Growth curves for the weight of cows

Contents

1. Introduction

The data used in this example contains the weights of 27 cows over 23 measurement occasions, spanning approximately 22 months. Data for this example is from Diggle, Liang & Zeger (1994, pp. 100-101). Eight of the cows (cases 1 through 8) received and iron supplement, and the animals were monitored for infection with M. paratuberculosis. This is a 2x2 experimental design, with four groups:

- Cows that received an iron supplement and were not reported as infected,
- Cows that received an iron supplement and were reported as infected,
- Cows that did not receive an iron supplement and were not reported as infected, and
- Cows that did not receive an iron supplement and were reported as infected.

In order to evaluate the effect of the iron supplement, we split the data into two groups: cows who received the supplement, and those that did not (see **cows_iron_logist.lsf** and **cows_niron_logist.lsf**). For each of the two groups, the following variables are available:

- Case: A grouping variable for individual cows' data
- Occasion: Indicates the order of measurements
- Log_wgt, weight: the natural log of the weight in kilograms
- Time: The point in time at which the measurement was made, as a fraction of 100 days.
- Infected: indicating whether a cow was infected, with 0 indicating infection and 1 no infection.
- Iron: indicating whether a cow received an iron supplement (Iron = 1) or not (Iron = 0)

The first few lines of the file **cows_iron_logist.lsf** are shown below.

LISREL for Windows - [cows_iron_logist.lsf]

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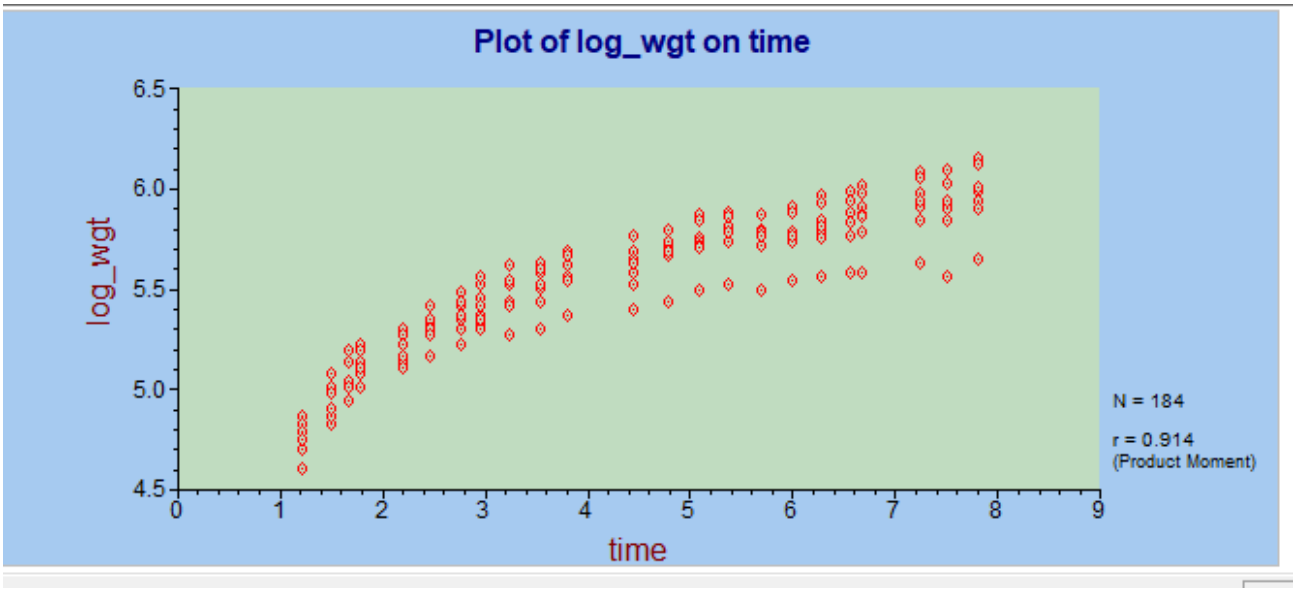
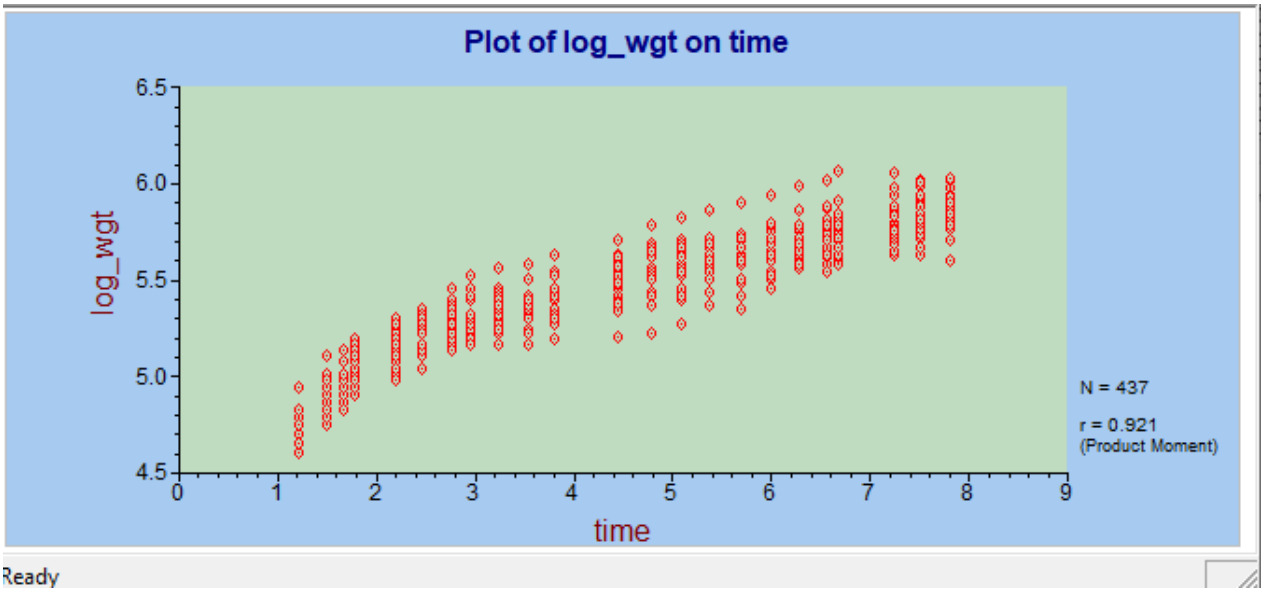
	case	occasion	log_wgt	weight	time	Infected
1	1.00	1.00	4.70	109.95	1.22	1.00
2	1.00	2.00	4.91	134.96	1.50	1.00
3	1.00	3.00	5.01	150.06	1.66	1.00
4	1.00	4.00	5.08	159.97	1.79	1.00
5	1.00	5.00	5.14	170.03	2.19	1.00
6	1.00	6.00	5.17	175.04	2.47	1.00
7	1.00	7.00	5.30	199.94	2.76	1.00
8	1.00	8.00	5.32	205.00	2.96	1.00
9	1.00	9.00	5.42	224.98	3.24	1.00
10	1.00	10.00	5.44	229.98	3.54	1.00
11	1.00	11.00	5.54	254.93	3.80	1.00
12	1.00	12.00	5.65	284.86	4.45	1.00
13	1.00	13.00	5.69	295.01	4.78	1.00
14	1.00	14.00	5.74	310.13	5.08	1.00
15	1.00	15.00	5.81	334.96	5.36	1.00

