

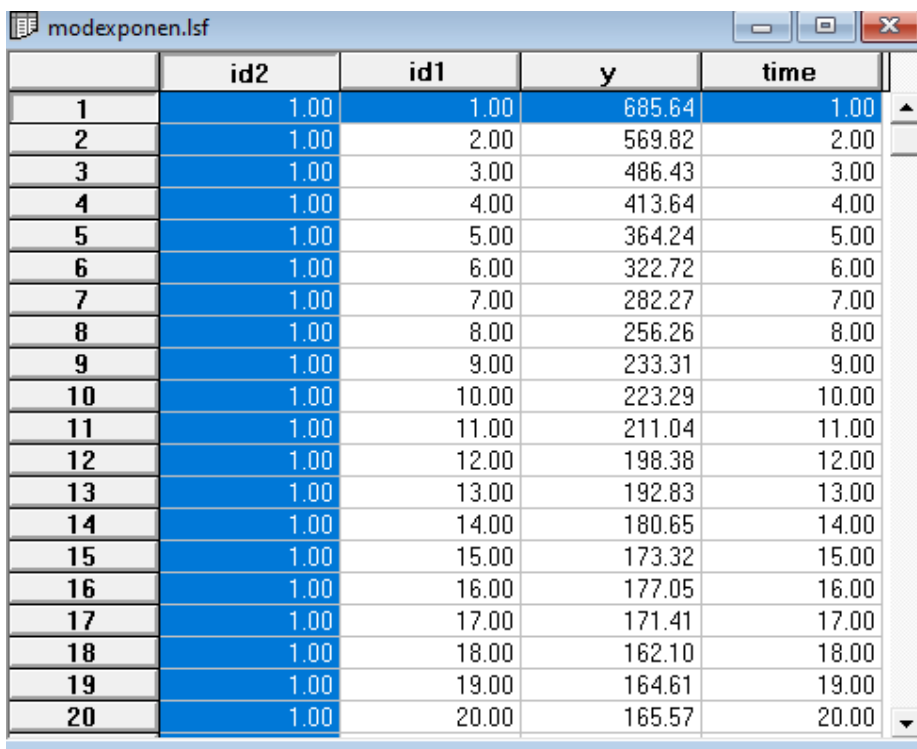
## Monomolecular curve for simulated data

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### 1. Introduction

In this example we consider the fitting of a monomolecular curve to simulated longitudinal data. Values were simulated for 20 time points and 80 cases. Data are given in **modexponen.lsf** and the data for the first case are shown below.



	id2	id1	y	time
1	1.00	1.00	685.64	1.00
2	1.00	2.00	569.82	2.00
3	1.00	3.00	486.43	3.00
4	1.00	4.00	413.64	4.00
5	1.00	5.00	364.24	5.00
6	1.00	6.00	322.72	6.00
7	1.00	7.00	282.27	7.00
8	1.00	8.00	256.26	8.00
9	1.00	9.00	233.31	9.00
10	1.00	10.00	223.29	10.00
11	1.00	11.00	211.04	11.00
12	1.00	12.00	198.38	12.00
13	1.00	13.00	192.83	13.00
14	1.00	14.00	180.65	14.00
15	1.00	15.00	173.32	15.00
16	1.00	16.00	177.05	16.00
17	1.00	17.00	171.41	17.00
18	1.00	18.00	162.10	18.00
19	1.00	19.00	164.61	19.00
20	1.00	20.00	165.57	20.00

The model is defined as

$$y = b_1 * [1 + s * \exp(b_2 - b_3 * \text{Time})] + e$$

with  $b_1 = 160.0$ ,  $b_2 = 1.50$ , and  $b_3 = 0.25$  used for simulation, where

