Test of hypotheses and power calculation

Once the validity of a model has been reasonably well established, various structural hypotheses about the parameters $\boldsymbol{\theta}(t \times 1)$ in this model may be tested. One can test hypotheses stating that:

- Certain θ's have particular values.
- Certain θ's are equal.

Each of these two types of hypotheses leads to a model with fewer parameters $\boldsymbol{\upsilon}$, where $\boldsymbol{\upsilon}(u \times 1)$ is a subset of the parameters in $\boldsymbol{\theta}$, $u < t$. In conventional statistical terminology, the model with parameters $\boldsymbol{\upsilon}$ is called the *null hypothesis* H_0 and the model with parameters $\boldsymbol{\theta}$ is called the *alternative hypothesis* H_1. Let χ_0^2 and χ_1^2 be the χ^2 goodness-of-fit measures for models H_0 and H_1, respectively. The test statistic for testing H_0 against H_1 is then

$$D^2 = \chi_0^2 - \chi_1^2,$$

which is used as χ^2 with $d = t - u$ degrees of freedom. The degrees of freedom can also be computed as the difference between the degrees of freedom associated with χ_0^2 and χ_1^2. To use the test statistic formally, one chooses a significance level α (probability of type 1 error) and rejects H_0 if D^2 exceeds the $(1-\alpha)$ percentile of the χ^2 distribution with d degrees of freedom.

The test statistic D^2 can only be used with GLS, ML, and WLS and is valid under the following assumptions:

- With GLS and ML: the observed variables have a multivariate normal distribution and the sample covariance matrix is analyzed with WLS if the correct weight matrix is used.
- The model H_0 is true.
- The sample is large.

A common type of hypothesis H_0 postulates that a single parameter θ be restricted to a specific value θ_0. The corresponding alternative hypothesis H_1 specifies θ as a free parameter in the same model. In this case, there are two alternative test statistics that can be used, both of which are easier to compute in that they require only one LISREL run rather than two as D^2 does. These alternatives are:

- Run LISREL under H_1 and use the t-value of $\hat{\theta}$ as a standard normal two-sided test statistic.
- Run LISREL under H_0 and use the modification index for $\theta = \theta_0$ as a χ^2 with one degree of freedom.

In very large samples, if H_0 is true, we have approximately:

$$t^2 = D^2 = \text{modification index.}$$

These three quantities will not be equal in small samples, however, and it can happen, although rarely, that only one or two are significant. In the general context of $D^2 = \chi_0^2 - \chi_1^2$, if H_0 cannot be rejected at a given level of significance α, it does not necessarily mean that H_0 is true, because the power of the test may be low. The *power* of a test is the probability of rejecting H_0 when H_1 is true (one minus the probability of type 2 error). The higher the power, the better the test.

The power of the D^2 test depends on:

- what the true model is
- the significance level α (power increases with α)
- the degrees of freedom (power decreases with degrees of freedom)
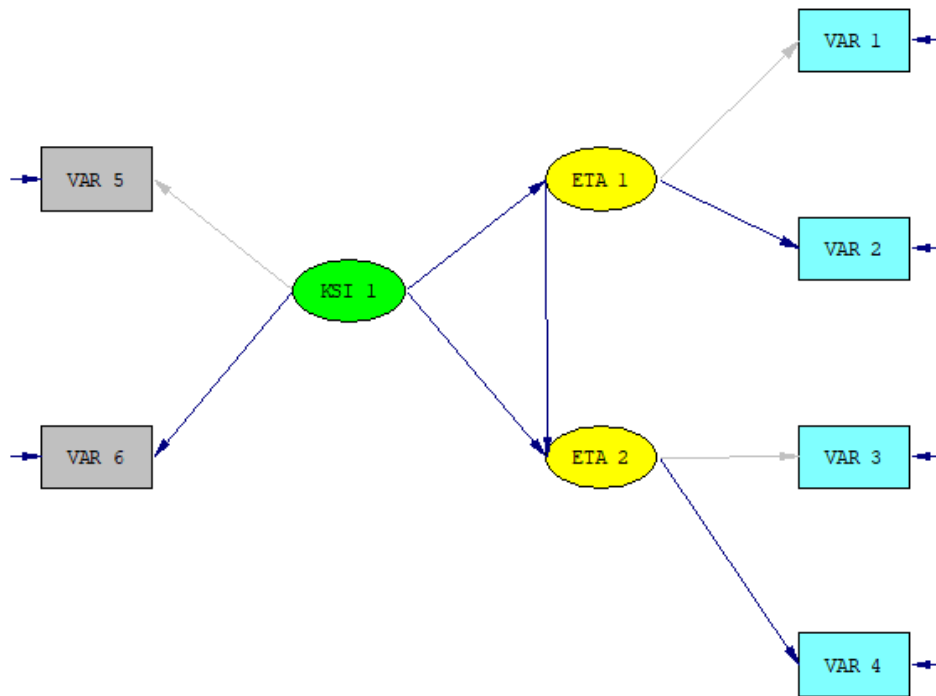- the sample size (power increases with sample size).