Ready

Scatterplots of the observed weight over time for the two groups are shown below. A scatterplot of the weights against time of measurement is obtained by using the **Graphs**, **Bivariate** option from the main menu bar. Log_wgt is specified as Y-variable, Time as x-variable.





Even when considered separately, there seems to be indications that a nonlinear curve may describe the data best.

2. Logistic model

We fit a logistic model to the data sets. The model we fit is formulated as

$$\log\_wgt_{it} = b_1 / \left[1 + s * Exp\left(b_2 - b_3 * Time_{it}\right)\right] + e_{it}$$
$$b_1 = \beta_1 + \gamma_{h1} * Infected_i + u_{1i}$$
$$b_2 = \beta_2 + \gamma_{h2} * Infected + u_{2i}$$
$$b_3 = \beta_3 + \gamma_{h3} * Infected + u_{3i}$$

Whether a cow is infected or not is used as covariate for all three b parameters, Syntax to fit this model for the cows that received the iron supplement is given **in cows_iron_logistic.prl:**

```
OPTIONS METHOD = ML CONVERGE = 0.001000 MAXITER = 30 QUADPTS = 75;
TITLE =  ;
 SY='cows_iron.LSF';
ID2 = case;
RESPONSE = log_wgt;
FIXED = time;
MODEL = logistic;
COVARIATES b1 = Infected
           b2 = Infected
           b3 = Infected;
```

The maximum likelihood solution at convergence is given below, with $s = 1$:

Coefficients	Beta	Std.Err.	Z-value	P > |z|
b1	5.88032	0.06140	95.76836	0.00000
b2	-0.91988	0.01222	-75.28364	0.00000
b3	0.47529	0.02427	19.58234	0.00000

Covariate Names	GAMMAH	Std.Err.	Z-value	P > |z|
Infected	0.22711	0.08811	2.57745	0.00995
Infected	-0.00050	0.02128	-0.02340	0.98133
Infected	-0.07267	0.03629	-2.00234	0.04525

Variance estimate	Level 1	Std.Err.	Z-value	P > |z|
Sigma**2	0.00193	0.00014	13.69150	0.00000

While all three β coefficients are significant, only the first γ coefficient is. We can obtain the expected values of log_wgt using the equations and the statistically significant estimates by calculating

$$E\left(\log\_ wgt_{it}\right) = \hat{b}_1 / \left[1 + Exp\left(\hat{b}_2 - \hat{b}_3 * Time_{it}\right)\right]$$