$$b_1 = \beta_1 + u_1$$

$$b_2 = \beta_2 + u_2$$

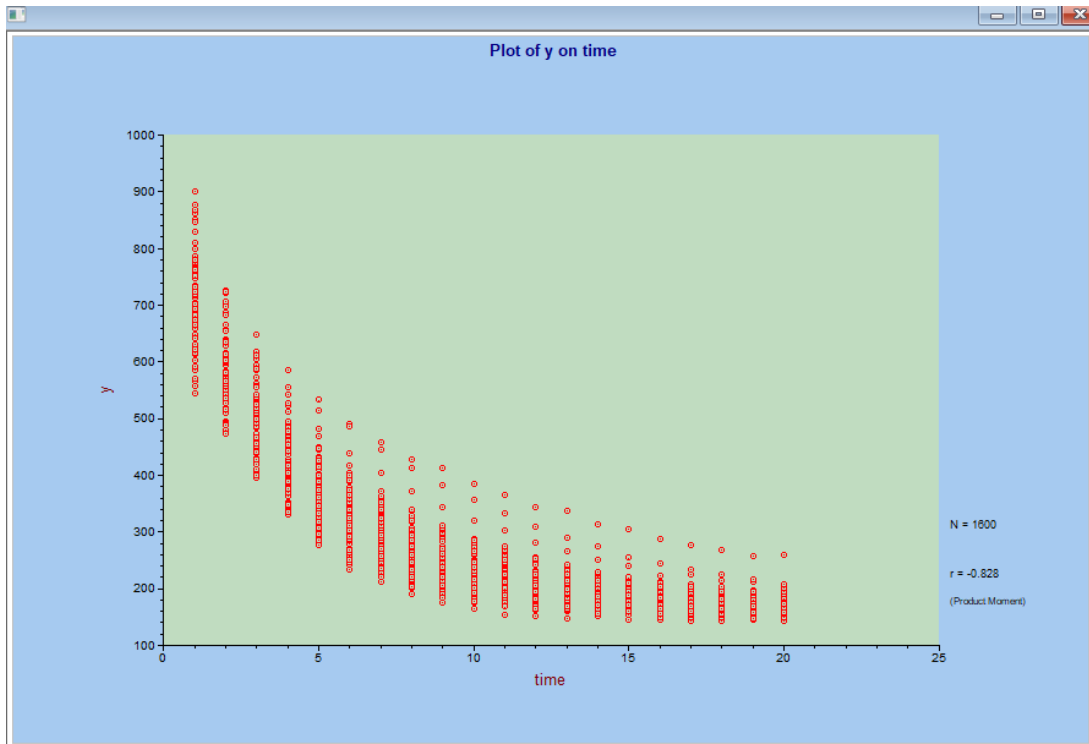
$$b_3 = \beta_3 + u_3$$

The covariance matrix of the parameters  $b_1$ ,  $b_2$ , and  $b_3$  used for simulation was

$$\begin{bmatrix} 85.000 & & \\ -1.000 & 0.050 & \\ 0.060 & 0.008 & 0.005 \end{bmatrix}$$

along with a value of 15 for  $\sigma^2$ .

A scatterplot of the  $y$  measurements over time is shown below. The relationship between  $y$  and time is nonlinear. When function values decrease monotonically over time as is the case here,  $s = 1$ .



## 2. Monomolecular curve

We now fit a monomolecular curve to the data in order to examine how the estimated and simulated coefficients compare. The syntax file for this model is shown in the syntax file **modexpon.prl**. The variable ID2 is used as level-2 identifier.

```
MODEXPON.prl
OPTIONS METHOD = ML CONVERGE = 0.00001 MAXITER =20 QUADPTS= 93;
TITLE = Simulated data modified exponential function ;
SY=modexponen.lsf;
ID1 = id1;
ID2 = id2;
RESPONSE = y;
FIXED = time;
MODEL = Monomolecular;
```

The ML solution is as follows.

```
MODEXPON.OUT
Maximum Likelihood Solution
```

Coefficients	Beta	Std.Err.	Z-value	P >  z
b1	160.75768	0.75635	212.54540	0.00000
b2	1.48017	0.01528	96.85824	0.00000
b3	0.25525	0.00538	47.44247	0.00000

Variance estimate	Level 1	Std.Err.	Z-value	P >  z
Sigma**2	16.37343	0.40653	40.27582	0.00000

