Estimating a correlation structure with WLS

Psychologist A consults Statistician K about her data analysis problem. She does not want to reveal her raw data but says that she has used PRELIS to compute the product-moment (Pearson) correlations for her five variables and that the PRELIS output suggests that the variables are non-normal. Fortunately, she has also computed the asymptotic covariance matrix of the product-moment correlations.

Psychologist A claims that her theory is that the correlations should be of the form

$$\mathbf{P}(\boldsymbol{\theta}) = \begin{bmatrix} 1 & & & & \\ \rho_1 & 1 & & & \\ \rho_2 & \rho_2 & 1 & & \\ \rho_2 & \rho_2 & \rho_3 & 1 & \\ \rho_2 & \rho_2 & \rho_3 & \rho_3 & 1 \end{bmatrix},$$

where $\boldsymbol{\theta}' = (\rho_1, \rho_2, \rho_3)$.

The observed correlation matrix is

$$\mathbf{R} = \begin{bmatrix} 1 & & & & \\ 0.526 & 1 & & & \\ 0.402 & 0.482 & 1 & & \\ 0.391 & 0.424 & 0.400 & 1 & \\ 0.417 & 0.489 & 0.274 & 0.442 & 1 \end{bmatrix}$$

The asymptotic covariance matrix of product-moment correlations is not of the form

$$ACov(s_{gh}, s_{ij}) = (1/N)(\sigma_{gi}\sigma_{hj} + \sigma_{gj}\sigma_{hi})$$

Hence, ML or GLS should not be used in this case. The only way to get correct χ^2, standard errors, standardized residuals, and modification indices is to use WLS with a weight matrix to the inverse of the asymptotic covariance matrix of the estimated correlations.

We assume that the correlation matrix is stored in the file **EX75.KML** and that the asymptotic covariance matrix is stored in the file **EX75.ACK**.

The LISREL command file (**EX75.LIS**) is:

```
ESTIMATION AND TESTING OF A CORRELATION STRUCTURE
DA NI=5 NO=200 MA=KM
KM FI=EX75.KML
AC FI=EX75.ACK
MO NX=5 NK=5 LX=ID PH=ST TD=ZE
EQ PH 3 1 PH 3 2 PH 4 1 PH 4 2 PH 5 1 PH 5 2
EQ PH 4 3 PH 5 3 PH 5 4
OU SE TV RS MI ME=WLS
```

The MO command specifies $\mathbf{\Lambda}_x = \mathbf{I}$ and $\mathbf{\Theta}_\delta = \mathbf{0}$. By default, $\mathbf{\Phi}$ is unconstrained. The VA command specifies $\mathbf{\Phi}$ to be a correlation matrix. The EQ command s specify the equality constraints in the correlation structure.

The output file gives the following solution, standardized residuals, and modification indices.

```
         PHI

                VAR 1      VAR 2      VAR 3      VAR 4      VAR 5
             --------   --------   --------   --------   --------
    VAR 1       1.000

    VAR 2       0.603      1.000
              (0.057)
               10.570

    VAR 3       0.513      0.513      1.000
              (0.039)    (0.039)
               13.020     13.020

    VAR 4       0.513      0.513      0.429      1.000
              (0.039)    (0.039)    (0.048)
               13.020     13.020      9.023

    VAR 5       0.513      0.513      0.429      0.429      1.000
              (0.039)    (0.039)    (0.048)    (0.048)
               13.020     13.020      9.023      9.023
```

```
 Goodness-of-Fit Statistics

 Degrees of Freedom                                          12
 Weighted Least Squares Chi-Square (C1)                      16.312 (P = 0.1774)

             Standardized Residuals

              VAR 1      VAR 2      VAR 3      VAR 4      VAR 5
           --------   --------   --------   --------   --------
    VAR 1      - -
    VAR 2    -2.860       - -
    VAR 3    -2.287     -0.711       - -
    VAR 4    -1.512     -1.239     -0.537       - -
    VAR 5    -1.943     -0.800     -3.209      0.411       - -

Modification Indices and Expected Change

         Modification Indices for PHI

              VAR 1      VAR 2      VAR 3      VAR 4      VAR 5
           --------   --------   --------   --------   --------
    VAR 1      - -
    VAR 2      - -        - -
    VAR 3     2.816      4.759       - -
    VAR 4     1.033      1.302      1.838       - -
    VAR 5     0.577      3.236     11.915      4.666       - -

         Expected Change for PHI

              VAR 1      VAR 2      VAR 3      VAR 4      VAR 5
           --------   --------   --------   --------   --------
    VAR 1      - -
    VAR 2      - -        - -
    VAR 3    -0.072      0.060       - -
    VAR 4    -0.066     -0.069      0.052       - -
    VAR 5    -0.035      0.041     -0.161      0.055       - -

 Maximum Modification Index is   11.91 for Element ( 5, 3) of PHI
```

The program gives correct results but *incorrect* degrees of freedom. The correct degrees of freedom is 7, the number of correlations (10) minus the number of parameters (3). By putting TD = FR instead of TD = ZE on the MO command, one will obtain identically the same results but with correct degrees of freedom. Incorrect degrees of freedom when TD = ZE are obtained because the program always assumes that a covariance structure is estimates, so that the diagonal elements of **S** are always counted in the degrees of freedom. The reason why TD = FR and TD = ZE give the same results, apart from the degrees of freedom, is that in both cases the same fit function of the parameters is minimized.

Both the standardized residuals and the modification indices show that $\hat{\rho}_{53}$ violates the equality constraint.

The negative sign of the standardized residual and of the estimated change of ρ_{53} suggests that ρ_{53} should be smaller than the estimated value 0.43. In fact, the estimated change in -0.16.

When the model is reestimated with ρ_{53} free, the goodness-of-fit χ^2 is 4.397 with six degrees of freedom. This represents a very good fit. The estimate of ρ_{53} is 0.268, which is in fact just 0.16 smaller than the previous value.

The analysis suggests that A's "theory" is in agreement with the data, except that ρ_{53} is not equal to ρ_{43} and ρ_{54}.