where

$$\hat{b}_1 = 5.88032 + 0.22711 * Infected$$

$$\hat{b}_2 = -0.91988$$

$$\hat{b}_3 = 0.47529$$

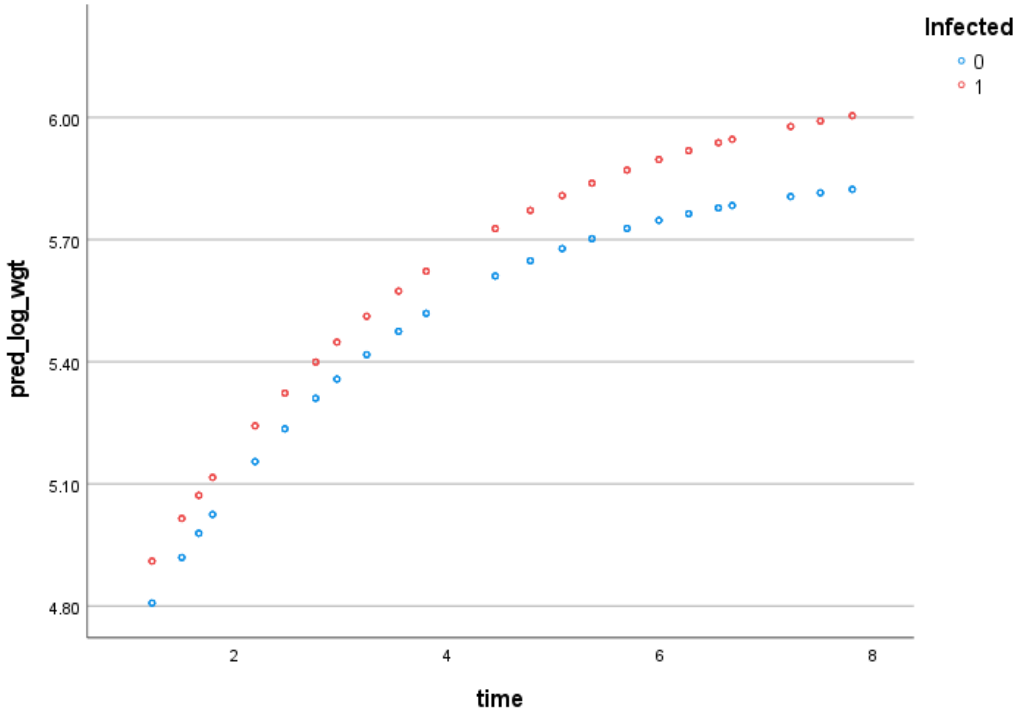
In the case of the cows who did not receive the iron supplement, the results obtained using the syntax file **cows_niron_logistic.prl** are as shown below.

Coefficients	Beta	Std.Err.	Z-value	P > |z|
b1	5.95419	0.04678	127.27455	0.00000
b2	-1.04021	0.01919	-54.21836	0.00000
b3	0.34438	0.02351	14.64948	0.00000

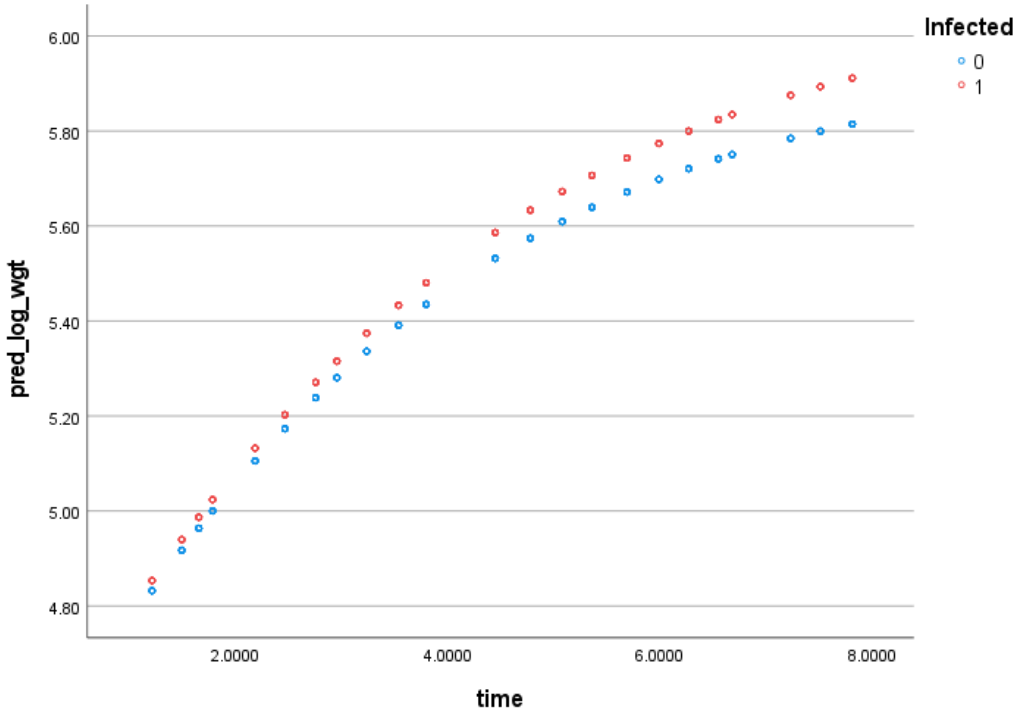
Covariate Names	GAMMAH	Std.Err.	Z-value	P > |z|
Infected	0.14176	0.06784	2.08969	0.03665
Infected	0.06697	0.02687	2.49182	0.01271
Infected	-0.02500	0.03353	-0.74581	0.45578

Variance estimate	Level 1	Std.Err.	Z-value	P > |z|
Sigma**2	0.00242	0.00012	20.75275	0.00000

In the graph of predicted log of the weight against time for cows who received the iron supplement the average predicted log weight is lower for the infected cows.



For cows that did not receive the iron supplement, the predicted average curve resembles that of the cows that received the iron supplement. However, there is less of a difference in the predicted log_wgt of infected and not infected cows.