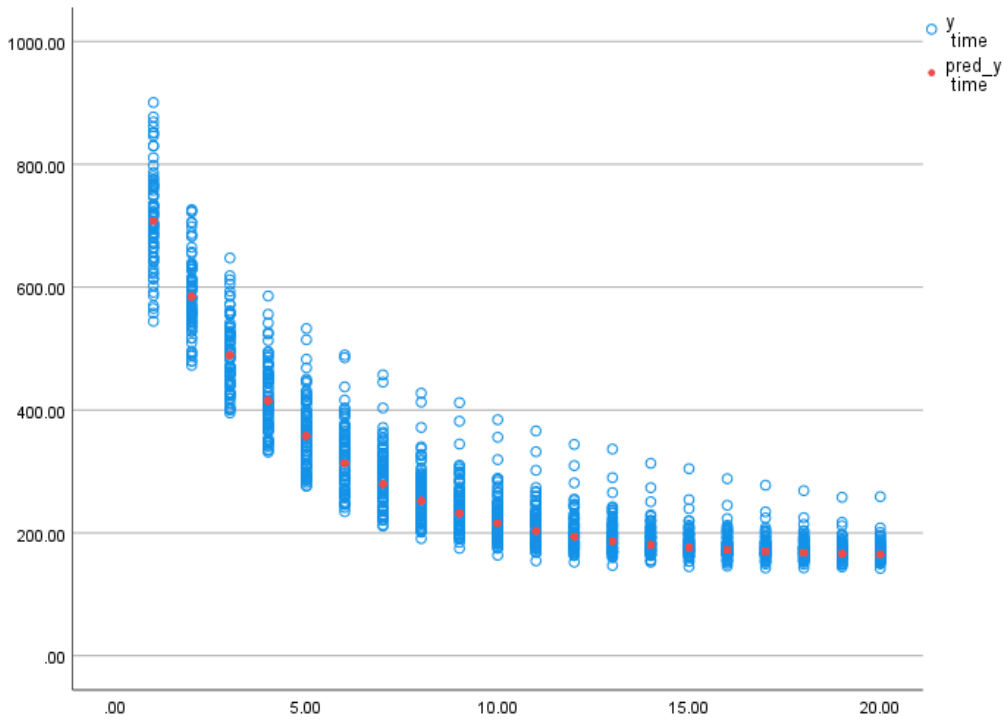
  

Covariances	Level 2	Std.Err.	Z-value	P >  z
u1,u1	86.57256	10.19262	8.49365	0.00000
u2,u1	-0.89759	0.16346	-5.49119	0.00000
u2,u2	0.03712	0.00418	8.88571	0.00000
u3,u1	0.09816	0.05212	1.88351	0.05963
u3,u2	0.00579	0.00114	5.09686	0.00000
u3,u3	0.00462	0.00052	8.91245	0.00000

Note: ML estimates of individual coefficients written to file THETA1.EST

The estimated beta coefficients are very close in value to those used in the simulation. This is also true of the estimated variance-covariance components.

A scatterplot of the predicted outcome and the simulated outcome over time is given below.



The program also writes the estimated parameters for each level-2 unit to an external text file named **thetai.est**. The contents of this file for the first few cases are as follows:

File	Edit	Format	View	Help
160.687		1.42615		0.241974
161.732		1.28529		0.964671E-01
145.737		1.58859		0.212490
163.643		1.42574		0.224722
163.045		1.69597		0.359848
158.225		1.64288		0.295048
161.169		1.75450		0.286779
161.196		1.64275		0.312356
165.252		1.48018		0.307195
162.210		1.48018		0.307218
165.069		1.24425		0.167672
169.022		1.42624		0.293932
175.295		1.37172		0.228737
167.276		1.64279		0.399079
143.186		1.77835		0.368954
147.654		1.53438		0.285893
158.687		1.85948		0.310339
174.947		1.15410		0.210215
153.525		1.29518		0.179571

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Using these results, the predicted outcome of, for example, the third case can be expressed as:

$$\hat{y} = 163.643 * [1 + \exp(1.42574 - 0.224722 * Time)]$$

When the observed and predicted outcomes are plotted, we see that the fitted curve describes the data well, as illustrated by the plots for the first 5 cases shown below.

