00	2.00	132.86	1.00
11	2.00	3.00	97.15	2.00
12	2.00	4.00	77.08	3.00
13	2.00	5.00	68.84	4.00
14	2.00	6.00	60.17	5.00
15	2.00	7.00	52.07	6.00
16	2.00	8.00	50.59	7.00

The model is defined as

$$y = b_1 * \exp(-b_2 \exp(-s * b_3 * Time)) + e$$

with b_1 = 54.0, b_2 = 2.0, and b_3 = 0.30, where

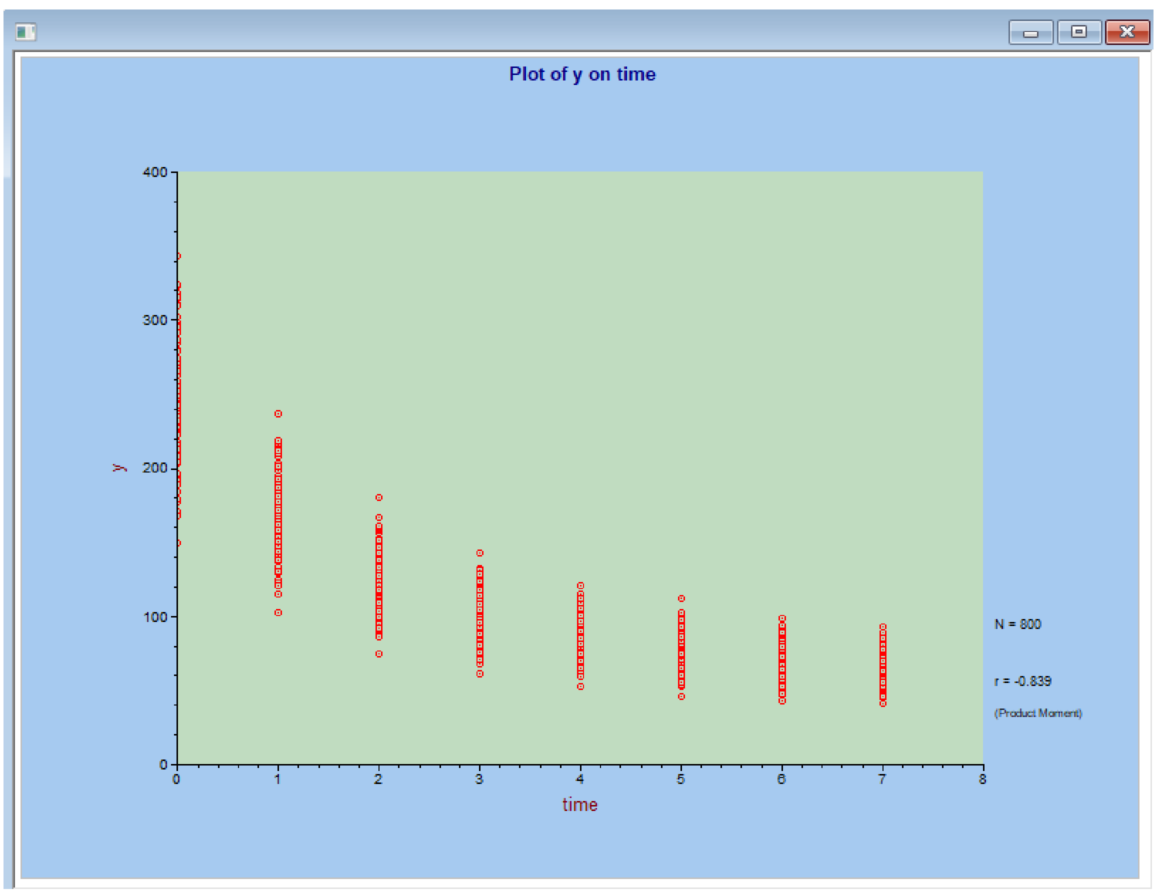
$$b_1 = \beta_1 + u_1$$
$$b_2 = \beta_2 + u_2$$
$$b_3 = \beta_3 + u_3$$

The covariance matrix of the parameters b_1, b_2, and b_3 used for simulation was

$$\begin{bmatrix} 90.000 & & \\ 0.570 & 0.192 & \\ 0.300 & 0.001 & 0.002 \end{bmatrix}.$$

The constant *s* has a value of 1 when the function values increase with a monotonic increase in time and -1 if it decreases. In this case *s* = -1.