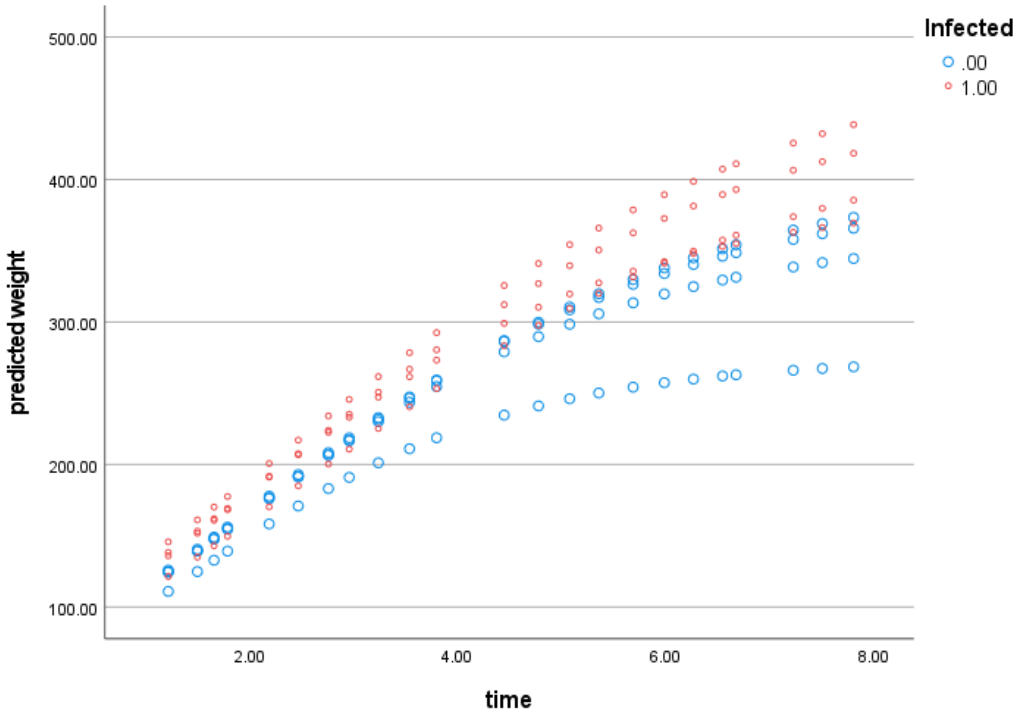
3. Individual curves

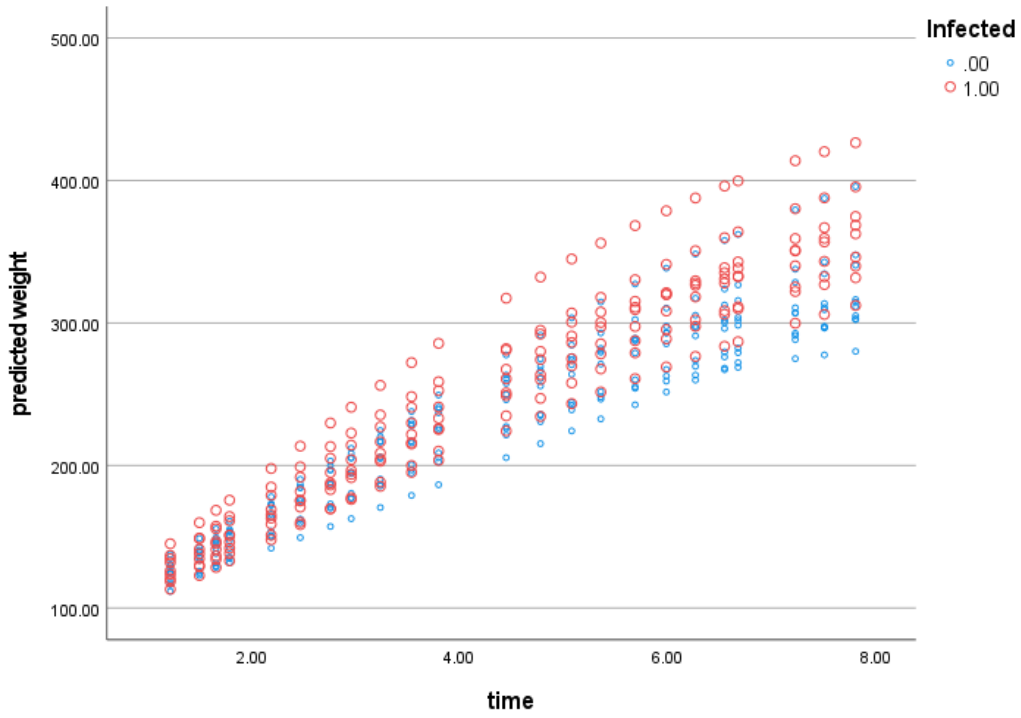
Next, we take a look at the individual curves for each cow. As part of the output, information on the estimated parameters for each level-2 unit (in this case each cow) are written to a text file named **thetai.est**.

The contents of this file for the cows that did not receive an iron supplement is shown below.

```
6.20744        0.334153        0.336121
6.12561        0.352455        0.251316
6.07614        0.310459        0.221710
5.91870        0.317795        0.424305
5.95873        0.331581        0.328260
6.23072        0.328895        0.267958
6.33570        0.334054        0.200759
6.06723        0.334073        0.315315
6.19928        0.348348        0.262870
5.70821        0.318236        0.412781
6.25340        0.328217        0.255344
5.98797        0.285586        0.230432
5.79172        0.324748        0.468958
5.82940        0.305504        0.411729
5.88616        0.308612        0.297895
5.86179        0.303065        0.354682
6.04331        0.304545        0.273781
6.34174        0.345223        0.153723
5.99964        0.309417        0.323113
```

