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The general LISREL model

The LISREL model, in its most general form, consists of two parts: the measurement model and the structural equation model.

The **measurement model** specifies how latent variables or hypothetical constructs depend upon or are indicated by the observed variables. It describes the measurement properties (reliabilities and validities) of the observed variables.

The **structural equation model** specifies the causal relationships among the latent variables, describes the causal effects, and assigns the explained and unexplained variance.

The full LISREL model for single samples is defined, for deviation about the mean, by the following three equations:

The structural equation model:

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\varsigma} \qquad (1)$$

The measurement model for **y**:

$$\mathbf{y} = \boldsymbol{\Lambda}_y\boldsymbol{\eta} + \boldsymbol{\varepsilon} \qquad (2)$$

The measurement model for **x**:

$$\mathbf{x} = \boldsymbol{\Lambda}_x\boldsymbol{\xi} + \boldsymbol{\delta} \qquad (3)$$

The terms in these models are defined as follows:

- $\mathbf{y}$ is a $p \times 1$ vector of observed response or outcome variables.
- $\mathbf{x}$ is a $q \times 1$ vector of predictors, covariates, or input variables.
- $\boldsymbol{\eta}$ is an $m \times 1$ random vector of latent dependent, or endogenous, variables
- $\boldsymbol{\xi}$ is an $n \times 1$ random vector of latent independent, or exogenous, variables
- $\boldsymbol{\varepsilon}$ is a $p \times 1$ vector of measurement errors in **y.**

$\boldsymbol{\delta}$ is a $q \times 1$ vector of measurement errors in **x**.

$\boldsymbol{\Lambda}_y$ is a $p \times m$ matrix of coefficients of the regression of **y** on $\boldsymbol{\eta}$.

$\boldsymbol{\Lambda}_x$ is a $q \times n$ matrix of coefficients of the regression of **x** on $\boldsymbol{\xi}$.

$\boldsymbol{\Gamma}$ is an $m \times n$ matrix of coefficients of the ξ-variables in the structural relationship.

B is an $m \times m$ matrix of coefficients of the η-variables in the structural relationship. **B** has zeros in the diagonal, and **I** – **B** is required to be non-singular.

$\boldsymbol{\varsigma}$ is a $m \times 1$ vector of equation errors (random disturbances) in the structural relationship between $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$.

Assumptions

The random components in the LISREL model are assumed to satisfy the following minimal assumptions:

$\boldsymbol{\varepsilon}$ is uncorrelated with $\boldsymbol{\eta}$

$\boldsymbol{\delta}$ is uncorrelated with $\boldsymbol{\xi}$

$\boldsymbol{\varsigma}$ is uncorrelated with $\boldsymbol{\xi}$

$\boldsymbol{\varsigma}$, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\delta}$ are mutually uncorrelated.

Covariance matrices:

$$Cov(\boldsymbol{\xi}) = \boldsymbol{\Phi}(n \times n) \qquad Cov(\boldsymbol{\varsigma}) = \boldsymbol{\Psi}(m \times m)$$

$$Cov(\boldsymbol{\varepsilon}) = \boldsymbol{\Theta}_{\varepsilon}(p \times p) \qquad Cov(\boldsymbol{\delta}) = \boldsymbol{\Theta}_{\delta}(q \times q)$$

The covariance matrix of the observations as implied by the LISREL model

The assumptions in the previous section imply the following form for the covariance matrix of the observed variables:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Lambda}_y \mathbf{A}\left(\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Gamma}' + \boldsymbol{\Psi}\right)\mathbf{A}'\boldsymbol{\Lambda}_y' + \boldsymbol{\Theta}_{\varepsilon} & \boldsymbol{\Lambda}_y \mathbf{A}\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Lambda}_x' \\ \boldsymbol{\Lambda}_x \boldsymbol{\Phi}\boldsymbol{\Gamma}'\mathbf{A}'\boldsymbol{\Lambda}_y' & \boldsymbol{\Lambda}_x \boldsymbol{\Phi}\boldsymbol{\Lambda}_x' + \boldsymbol{\Theta}_{\delta} \end{bmatrix} \tag{4}$$

where $\mathbf{A} = (\mathbf{I} - \mathbf{B})^{-1}$.

Fixed, free, and constrained parameters

The general LISREL model is specialized by fixing and constraining the parameters that comprise the elements in $\mathbf{\Lambda}_y$, $\mathbf{\Lambda}_x$, $\mathbf{B}$, $\mathbf{\Gamma}$, $\mathbf{\Phi}$, $\mathbf{\Psi}$, $\mathbf{\Theta}_\varepsilon$ and $\mathbf{\Theta}_\delta$. The elements are of three kinds:

Fixed parameters – assigned specified values

Constrained parameters – unknown but equal to or functions of one or more other unknown parameters

Free parameters – unknown and not constrained to be equal to other parameters

LISREL notation for numbers of variables

Variables	Number	Notation
y	p	NY
x	q	NX
η	m	NE
ξ	n	NK