A scatterplot of the *y* measurements over time is shown below. The relationship between *y* and time is nonlinear.



2. Gompertz curve

We now fit a gompertz curve to the data in order to examine how the estimated and simulated coefficients compare. The syntax file for this model is shown in the syntax file **gompertz.prl**. The variable Case is used as level-2 identifier (ID2).

Gompertz.prl

```
!----------------------------------------------------------
!   Model simulated data : Gompertz, 100 cases
!   y= b1*exp(-b2*exp(-b3*t)), t= 0,1,2,3,4,5,6,7
!      where:
!      b1=54.0,  b2=-2.00,  b3=0.30
!      Covariance matrix of parameters (b1,b2,b3)
!         90.000
!          0.570       0.192
!          0.300       0.001      0.002
!       Level 1 error variance : 3.00
!----------------------------------------------------------
OPTIONS METHOD = ML CONVERGE = 0.0000010 MAXITER = 30 QUADPTS = 33;
TITLE = Simulated data Single Gompertz;
SY=GOMPERTZ.LSF;
ID1 = id1;
ID2 = id2;
RESPONSE = y;
FIXED = time;
MODEL = Gompertz;
```

The ML solution is as follows.

Gompertz.OUT

```
 --------------------------------------------------------------------
   Coefficients          Beta        Std.Err.      Z-value      P > |z|
 --------------------------------------------------------------------
             b1       54.40032       0.67215     80.93528      0.00000
             b2       -1.50269       0.00238   -631.75649      0.00000
             b3        0.29973       0.00101    296.18257      0.00000


 --------------------------------------------------------------------
   Variance estimate    Level 1      Std.Err.      Z-value      P > |z|
 --------------------------------------------------------------------
    Sigma**2            2.53837       0.09325     27.22129      0.00000


 --------------------------------------------------------------------
   Covariances          Level 2      Std.Err.      Z-value      P > |z|
 --------------------------------------------------------------------
          u1,u1       86.91155       9.03585      9.61852      0.00000
          u2,u1        0.04520       0.02382      1.89732      0.05779
          u2,u2        0.00003       0.00011      0.27381      0.78423
          u3,u1        0.01416       0.00997      1.41955      0.15574
          u3,u2        0.00001       0.00004      0.24440      0.80692
          u3,u3        0.00004       0.00002      1.75930      0.07853

  Note: ML estimates of individual coefficients written to file THETAI.EST
```

The beta coefficients reported largely correspond to the values used in simulation. The value of sigma**2 is lower than the value of 3.00 used in simulation. While the size of the estimated variances of the three *b* coefficients echo those used in simulation, there is noticeable differences in the covariances.