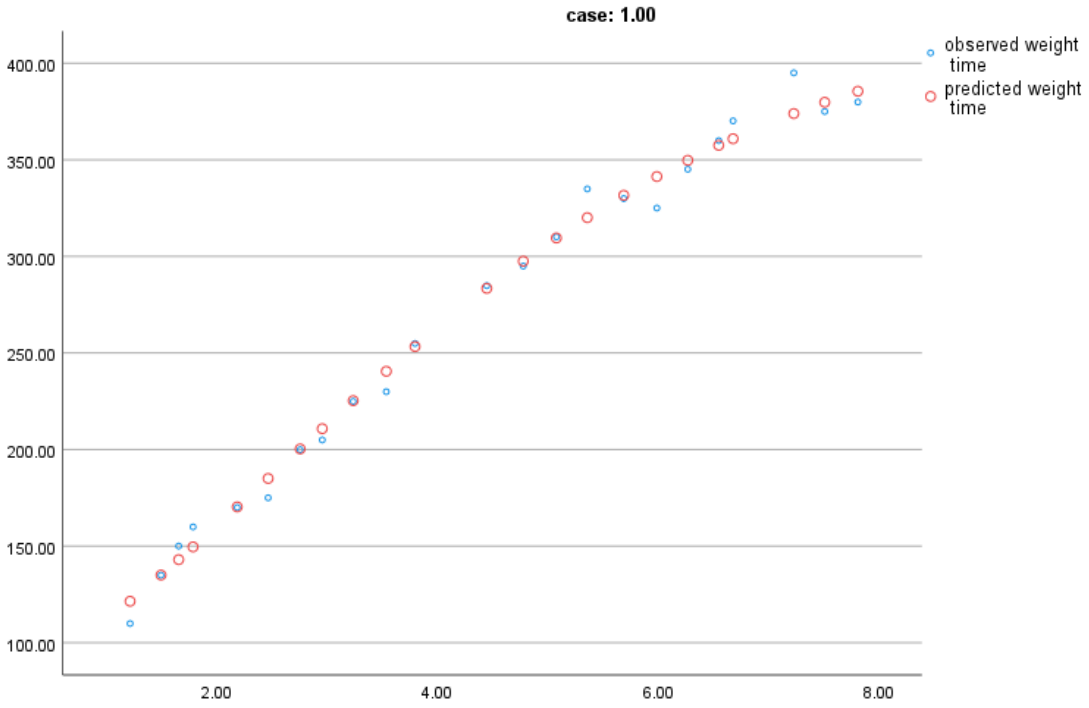
These estimates can be used to get individual curves when used in combination with the observed data. The predicted log_wgt for the individual cows are given below for both groups, first for the cows that received the supplement and then for those that did not.