The procedure for computing the power of the D^2 test is as follows:

1. Specify the model completely under H_1. The power can only be computed for a completely specified model, i.e., all parameters must be numerically specified and satisfy H_1.
2. Compute the covariance matrix Σ for the model specified in Step 1. Σ can be computed using LISREL. To do this, use any input covariance matrix and set IT = 1 and SI = *filename* on the OU command. Σ will then be saved in the file specified by *filename*.
3. Run LISREL under H_0 using the Σ computed in step 2 as input covariance matrix. The χ^2 measure obtained in this step is the noncentrality parameter λ.
4. Using tables of the noncentral χ^2 distribution with noncentrality parameter λ and degrees of freedom $d = t - u$, determine the probability of obtaining a value larger than the $(1-\alpha)$ percentile of the central χ^2 distribution with d degrees of freedom.

To illustrate tests of hypotheses and power calculation, we use an artificial example. Suppose we have a random sample of 100 cases observed on x_1, x_2, y_1, y_2, y_3 and y_4 with the following sample covariance matrix:

$$S = \begin{bmatrix} 1.531 \\ 0.579 & 0.991 \\ 0.644 & 0.502 & 1.449 \\ 0.262 & 0.276 & 0.445 & 0.718 \\ 0.418 & 0.286 & 0.492 & 0.249 & 1.111 \\ 0.464 & 0.279 & 0.523 & 0.276 & 0.657 & 1.144 \end{bmatrix}.$$

We are interested in the model shown below.

In particular, we are interested in whether $\gamma_{21} = 0$, i.e., whether the effect of ξ on η_2 is only indirect via η_1.

To test the hypothesis $H_0 : \gamma_{21} = 0$, we first run LISREL under H_1 using the following command file (**EX82A.LIS**).

```
EXAMPLE 8.2 RUN 1
Using sample covariance matrix to estimate Model H1
DA NI=6 NO=100
CM FI=EX82.COV
MO NY=4 NX=2 NE=2 NK=1 BE=SD
FR LY(2,1) LY(4,2) LX(2)
VA 1 LY(1,1) LY(3,2) LX(1)
OU SE TV
```

This run gives

```
                    Goodness-of-Fit Statistics

Degrees of Freedom for (C1)-(C2)                      6
Maximum Likelihood Ratio Chi-Square (C1)              1.987 (P = 0.9209)
```

indicating that model H_1 fits well. The estimate of γ_{21} is 0.391 with a t-value of 1.80. Using this as a test statistic, H_0 cannot be rejected at the 5 percent level since the t-value does not exceed 1.96, the 97.5 percentile of the standard normal distribution. Next, we run LISREL under H_0 to obtain the other two test statistics. Except for the title lines we can use the same command file and just add the command FI GA(2,1) and add MI to the OU command (**EX82B.LIS**).

This run gives $\chi^2 = 4.52$ with 7 degrees of freedom; the modification index for γ_{21} is 2.695. The D^2 statistic is computed as $D^2 = 4.52 - 1.97 = 2.55$. Thus both D^2 and the modification index are non-significant at the 5 percent level since they do not exceed 3.84, the 95th percentile of the χ^2 distribution with one degree of freedom.