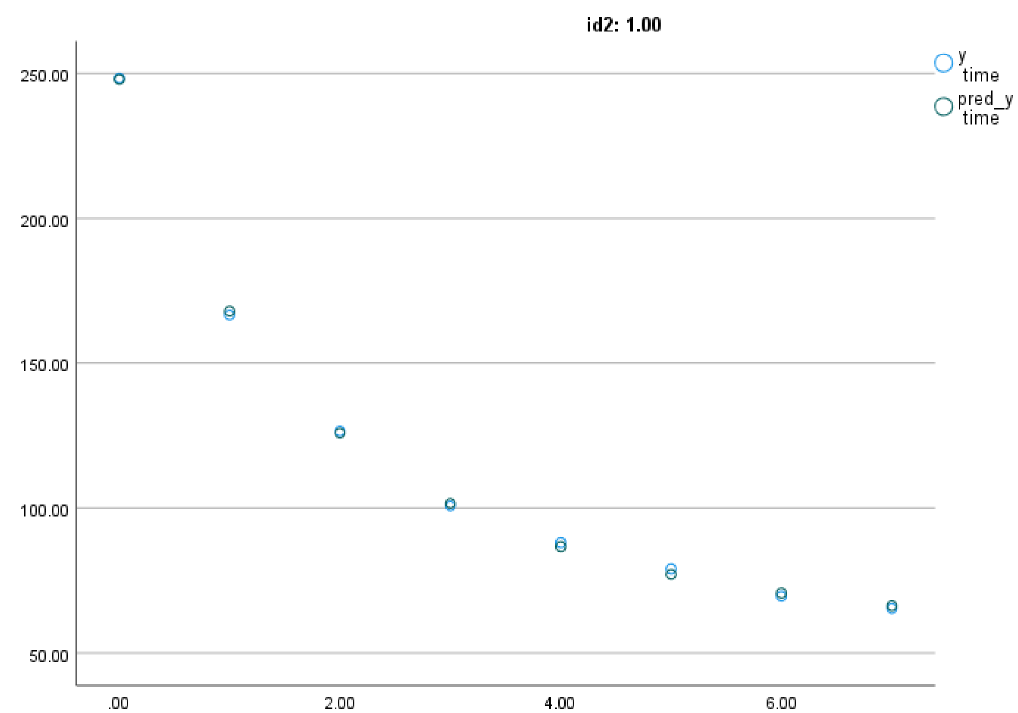
The program also writes the estimated parameters for each level-2 unit to an external text file named **thetai.est**. The contents of this file for the first few cases are as follows:

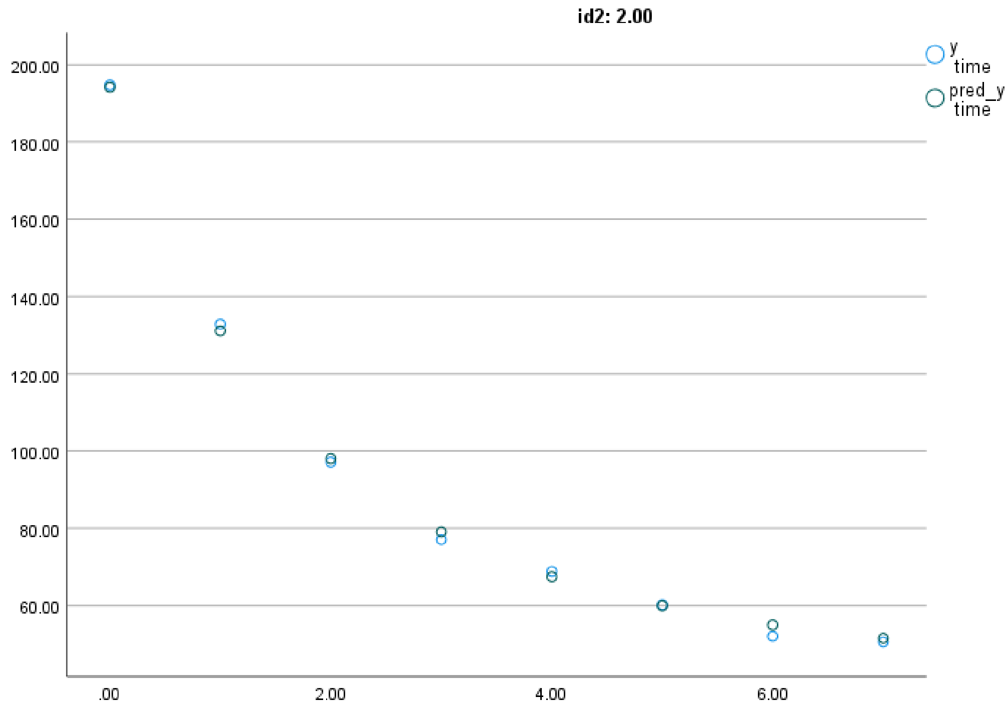
55.1960	-1.50225	0.299920
42.9573	-1.50868	0.301638
50.5869	-1.50493	0.293550
46.0998	-1.50701	0.296603
50.9108	-1.50450	0.291341
54.4046	-1.50275	0.303915
37.6454	-1.51135	0.296877
55.6144	-1.50203	0.309420
52.5886	-1.50370	0.292706
50.7248	-1.50427	0.303589
69.5232	-1.49479	0.305350
66.7456	-1.49629	0.297418
57.3488	-1.50127	0.293293
64.1035	-1.49752	0.304034
54.5461	-1.50238	0.301071
70.6254	-1.49443	0.307217
69.5894	-1.49489	0.300505
53.2464	-1.50327	0.300146
67.0046	-1.49607	0.293826
63.5495	-1.49755	0.303348
47.4135	-1.50618	0.301047
42.0546	-1.50908	0.298040