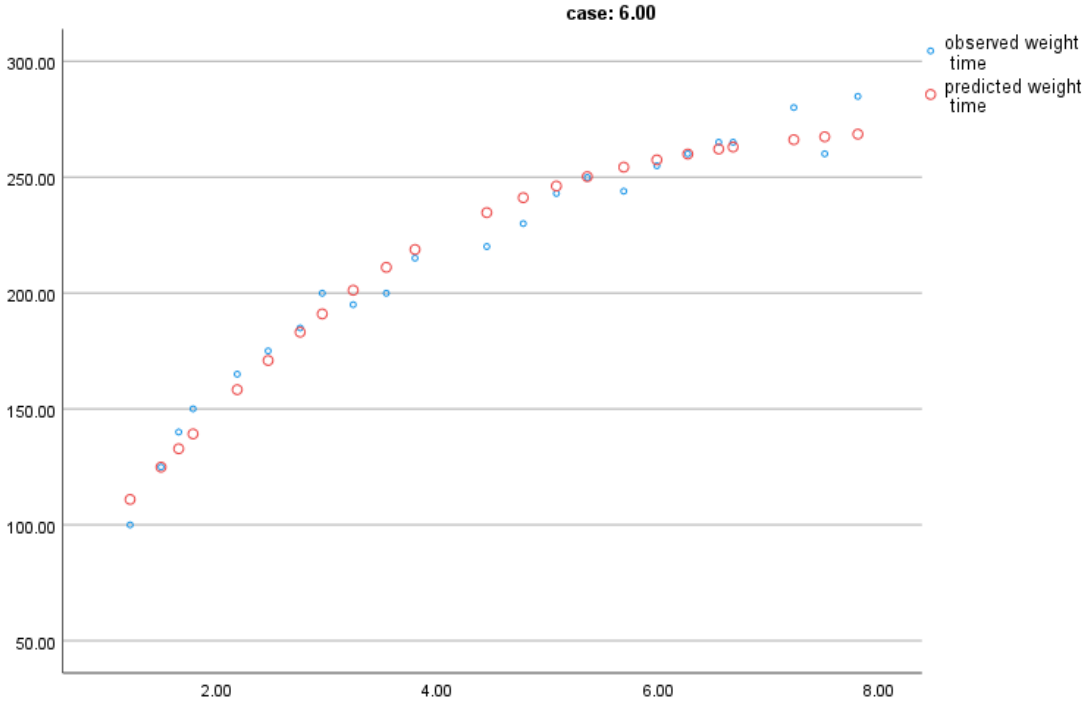




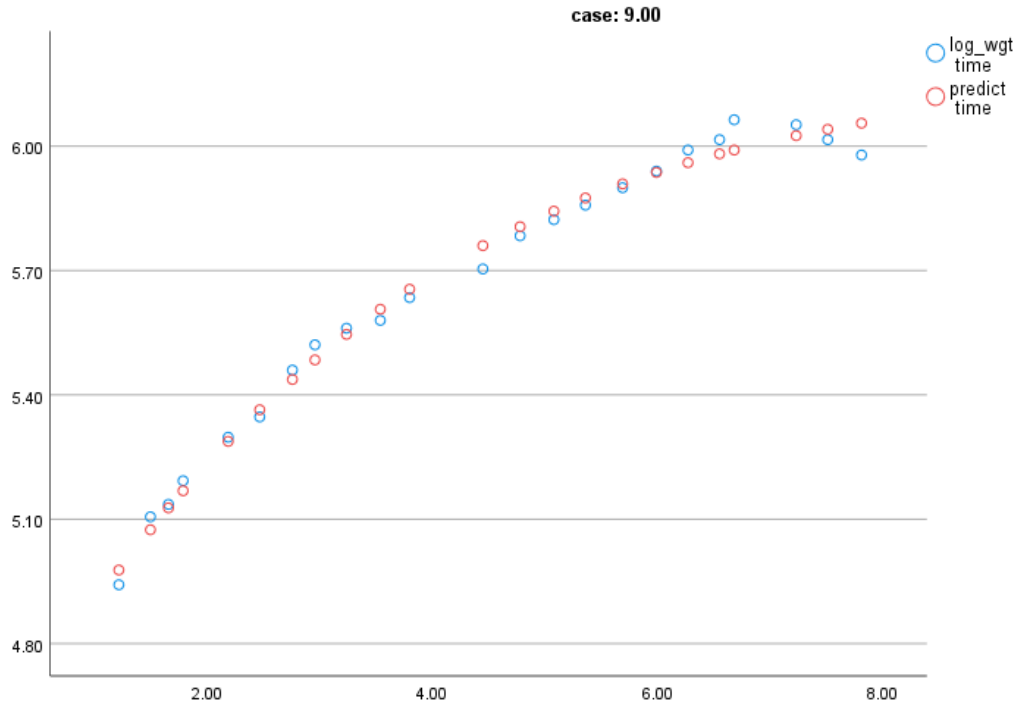
When the combined graph for cows who did not receive the iron supplement is compared to the similar graph for cows who received iron, we see that the difference in predicted weight between infected and non-infected cows who received the supplement is more pronounced than is the case for cows with no iron supplement.

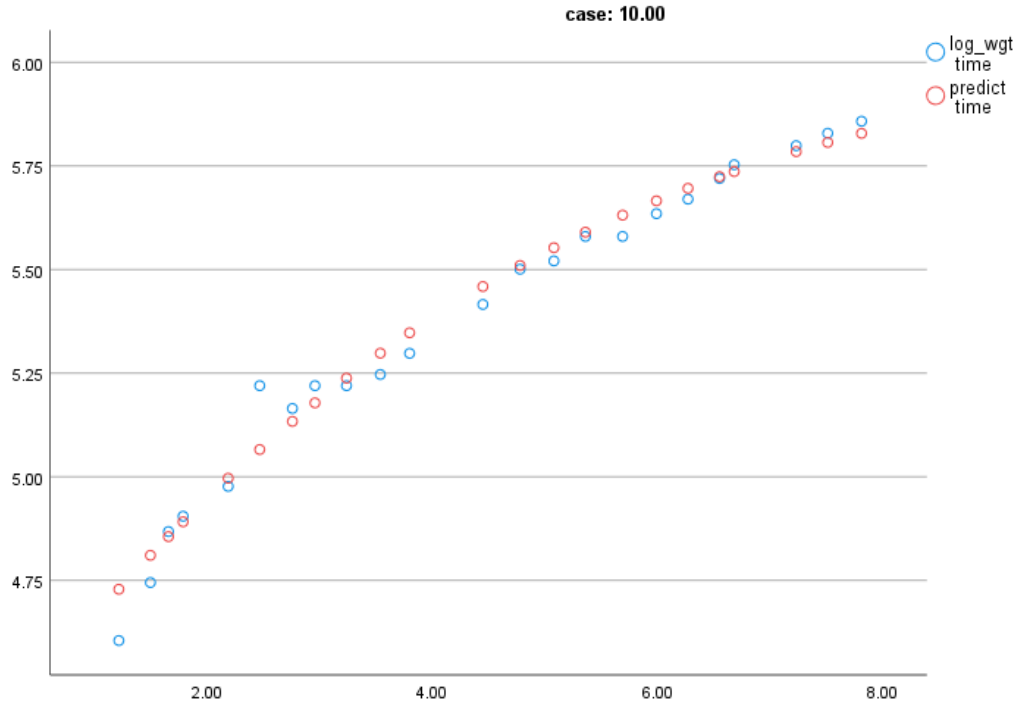
Here we show a few individual curves for individual cows. The first two graphs are for cows that received an iron supplement: the first was reported as infected, the second was not infected.


case: 1.00
400.00
350.00
300.00
250.00
200.00
150.00
100.00
2.00
4.00
6.00
8.00
observed weight time
predicted weight time