To calculate the power of the D^2 test we follow the steps outlined previously:

Step 1:

We are interested in testing $H_0 : \gamma_{21} = 0$ against the alternative $H_1 : \gamma_{21} = 0.3$, say. However, all the parameters of the H_1 model must be specified numerically. We choose these such that the parameter matrices in LISREL are:

$$\Lambda_y = \begin{bmatrix} 1 & 0 \\ 0.7 & 0 \\ 0 & 1 \\ 0 & .6 \end{bmatrix} \quad \Theta_\varepsilon = diag(.5,.5,.5,.5)$$

$$\Lambda_x = \begin{bmatrix} 1 \\ .8 \end{bmatrix} \quad \Theta_\delta = diag(.5,.5)$$

Step 2:

To illustrate how Σ can be computed, we use a command file with an identity matrix as input covariance matrix. The sample covariance matrix or any other positive definite matrix may also be used.

```
EXAMPLE 8.2 RUN 3
Computing Sigma
DA NI=6 NO=100
CM
1 0 1 2*0 1 3*0 1 4*0 1 5*0 1
MO NY=4 NX=2 NE=2 NK=1 BE=FU,FI GA=FI PH=FI PS=FI TE=FI TD=FI
MA LY
1 0 .7 0 0 1 0 .6
MA LX
1 .8
MA BE
0 0 .4 0
MA GA
.5 .3
MA PH
1
MA PS
.5 .5
MA TE
4*.5
MA TD
.5 .5
OU SI=EX82.SIG
```
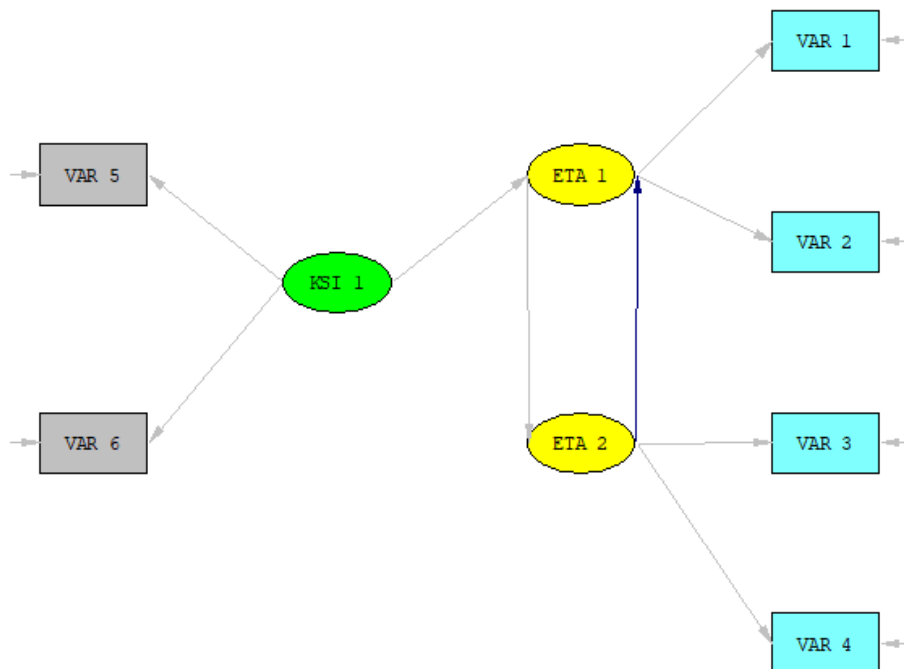
The resulting Σ in file **EX82.SIG** is

```
0.12500D+01  0.52500D+00  0.86750D+00  0.45000D+00  0.31500D+00  0.13300D+01
0.27000D+00  0.18900D+00  0.49800D+00  0.79880D+00  0.50000D+00  0.35000D+00
0.50000D+00  0.30000D+00  0.15000D+01  0.40000D+00  0.28000D+00  0.40000D+00
0.24000D+00  0.80000D+00  0.11400D+01
```

Step 3:

The command file for this step (**EX82D.LIS**) is the same as in run 1 except that the file **EX82.SIG** is used instead of **EX82.COV**, i.e., Σ is analyzed instead of **S**. This results in $\chi^2 = 3.12$, which is the required non-centrality parameter λ. Note that this value is approximately the same as the modification index for γ_{21}.