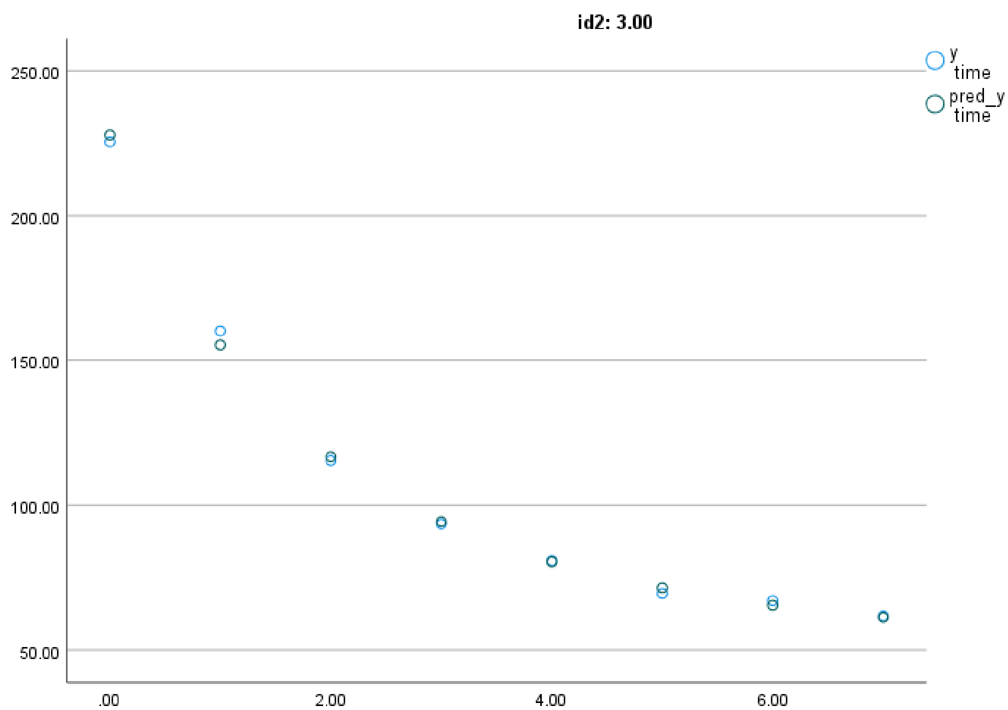
Using these results, the predicted outcome of, for example, the third case can be expressed as:

$$\hat{y} = 50.5869 * \exp(1.50868 \exp(-0.293550 * Time))$$

When the observed and predicted outcomes are plotted, we see that the fitted curve describes the data well, as illustrated by the plots for the first 3 cases shown below.




id2: 2.00
y time
pred_y time
200.00
180.00
160.00
140.00
120.00
100.00
80.00
60.00
.00
2.00
4.00
6.00


id2: 3.00
y time
pred_y time
250.00
200.00
150.00
100.00
50.00
.00
2.00
4.00
6.00