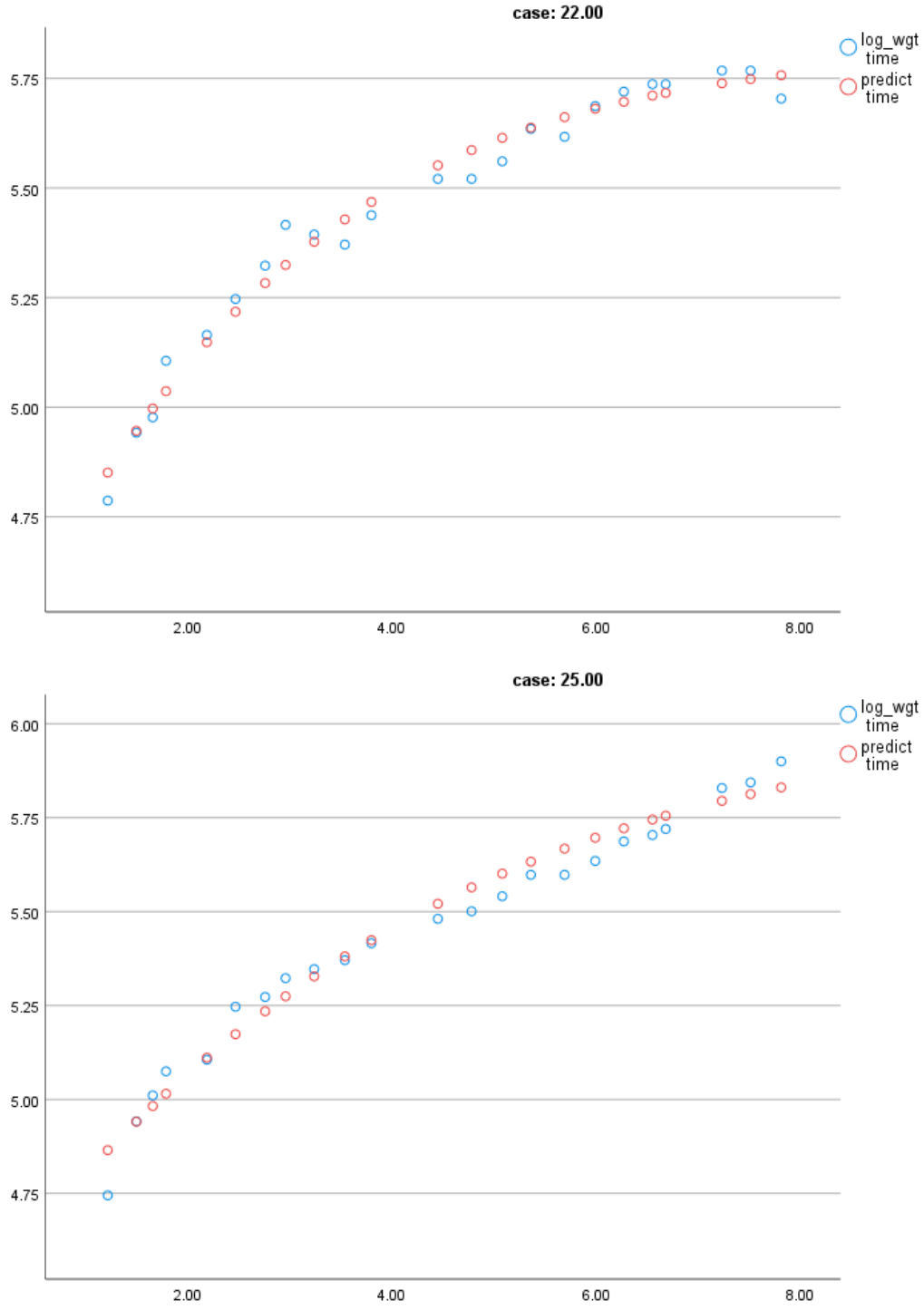

case: 6.00
300.00
250.00
200.00
150.00
100.00
50.00
2.00
4.00
6.00
8.00
observed weight time
predicted weight time

Turning to the cows that did not receive the iron supplement, we look at the graphs for 4 cows: 2 infected, 2 not infected. The first two graphs are for cows 9 and 10 (infected)



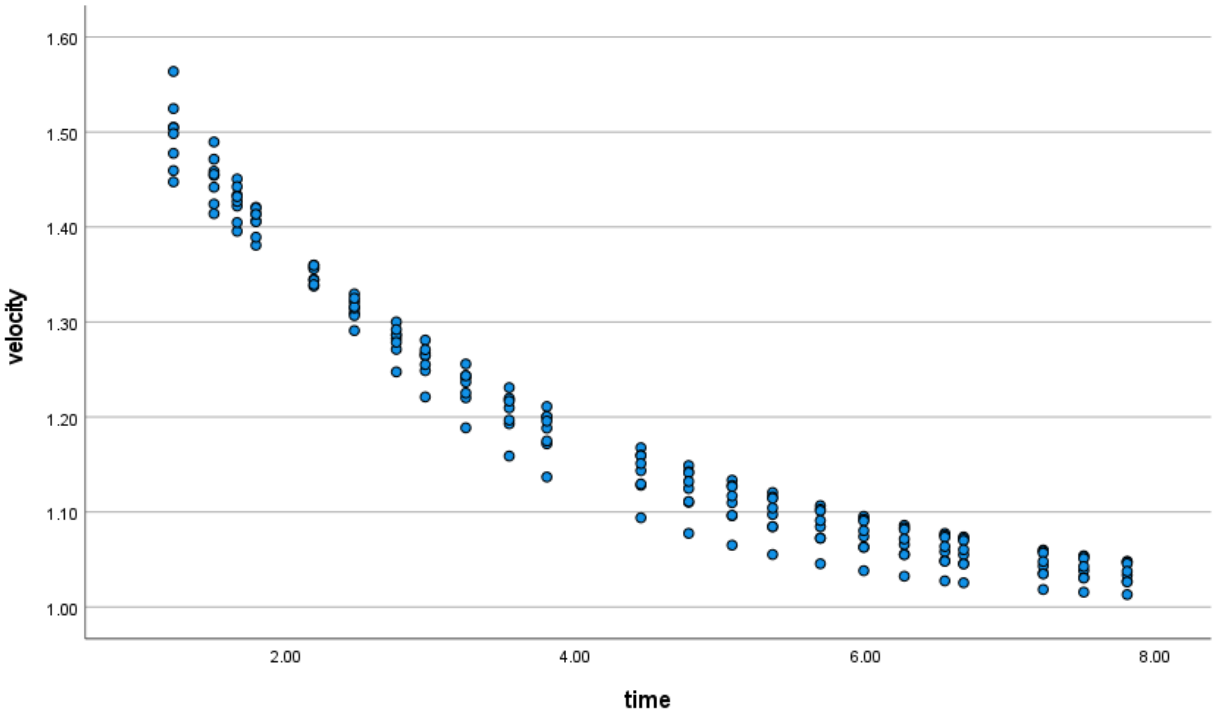


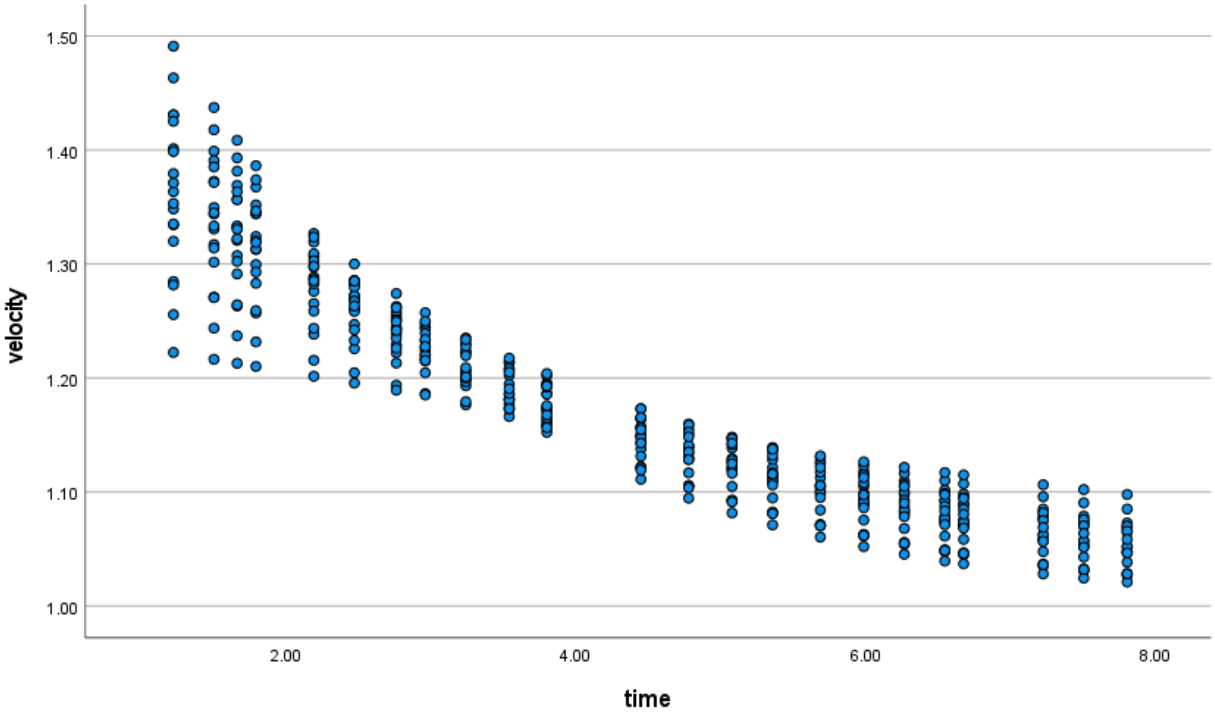
while the next two were not (cases 22 and 25)



In general, we find close agreement between the observed and predicted weight over time regardless of infection.

Velocity curves are obtained by calculating the first-order derivatives of the predicted values with respect to time. The velocity curves for the two groups are given below. Regardless of which of the four groups formed by the experimental design that an individual cow belonged to, the velocity, in other words the rate of change in weight gain, consistently decreased as the cow approached its final weight.





4. Comparing the groups

When considering the entire group of cows, the following questions are of interest:

1. What is the difference in the average expected weight
2. Gain of cows at the end of the time period?
3. What is the impact of the iron supplement for infected and non-infected cows?

To answer these questions, we calculate the average expected weight gain for the cows using the estimates given in the files **thetai.est** as obtained under the logistic models for the two groups. Results for the 4 subgroups are given below.

Iron	Infected	Group	Observed average starting weight	predicted average starting weight	observed average weight gain	predicted average weight gain
yes	**yes (Infected = 0)**	1	112.4860	121.5323	246.2350	216.5963
	no (Infected = 1)	2	123.7574	135.4021	302.4275	267.6024
no	**yes (Infected = 0)**	3	112.4743	124.6543	219.5419	197.1039
	no (Infected = 1)	4	117.7517	127.8647	246.1362	234.2021

When we look at the observed and predicted average weight gain, we note the largest difference of approximately 83 kg in observed average weight gain, and 70 kg in predicted average weight gain, were between groups 2 and 3.

Cows that did not receive the iron supplement and were infected (group 3) had the lowest observed and predicted average weight gain, even though the observed average starting weight for this group was the same as for infected cows that received the supplement (group 1). When we compare the observed average weight gain for groups 1 and 3, a difference of approximately 27 kg was observed. For the model fitted, the predicted difference is 19 kg.

As a matter of fact, the average observed weight gain for the non-infected cows without supplement correspond to the observed average weight gain of infected cows who received the iron supplement. This leads us to conclude that the iron supplement may be useful in counteracting the effect on infection to some extent. The group that clearly gained the most were the non-infected cows that received the iron supplement, as these cows had the highest average values for both observed and predicted starting weight and weight gain. This seems to indicate that the most profitable herd from the farmer's perspective would be one that is not infected and receives the iron supplement.

Turning to the predicted average weight gain, we note that the predicted average starting weight and weight gain under the model tend to overestimate the starting weight on average, while underestimating the average weight gain. That said, we have to keep in mind that we are working with a small sample, having but 8 cows that received the iron supplement.